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Free bending vibration analysis of thin bidirectionally exponentially graded orthotropic rectangular plates resting on two-parameter elastic foundations

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ABSTRACT

The vibration of biderctionally exponentially graded orthotropic plates (BEGOPs) resting on the two-parameter elastic foundation is studied. Pasternak elastic foundation (PEF) model is used as two-parameter foundation model. The heterogeneity of the orthotropic exponentially changes depending on the axial and thickness coordinates. The motion equation is derived based on the classical plate theory and solved by using Galerkin method. To validate of current results was made a comparison with the previous studies. The effects of material gradient and orthotropy, and the two-parameter elastic foundations on the dimensional frequency parameters (DFPs) are investigated.

1. Introduction

The wide use of modern composites in various products of modern technology required not only the development of traditional methods for the analysis of thin-walled plates, but also the formulation of new tasks and revealed the need to take into account the new main factors that determine the bearing capacity of structures. Among these factors, anisotropy and heterogeneity of the material occupy an important place. These factors introduce additional complexity into the study of the vibration and stability problems of composite structures. A great contribution to the theory of anisotropic plates was made the work Reddy [1].

Inhomogeneous structures are often used in technical designs that take full advantage of continuous and gradual changes in the physical and mechanical properties of the material. Such structures are widely used in aviation, aerodynamic structure, space vehicles, light-alloy structure of cars and in other engineering structures. Compared to homogeneous orthotropic plates, the adoption of continuous change of material properties can provide important benefits. Indeed, the increase in the number of constructive variables extends the possibilities of advanced composite materials, as well as stability and vibration behaviors may be significantly altered. The reason for the appearance of heterogeneity of the material can be, manufacturing technology, thermal and mechanical treatment, heterogeneity of compositions and a number of other reasons. As a result of the above reasons, the inhomogeneity can simultaneously depend on the spatial coordinates. The basic knowledge on the changes of the material properties is given in the work of Lomakin [2]. Efforts related to the determination of various types of functionally graded anisotropic materials have been the focus of research in recent years [3–6]. Using above mentioned models, several important problems were solved about the oscillations of the functionally graded orthotropic plates [7–12].

In many practical applications, composite plates are in contact with soils or other solid particles and can have significant and unavoidable effect on their behaviors. To correctly determine the influence of the elastic foundation, there are various models, among which one of the effective model was proposed by Pasternak, which is called a two-parameter elastic foundation [13]. Besides, a comprehensive review of elastic foundation models is discussed in the Ref. [14]. The vibration of homogeneous orthotropic plates resting on the two-parameter elastic foundations, which has practical applications in civil, mechanical, marine and aerospace engineers have been studied using various analytical and numerical methods [15–21].

In recent years, the urgency of solving the stability and vibration problems of functionally graded composite plates has increased dramatically. This is explained, first of all, by the continuous expansion of the introduction of inhomogeneous composite plates into load-bearing elements of structures working in contact with different environments. The numerous studies on the vibration of functionally graded orthotropic plates resting on the Pasternak elastic foundation have been published in the literature [22–29]. In the majority of the above mentioned studies, the change in the elastic properties of FG orthotropic

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materials was carried out as the function of thickness or axial coordinates, separately. The main contribution to this study is made by the development and implementation of the vibration analysis for thin exponentially graded (EG) orthotropic plates which the material properties vary depending on the axial and thickness coordinates together and resting on the Pasternak elastic foundation.

2. Formulation of the problem

The configuration of rectangular biderctionally exponentially graded orthotropic plate (BEGOP) with the length a, the breadth b and the thickness h and resting on the Pasternak elastic foundation (PEF) is illustrated in Fig. 1. The plate referred to a system of rectangular coordinate system Oxyz. The mid-plane being $z = 0$ and the origin is at one corners of the orthotropic plate. The x and y axes are taken along the principle directions of orthotropy and z axis is normal to the them. The reaction of the PEF is related to the deflection, w, with the following relationship [13,14].

$$R = K_w w - K_p\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{1}$$

where $K_w(N/m^3)$ and $K_p(N/m^2)$ are spring and shear moduli of the two-parameter elastic foundation [15–27].

It is assumed that the material properties of the orthotropic plate vary in the axial and thickness directions, as follows:

$$E_1 = E_1^0 f_1(X) f_2(Z), \quad E_2 = E_2^0 f_1(X) f_2(Z), \quad G_{12} = G_{12}^0 f_1(X) f_2(Z), \quad \rho = \rho^0 \psi_1(X)\psi_2(Z) \tag{2}$$

where E_1^0 and E_2^0 are the Young's modulus in the x and y directions, respectively; G_{12}^0 is the shear modulus and ρ^0 is the density of the homogeneous orthotropic plate. Furthermore, $f_1(X)$ and $\psi_1(X)$ are exponential functions characterize the change of the Young's and shear moduli, and density in the x direction, respectively; $f_2(Z)$ and $\psi_2(Z)$ are exponential functions characterize the change of the Young's and shear moduli, and density, respectively, in the z direction and the following definitions apply [3–9]:

