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Revealing optical soliton solutions of Schrödinger equation having parabolic law and anti-cubic law with weakly nonlocal nonlinearity

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ABSTRACT

In this study, we purpose to ensure optical soliton solutions of the nonlinear Schrödinger equation having parabolic and anti-cubic (AC) laws with a weakly non-local nonlinearity by using the new Kudryashov method. As far as we know this model has not been presented and studied before. Furthermore, what differs this study from other studies is, not only obtains a variety of analytical solutions of the examined model but also substantiates the effects of the parabolic and anti-cubic laws with a weakly non-local nonlinearity on soliton behaviour, by choosing the particular soliton forms, which are dark, bright and W-like. Eventually, we depict some of the derived solutions in contour, 2D and 3D diagrams selecting the appropriate values of parameters by means of Matlab to demonstrate the importance of the given model. It is indicated that parabolic and AC parameters taking into consideration the weak non-local contribution have a very remarkable impact on the soliton structure, and the impact alters connected with the parameters and the soliton form. Besides, enabling and retaining the critical balance between the parameters and the soliton form and the interactive relation of the parameters with each other comprises major challenges.

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Soliton molecule; W-like soliton; nonlinear effect; chromatic dispersion

1. Introduction

Nonlinear partial differential equations have widespread implementation in nonlinear physics branches such as nonlinear fibre optics, plasma physics, mechanical waves, fluid dynamics and optics; thus, it has captivated much interest from research specialists in the last two quarters [1–20]. These phenomena have been mostly modelled utilizing different forms of the nonlinear Schrödinger equation (NLSE) that defines the propagation of soliton. The NLSE is a very notable equation and it is also used in a very wide range from water waves to optics. In [21], higher order NLSE having derivative non-Kerr nonlinearity is investigated via the improved modified extended tanh-function. [22] presents optical soliton solutions of NLSE with polynomial law and quadratic–cubic law of refractive index. [23] examines the stationary solitons of the generalized NLSE in the presence of chromatic dispersion and polynomial of powers having an arbitrary refractive index. In [24], the F-expansion scheme is employed to the (1+1)-dimensional NLSE with Kerr law nonlinearity in order to achieve highly dispersive optical soliton solutions. The conformable space-time fractional perturbed NLSE having various laws of nonlinearity was examined in [25]. In [26], (3+1) dimensional NLSE with sixth and fourth-order dispersive terms having cubic-quintic-septic nonlinearities was examined. [27–29] tackle the NLSE having Kudryashov's sextic power-law and optical solutions. In [30], Mathanaranjan explored the soliton solution of the conformable space-time fractional cubic-quartic NLSE with diverse laws of nonlinearity. In [31], the semi-inverse variational principle was implemented to the perturbed NLSE with cubic–quintic–septic refractive index. [32] includes various optical soliton solutions of the (3+1) -dimensional NLSE. Many kinds of laws of nonlinearity of the Lakshmanan–Porsezian-Daniel model were examined in detail [33–36]. [37] addresses the cubic–quartic NLSE with quadratic–cubic nonlinearity. Zayed et al. perused the dimensionless structure of the stochastic Sasa–Satsuma model in detail [38]. [39] present the cubic-quartic bright optical soliton of perturbed Fokas Lenells equation. In addition, a number of procedures have been explored in the literature to acquire soliton solutions to such problems. Some of these methods are as follows: Sine–Gordon equation scheme [20, 40], F-expansion technique [40], Adomian decomposition procedure [41], Laplace–Adomian decomposition method [42], Kudryashov's method [43],

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the modified Kudryashov's approach [44], the scheme of undetermined coefficients [45], nonstandard finite difference technique [46], the trial equation scheme [47] and many more.

The first target of this article is to generate analytical optical soliton solutions of the (1+1)-dimensional NLSE having parabolic law with a weakly non-local nonlinearity given as [48]:

$$i\vartheta_t + \rho\vartheta_{xx} + \left(b_1|\vartheta|^2 + b_2|\vartheta|^4 + b_3\left(|\vartheta|^2\right)_{xx}\right)\vartheta = 0, \tag{1}$$

in which the complex-valued function $\vartheta(x,t)$ defines the soliton profile, and x and t expresses the spatial and temporal coordinates, respectively. The first term $i\vartheta_t$ is the temporal evolution whereas the second term $\rho\vartheta_{xx}$ represents the group-velocity dispersion (GVD). The next two nonlinear terms are members of parabolic law nonlinearity [49–59] with the coefficients b_1, b_2. These two nonlinear terms are conjugated for the cumulative nonlinear effect that is based on these two effects. The last nonlinear effect stands for the coefficient of b_3 that is from weakly non-local nonlinearity [60–69]. Moreover, ρ, b_1, b_2 and b_3 are real values.

The second objective of this paper is to examine the (1+1)-dimensional NLSE having anti-cubic law with a weakly non-local nonlinearity introduced as:

$$i\vartheta_t + \rho\vartheta_{xx} + \left(b_1|\vartheta|^{-4} + b_2|\vartheta|^2 + b_3|\vartheta|^4 + b_4\left(|\vartheta|^2\right)_{xx}\right)\quad\vartheta = 0, \tag{2}$$

where the three coefficients b_1, b_2 and b_3 that are from anti-cubic nonlinear forms [70–78].

What encourages us to do this study is that the models have not been examined before in the literature. Additionally, the non-locality of nonlinear response in wave propagation problems is a significant determinant in a variety of mathematical and physical contexts. Impacts of non-locality are accomplishable in those media where non-locality originates in the single continuum of nonlinearity such as parabolic and anti-cubic law. These captivating models arise when two or more competitive nonlinearities make a contribution to the procedure of nonlinearity. [48, 79] examine the soliton solutions of the dimensionless structure of the NLSE in parabolic law with a weakly non-local nonlinearity. [80] investigates the interactive relation of dark solitons with an arbitrary degree of non-local nonlinearity. [81] presents the properties of pure-quartic optical soliton solutions in a nonlinear media with a weakly non-locality.

The paper is configured as follows: Section 2 includes the mathematical analysis of the equations under consideration. The NKM is mathematically examined in Section 2. NKM is performed to the examined model which is given by Equations (1), (2), respectively in Section 3. Diagrams of the obtained soliton solutions are indicated graphically and the consequences that we attained are interpreted in Section 4. The conclusion of the article is referred to in Section 5.

2. Mathematical analysis

2.1. Ordinary differential equation shape of Equation (1)

We take into account the following transformation of Equation (1) as:

$$\vartheta(x,t) = \vartheta(\zeta)\,e^{i(-\kappa x+\omega t+\theta_{0},)}, \quad \zeta = x - \nu t, \tag{3}$$

in which ν, κ, ω, and θ_0 are real constants. Herein, ν expresses the velocity, κ, ω and θ_0 stand for the wave number, the frequency and the phase number, respectively. Employing Equation (3) to Equation (1), and dividing the generated relation into the real and imaginary components, we get :

$$\left(2b_3\vartheta^2 + \rho\right)\vartheta'' + 2b_3\vartheta\left(\vartheta'\right)^2 - \left(\omega - b_2\vartheta^4 + \rho\kappa^2 - \vartheta^2 b_1\right)\