$$f_1(X) = \alpha_1^X, f_2(Z) = \alpha_2^{Z+0.5}, \ \psi_1(X) = \beta_1^X, \ \psi_2(Z) = \beta_2^{Z+0.5} \tag{3}$$

in which $X = x/a$ and $Z = z/h$ are the dimensionless variables; $\alpha_1 = \frac{f_1(1)}{f_1(0)}$ is the variation parameter of Young's and shear moduli in the x direction in which $f_1(0)$ and $f_1(1)$ are the values of function, $f_1(X)$, on the $x = 0$ and $x = a$ edges of the plate, respectively. $\beta_1 = \frac{\psi_1(1)}{\psi_1(0)}$ is the variation parameter of the density in the x direction in which $\psi_1(0)$ and $\psi_1(1)$ are the values of the function, $\psi_1(X)$, on the $x = 0$ and $x = a$ edges of the plate, respectively. $\alpha_2 = \frac{f_2(0.5)}{f_2(-0.5)}$ is the variation parameter of Young's and shear moduli in the z direction in which $f_2(-0.5)$ and $f_2(0.5)$ are the values of the function, $f_2(Z)$, on the $Z = -0.5$ and $Z = 0.5$ planes of the plate, respectively. Furthermore, $\beta_2 = \frac{\psi_2(0.5)}{\psi_2(-0.5)}$ is the variation parameter of the density in the z direction in which $\psi_2(-0.5)$ and $\psi_2(0.5)$ are the values of the function, $\psi_2(Z)$, on the $Z = -0.5$ and $Z = 0.5$ planes of the plate, respectively. Poisson's ratios of orthotropic material ν_{12} and ν_{21} are constant and the following inequality is satisfied: $\nu_{21}E_1^0 = \nu_{12}E_2^0$.

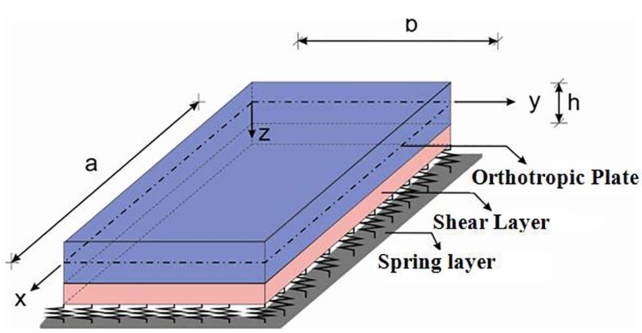

Fig. 1. The rectangular BEGOP on the PEF and the coordinate system.

3. Basic equation

Based on the classical plate theory (CPT), the relationships between the stresses and strains at an arbitrary point of the BEGOPs are written in the following form [2–7]:

$$\sigma_{11} = \frac{E_1^0 \alpha_1^X \alpha_2^{Z+05}}{1-\nu_{12}\nu_{21}}(\varepsilon_{11} + \nu_{12}\varepsilon_{22}), \sigma_{22} = \frac{E_2^0 \alpha_1^X \alpha_2^{Z+05}}{1-\nu_{12}\nu_{21}}(\varepsilon_{22} + \nu_{21}\varepsilon_{11})\sigma_{12} = G_{12}^0 \alpha_1^X \alpha_2^{Z+05}\varepsilon_{12} \tag{4}$$

Let us assume that the Kirchhoff-Love hypotheses are valid for the BEGOPs, and have [1]

$$\varepsilon_{11} = e_{11} - z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_{22} = e_{22} - z\frac{\partial^2 w}{\partial y^2}, \quad \varepsilon_{12} = e_{12} - 2z\frac{\partial^2 w}{\partial x \partial y} \tag{5}$$

where e_{11}, e_{22}, e_{12} are the strains in the mid-plane.

The force and moment resultants are expressed by the following relations [1]:

$$(T_{ij}, M_{ij}) = \int_{-h/2}^{h/2} \sigma_{ij}[1,z]dz, \ (i,j = 1,2) \tag{6}$$

Since there are no external forces in the plane of the plate ($T_{ij} = 0$, $i,j = 1,2$) (it assumed that the plate experiences a pure bending), it is therefore assumed that the resultant forces are everywhere equal to zero. In this case, the following conditions can be written:

$$\upsilon_1(e_{11} + \nu_{12}e_{22}) - \upsilon_2(\chi_{11} + \nu_{12}\chi_{22}) = 0\upsilon_1(e_{22} + \nu_{21}e_{11}) - \upsilon_2(\chi_{22} + \nu_{21}\chi_{11}) = 0\upsilon_1 e_{12} - \upsilon_2\chi_{12} = 0 \tag{7}$$

where χ_{11}, χ_{22} and χ_{12} are the curvatures of the middle plane and the following definitions apply:

$$\upsilon_1 = \frac{h(\alpha_2 - 1)}{ln\ \alpha_2}, \quad \upsilon_2 = \frac{h^2[2(1-\alpha_2) + (\alpha_2 + 1)ln\ \alpha_2]}{2ln^2\ \alpha_2} \tag{8}$$

Taking into account relations (4), (5) and (7) in the expression (6), we obtain the following expressions for the moments:

$$M_{11} = D_1^0 \Lambda \alpha_1^X\left(\frac{\partial^2 w}{\partial x^2} + \nu_{12}\frac{\partial^2 w}{\partial y^2}\right), M_{22} = D_2^0 \Lambda \alpha_1^X\left(\frac{\partial^2 w}{\partial y^2} + \nu_{21}\frac{\partial^2 w}{\partial x^2}\right)M_{12} = 2D_T^0 \Lambda \alpha_1^X \frac{\partial^2 w}{\partial x \partial y} \tag{9}$$

where D_1^0, D_2^0, D_T^0 are flexural rigidities of the homogeneous orthotropic plate (HOP), Λ is the parameter and the following notations apply:

$$D_1^0 = \frac{E_1^0 h^3}{12(1-\nu_{12}\nu_{21})}, D_2^0 = \frac{E_2^0 h^3}{12(1-\nu_{12}\nu_{21})}, D_T^0 = \frac{G_{12}^0 h^3}{12}, \Lambda = 12\frac{\alpha_2\ ln^2\alpha_2 - (\alpha_2 - 1)^2}{(\alpha_2 - 1)ln^3\alpha_2} \tag{10}$$

Taking into account Eqs. (1) and (2), the partial differential equation of the motion for the BEGOPs on the PEF can be written as [6,8]:

$$\frac{\partial^2 M_{11}}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_{22}}{\partial y^2} - K_w w + K_p\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \rho_1 h \beta_1^X \frac{\partial^2 w}{\partial t^2} = 0 \tag{11}$$

where the following definition applies:

$$\rho_1 = \rho^0 \frac{\beta_2 - 1}{ln\ \beta_2} \tag{12}$$

Substituting (9) into Eq. (11), after elementary transformations we obtain:

$$\Lambda\alpha_1^X\left\{\left[D_1^0\frac{\partial^4 w}{\partial x^4}+D_2^0\frac{\partial^4 w}{\partial y^4}+(D_1^0\nu_{21}+\nu_{12}D_2^0+4D_T^0)\frac{\partial^4 w}{\partial x^2\partial y^2}\right]+2D_1^0\frac{ln\ \alpha_1}{a}\left(\frac{\partial^3 w}{\partial x^3}+\nu_{12}\frac{\partial^3 w}{\partial x\partial y^2}\right)+D_1^0\frac{ln^2\ \alpha_1}{a^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu_{12}\frac{\partial^2 w}{\partial y^2}\right)+4D_T^0\frac{ln\ \alpha_1}{a}\frac{\partial^3 w}{\partial x\partial y^2}\right\}-K_w w+K_p\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\rho_1 h\beta_1^X\frac{\partial^2 w}{\partial t^2}=0 \tag{13}$$

The Eq. (13) is the motion equation of BEGOPs resting on the Pasternak-Winkler elastic foundations.

4. The solution of equation of motion

We assume that the boundary conditions for the bending of continuous the BEGOP coincide with the usual ones in the homogeneous isotropic plate.

We take the harmonic solution of Eq. (13) in the form [15,16]

$$w(x,y,t)=A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{i\omega t} \tag{14}$$

which satisfies the movable simply-supported boundary conditions edges of the BEGOPs, here $i=\sqrt{-1}$, m and n positive integers and A is the unknown amplitude.

Substituting (14) into Eq. (13) and applying Galerkin method, after integrating we obtain expression for the frequency (in rad/s) of BEGOPs resting on the PEF.

$$\omega_{wp}=\sqrt{\frac{\Pi_1\Pi_2}{\rho_1 h\Pi_{1\rho}\Pi_{2\rho}}[\eta_1^4 D_1^0+\eta_2^4 D_2^0+\eta_1^2\eta_2^2(\nu_{21}D_1^0+\nu_{12}D_2^0+4D_T^0)]+\frac{K_w+K_p(\eta_1^2+\eta_2^2)}{\rho_1 h\Pi_{1\rho}\Pi_{2\rho}}} \tag{15}$$

where the following definitions apply:

$$\Pi_1=\frac{1-\alpha_1}{[0.25\pi^{-2}ln^2\alpha_1+1]ln\alpha_1},\Pi_{1\rho}=\frac{1-\beta_1}{[0.25\pi^{-2}ln^2\beta_1+1]ln\beta_1},\Pi_2=12\frac{\alpha_2 ln^2\alpha_2-(\alpha_2-1)^2}{(\alpha_2-1)ln^3\alpha_2},\Pi_{2\rho}=12\frac{\beta_2\ ln^2\beta_2-(\beta_2-1)^2}{(\beta_2-1)ln^3\beta_2}. \tag{16}$$

where $\eta_1=m\pi/a$ and $\eta_2=m\pi/b$.

The expression for the dimensionless frequency parameter (DFP) for BEGOPs resting on the PEF is defined as:

$$\omega_{1wp}=\omega_{wp}\frac{a^2}{h}\sqrt{\rho^0/E_1^0} \tag{17}$$

As $\Pi_1=\Pi_{1\rho}=1$ and $K_w=K_p=0$ from expression (15) is obtained the frequency for the unconstrained plates made of BEGOPs in which elastic properties vary only in the thickness direction, z.

As $\Pi_1=\Pi_2=\Pi_{1\rho}=\Pi_{2\rho}=1$ and $K_w=K_p=0$, the frequency coincides with the frequency of the HOP without the elastic foundation and expressed as:

$$\omega_0=\sqrt{\frac{\eta_1^4 D_1^0+\eta_2^4 D_2^0+\eta_1^2\eta_2^2(D_1^0\nu_{21}+D_2^0\nu_{12}+4D_{12}^0)}{\rho^0 h}} \tag{18}$$

5. Results and discussion

5.1. Comparative studies

In first example, the values of the DFP, $\omega_1=\omega(a^2/h)\sqrt{\rho^0/E_2^0}$, for square HOPs without elastic foundations for $a/h=100$ are compared with the Levy type solution of Thai and Kim [22] and presented in Table 1. The following material properties were used in the comparison:

Table 1
Comparison the values of DFP for square HOPs with the results of Thai and Kim [22].

	$\omega_1=\omega a^2/h\sqrt{\rho^0/E_2^0}$	
E_1^0/E_2^0	Thai and Kim [22]	Present study
10	10.4963	10.4963
25	15.2278	15.2278
40	18.8052	18.8052

$E_1^0/E_2^0=10, 25, 40$; $G_{12}^0=0.5E_2^0$; $\nu_{12}=0.25$; $\rho_0=1\ \text{kg/m}^3$.

The values obtained in this study are in a good agreement with those obtained in the study of the Thai and Kim [22].In second example, the values of dimensionless frequency parameter, $\omega_1=\omega a^2\sqrt{\rho^0 h/D^0}$, for the functionally graded (along the x axis) isotropic rectangular plate are compared with the results of Xia et al. [6]. Here $D^0=\frac{E^0h^3}{12(1-\nu_0^2)}$ is flexural rigidity for homogeneous isotropic plates. Consider the functionally graded isotropic rectangular plate with geometric dimensions $a/h=20$ and $b=a/2$, and the Young's modulus of isotropic plate changes along the x direction according to $E=E^0f(X)$. Here $f(X)=1+\frac{x}{a}(k-1)$ is the functionally graded function, $k=\frac{f(1)}{f(0)}$, in which $f(0)$ and are the values of the $f(X)$ function at the left and right ends of the plate, respectively. The material properties of homogeneous isotropic rectangular plate are taken to be $E^0=E_1^0=E_2^0=3\times10^7$ Pa, $\nu_0=\nu_{12}=\nu_{21}=0.3$, $G_{12}^0=\frac{E^0}{2(1+\nu_0)}$ and $\rho_0=1\ \text{kg/m}^3$. It can be seen from Table 2 that the values of the DFP of the functionally graded (along the x axis) isotropic plate are in good agreement with those obtained by Xia et al. [6]. The difference between the results is due to the fact that our study uses the CPT, while in the other study the shear deformation plate theory is used.

In Table 3 presents a comparative study of the DFP, ω_1, for the flexural modes of thin homogeneous isotropic square plates resting on the PEF. Dimensionless parameters are specified as

$$\omega_1=\frac{\omega b^2}{\pi^2}\sqrt{\frac{\rho^0 h}{D^0}},\overline{K}_w=\frac{K_w a^4}{D^0},K_P=\frac{K_P a^2}{D^0},\nu_0=0.3,\rho^0=1\ \text{kg/m}^3,a/b=1,b/h=100$$

The data are taken from study of Ferreira et al. [19]. It can be seen

Table 2
Comparison of the values of DFP for functionally graded isotropic plates with the results of Xia et al. [6].

	$\omega_1=\omega a^2\sqrt{\rho^0 h/D^0}$	
$f_1(1)/f_1(0)$	Xia et al. [6]	Present study
1	48.540	49.348
3	66.722	69.788

Table 3
Comparison of DFPs for the flexural modes of thin square plates on the PEF.

$\overline{K}_w$	$\overline{K}_p$	References	ω_1
10^2	10	Xiang et al. [15]	2.6551
		Zhou et al. [18]	2.6551
		Ferreira et al. [19]	2.6559
		Present study	2.6558
5×10^2	10	Xiang et al. [15]	3.3400
		Zhou et al. [18]	3.3398
		Ferreira et al. [19]	3.3406
		Present study	3.34057

from Table 3, our results are excellent agreement with those of Xiang et al. [15], using a Mindlin approach, Zhou et al. [18], who used a 3D Ritz approach and Ferreira et al. [19] using radial basic functions.

5.2. Study of influences of material gradient, orthotropy and PEF on the DFPs

This section presents new numerical calculations and analyzes related to the influences of material gradient in x and z directions and material orthotropy on the DFPs of the rectangular plates resting on the PEF. In Figs. 2–5, the elastic properties for the homogeneous Carbon Fiber Reinforced Polymer (CFRP) are taken as [1]

$$E_1^0 = 138.6\text{ GPa}, E_2^0 = 8.27\text{ GPa}; G_{12}^0 = 4.12\text{ GPa}, \nu_{12} = 0.26, \rho^0 = 1824\text{ kg/m}^3$$

Figs. 2 and 3 show the distribution of the DFPs of homogeneous (H) and EG profiles of CFRP rectangular plates in accordance with the ratio a/h. According these figures, the curves related to $(K_w,K_p) = (2.5\times10^6, 2.5\times10^4)$ and $(K_w,K_p) = (0,0)$ represent the plates with and without the PEF. Here $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$ refers to homogeneous orthotropic plate. The geometric parameters of the plate are: $a/b = 0.5$ and $b = 1m$. It is concluded from these figures that the values of the DFPs for H and EG profiles of CFRP rectangular plates without the PEF do not depend on the ratio, a/h, whereas, the values of the DFPs increase with the increasing ratio a/h, as the considering the effect of the PEF.In absence of the PEF, the influence of the biderctionally

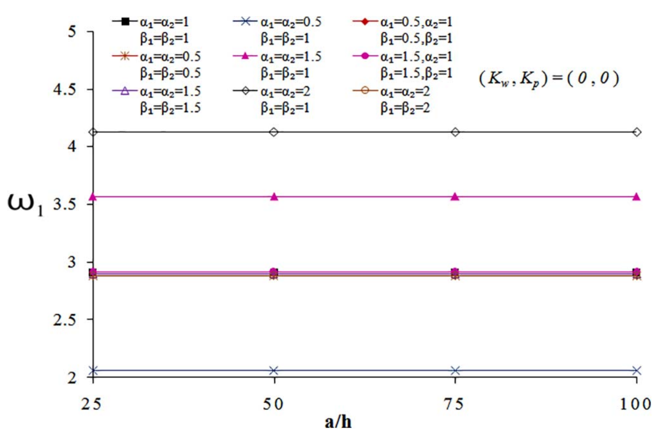

Fig. 2. Distribution of DFPs of unconstrained HOP and BEGOPs in accordance with the ratio a/h.

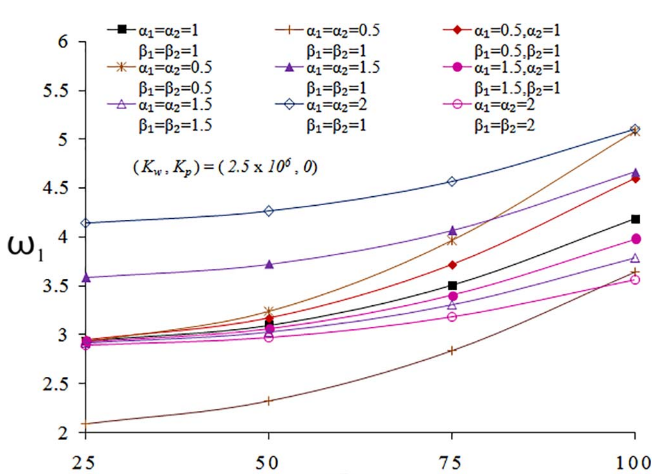

Fig. 3. Distribution of DFPs of HOP and BEGOPs resting on the PEF in accordance with the ratio a/h.

graded (BG) profiles of the material properties on the DFPs does not depend on the variation of the ratio, a/h. For instance, as the Young's moduli vary only in the x direction, i.e., for $\alpha_1 = 0.5, 1.5, 2.0;\ \alpha_2 = \beta_1 = \beta_2 = 1$, the effects of graded profiles on the DFPs are (−15.55%), 10.81% and 19.43%, respectively, as the a/h increases from 25 to 100. The efficiencies of graded profiles are (−29.11%), 22.55%, 41.74%, respectively, as the a/h increases from 25 to 100 for $\alpha_1 = \alpha_2 = 0.5, 1.5, 2.0;\ \beta_1 = \beta_2 = 1$, i.e., as the Young's moduli change together in the x and z directions, and the density remains constant. When the Young's moduli and density of the CRFP plate change in the x direction and do not change in the z direction, i.e., for $\alpha_1 = \beta_1 = 0.5, 1.5, 2.0; \alpha_2 = \beta_2 = 1$, the effects of graded profiles on the DFPs are negligible, while as $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0.5, 1.5, 2.0$, thee effects are weak and is around (−1.17%).

If the considering effect of the PEF, the influence of the EG profiles on the DFPs decrease with the increasing of the ratio, a/h. For instance, as the Young's moduli vary only in the x direction, i.e., for $\alpha_1 = 0.5, 1.5, 2.0;\ \alpha_2 = \beta_1 = \beta_2 = 1$, the effects of graded profiles on the DFPs decrease from (−15.16%) to (−5.72%), from 10.58% to 4.31% and from 18.99% to 7.92%, respectively, since the a/h increases from 25 to 100. The efficiencies of graded profiles diminish from (−28.28%) to (−10.14%), from 22.08% to 9.26% and from 40.86% to 17.88% for $\alpha_1 = \alpha_2 = 0.5, 1.5, 2.0;\ \beta_1 = \beta_2 = 1$, respectively, i.e., as the Young's moduli change together in the x and z directions, and the density remains constant. When the Young's moduli and density of the CRFP plate change in the x direction and do not change in the z direction, i.e., for $\alpha_1 = \beta_1 = 0.5, 1.5, 2.0; \alpha_2 = \beta_2 = 1$, the effects of graded profiles on the DFPs increase from 0.51% to 11.69%, from (−0.2%) to (−5.87%) and from (−0.34%) to (−9.6%), respectively, since the a/h increment from 25 to 100. When the Young's moduli and density of the CRFP plate change together in the x and z directions, i.e., for $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0.5, 1.5, 2.0$, the effects of graded profiles on the DFPs increment from 0.03% to 25.33%, from (−0.81%) to (−11.2%) and from (−1.76%) to (−17.8%), respectively, since the a/h increases from 25 to 100.

In all plates with the homogeneous and graded profiles, the influence of the PEF on the values of the DFP becomes more pronounced, as the ratio a/h increases. Depending on the choice of BG profiles, the influence of the PEF on the values of the DFP of plates varies considerably. When the HOP is compared with the BEGOPs, the effect of the PEF on the DFP values for graded profiles of rectangular plates is more pronounced at $\alpha_i,\beta_i \leqslant 1$, whereas this effect becomes less pronounced at $\alpha_i,\beta_i > 1$. In addition, at the same values of the parameters α_i and β_i, the effect of the PEF on the values of DFP weakens the originality of the graded profiles among themselves.In Figs. 4 and 5, the characteristics curves of the DFPs of HOP and BEGOPs with and without the PEF $((K_w,K_p) = (5\times10^6, 5\times10^4)$ and $(K_w,K_p) = (0,0))$ in accordance with the ratio a/b are plotted for the nine values of the

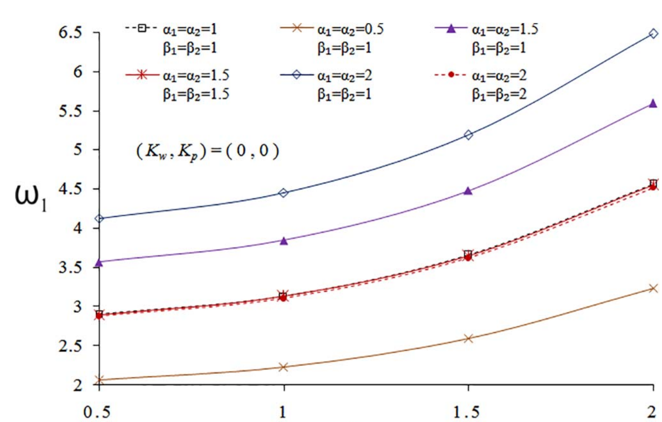

Fig. 4. Distribution of DFPs of unconstrained HOP and BEGOPs in accordance with the ratio a/b.

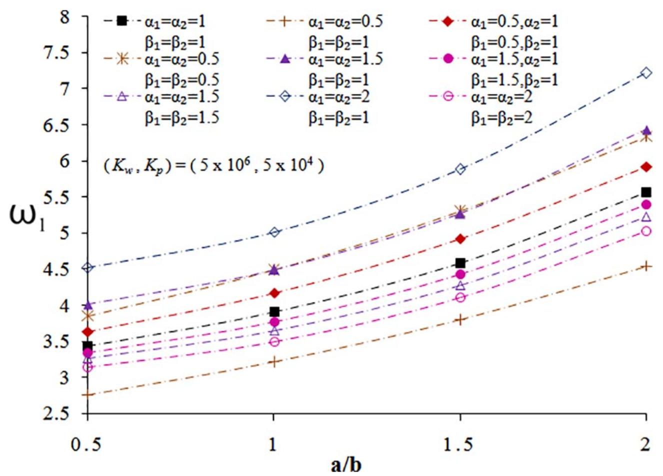

Fig. 5. Distribution of DFPs of HOP and BEGOPs resting on the PEF in accordance with the ratio a/b.

Young's moduli, shear modulus and density variation parameters and for $a/h = 50, b = 1m$. It should be remembered that if one of the variation parameters of Young's and shear moduli, and density is equal to one, this means that this property of the material is homogeneous. The Young's and shear moduli, and density variation parameters that are not considered in the analysis are considered to be equal to one. It can be inferred that the magnitudes of the DFPs of HOP and BEGOPs with and without the PEF increase with the increasing of the aspect ratio, a/b.

In absence of the PEF, the effects of graded profiles on the values of the DFPs monotonically and slightly change, since a/b increases from 0.5 to 2. For example, as $\alpha_1 = \alpha_2 = 0.5, \beta_1 = \beta_2 = 1$, the effect of graded profiles on the DFPs for the plate without the PEF very slightly decreases from (−15.55%) to (−15.39%); as $\alpha_1 = 2, \alpha_2 = 1, \beta_1 = \beta_2 = 1$, the effect very slightly increases from 19.43% to 19.67% and as $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0.5, 2.0$, the effect very slightly decreases from (−1.17%) to (−0.96%), respectively, since the ratio a/b increases to 2.0.Also Figs. 4 and 5 reveal that the effects of graded profiles on the values of the DFPs for plates resting on the PEF relatively rapidly and discontinuous change, since a/b increases from 0.5 to 2. For example, as $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0.5, 1.5, 2.0$, the effects of graded profiles on the DFPs for plates on the PEF increase from 11.88% to 15.51%, from (−5.26%) to (−6.67%) and from (−8.48%) to (−10.59%), respectively, since the ratio a/b increases from 0.5 to 1.5, then these effects decrease to 13.86%, (−5.97%) and (−9.51%), respectively, since the ratio a/b increases to 2.0. As $\alpha_1 = 2, \alpha_2 = 1, \beta_1 = \beta_2 = 1$, the effect of graded profile on the DFP first decreases from 14.23% to 12.83%, since the ratio a/b increases from 0.5 to 1.5, then this effect increases to 13.59%, since the ratio a/b increases to 2.0.

Likewise, the effects of PEF on the values of the DFPs for HOP and BEGOPs first increase, since a/b increases from 0.5 to 1.5 and then decrease as the ratio a/b increases to 2.0. For example, the effect of PEF on the DFP first increases from 18.19% to 25.64%, since the ratio a/b increases from 0.5 to 1.5, and then this effect decreases to 21.95%, since the ratio a/b increases to 2.0 for the HOP. The effects of the PEF on the DFPs for BEGOPs with $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0.5, 1.5, 2.0$ increase from 33.8% to 44.53%, from 12.44% to 18.51% and from 9.45% to 13.46%, respectively, since the ratio a/b increases from 0.5 to 1.5, and then these effects decrease to 40.2%, 15.05% and 11.42%, respectively, since the ratio a/b increment to 2.0.To study the effects of one- and two-parameter elastic foundations on the DFPs for HOP and BEGOPs are plotted curves in accordance with the ratio of the orthotropy E_1/E_2 for different values of α_i and $\beta_i (i = 1,2)$ parameters with $a/h = 50$ and $a/b = 0.5$ and presented in Figs. 6–8. The parameters of Winkler and Pasternak elastic foundations are considered: $(K_w, K_p) = (2.5 \times 10^6, 2.5 \times 10^4)$, $(K_w, K_p) = (2.5 \times 10^6, 0)$ and $(K_w, K_p) = (0,0)$. The elastic properties for the homogeneous orthotropic materials are taken as $E_1^0 = 200$ GPa, $E_2^0 = E_1^0/k; k = 5, 20, 35, 50$, $G_{12}^0 = 0.5E_2^0$, $\nu_{12} = 0.3$, $\rho^0 = 1$ kg/m^3. As it seen from Figs. 6–8, the magnitudes of the DFPs of HOP and BEGOPs with and without elastic foundations decrease with the increasing of the orthotropy ratio, E_1/E_2. The values of the DFP are increasing when the Winkler and Pasternak elastic foundation effects are taken into consideration. The effect of PEF on the DFP values appears to be more pronounced than the Winkler elastic foundation (WEF).In absence of

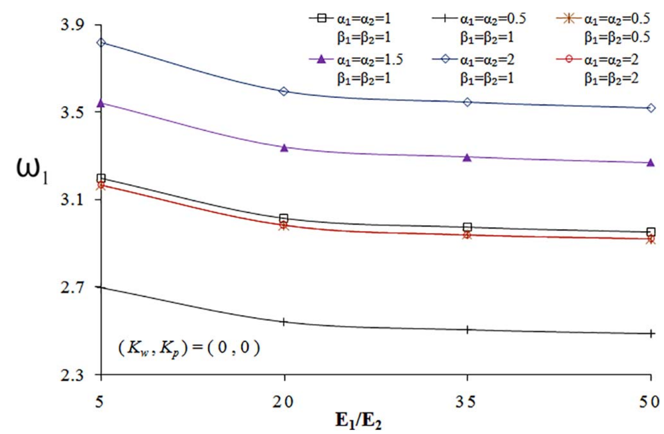

Fig. 6. Distribution of DFPs of HOP and BEGOPs without elastic foundations in accordance with the orthotropy ratio E_1/E_2.

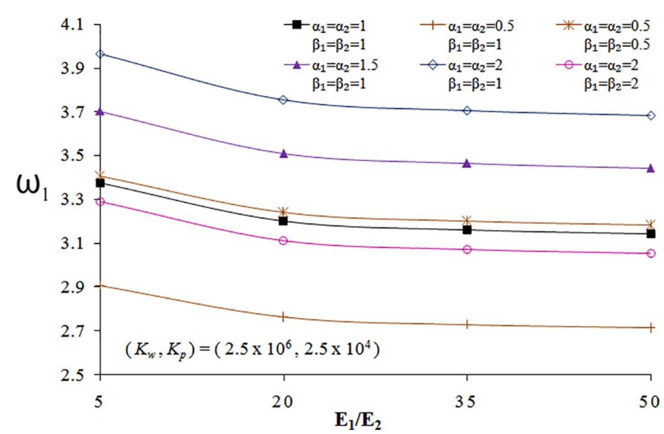

Fig. 7. Distribution of DFPs of HOP and BEGOPs resting on the PEF in accordance with the orthotropy ratio E_1/E_2.

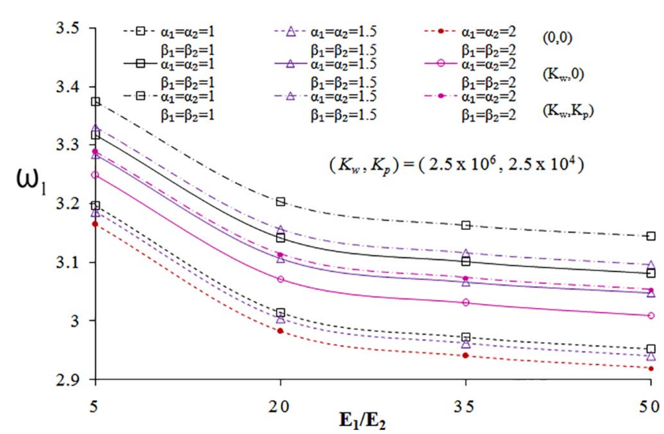

Fig. 8. Distribution of DFPs of HOP and BEGOPs with and without the one and two-parameter elastic foundations in accordance with the orthotropy ratio E_1/E_2.

Table 4
Distribution of the DFP values for BEGOPs resting on the PEF depending on the wave numbers (*m, n*) for three different values of the parameter α_1.

	ω_1			
	$\alpha_1 = \alpha_2 = 1$ $\beta_1 = \beta_2 = 1$	$\alpha_1 = 0.5, \alpha_2 = 1$ $\beta_1 = \beta_2 = 1$	$\alpha_1 = 1.5, \alpha_2 = 1$ $\beta_1 = \beta_2 = 1$	$\alpha_1 = 2, \alpha_2 = 1$ $\beta_1 = \beta_2 = 1$
(*m, n*)	$(K_w,K_p) = (0,0)$			
(1,1)	3.139	2.653	3.480	3.752
(1,2)	4.561	3.859	5.058	5.458
(1,3)	7.649	6.469	8.481	9.148
(2,1)	11.652	9.882	12.933	13.975
(2,2)	12.557	10.652	13.938	15.064
(2,3)	14.617	12.402	16.226	17.538
(3,1)	25.916	21.996	28.772	31.107
(3,2)	26.680	22.646	29.621	32.026
(3,3)	28.253	23.983	31.368	33.919
(*m, n*)	$(K_w,K_p) = (2.5 \times 10^6, 2.5 \times 10^4)$			
(1,1)	3.543	3.121	3.849	4.096
(1,2)	4.916	4.273	5.381	5.758
(1,3)	7.936	6.806	8.742	9.390
(2,1)	11.795	10.051	13.062	14.095
(2,2)	12.717	10.840	14.082	15.197
(2,3)	14.792	12.607	16.384	17.685
(3,1)	26.002	22.098	28.850	31.179
(3,2)	26.776	22.759	29.708	32.106
(3,3)	28.364	24.113	31.468	34.009

the elastic foundations and when the orthotropy ratio, E_1/E_2, varies from 5 to 50, the effect of graded profiles on the PEF values shows a slight change, with the greatest effect varying from 19.61% to 19.51%, while if the considering effects of Winkler and Pasternak elastic foundations, the effects of the graded profiles varying from 18.3% to 18.11% and from 17. 75% to 17.33%, respectively, for $\alpha_1 = 2, \alpha_2 = \beta_1 = \beta_2 = 1$. As can be seen from Figs. 6–8, the elastic foundations effects reduce the influences of graded profiles on the DFP values. Table 4 presents the variation of the DFP values for HOP and BEGOPs resting on the PEF depending on the wave numbers (m, n) for three different values of the parameter α_1 (=0.5, 1.5 and 2) with $a/h = 50$, $a/b = 1$, $b = 1$m, $\alpha_2 = \beta_1 = \beta_2 = 1$, $(K_w, K_p) = (2.5 \times 10^6, 2.5 \times 10^4)$ and $(K_w, K_p) = (0,0)$. It can observe that the change of the effect of the gradient profile in the x direction on the DFP values for the unconstrained BEGOPs is very small, since the wave numbers (m, n) increase. If we consider the effect of PEF, the effect of graded profiles increases significantly, depending on the increase of wave numbers (m, n). For example, with $\alpha_1 = 0.5$, 1.5, and 2, these effects increase from (−11.91%) to (−15%), from 8.64% to 10.94% and from 15.61% to 19.9%. As the number of waves is greater than two (m, n) > (2,2), the effect of the PEF on the values of the DFPs is significantly reduced.

6. Conclusions

Based on the CPT, the free vibration of BEGOPs resting on the two-parameter elastic foundations is studied. The PEF model is used as two-parameter elastic foundation model. The biderctionally graded profiles of the orthotropic materials vary depending on the axial and thickness coordinates. The motion equation is derived based on the classical plate theory and solved by using Galerkin method. To validate of current results was made a comparison with the previous studies. The effects of material gradient, orthotropy and the two-parameter elastic foundations on the DFPs are investigated.

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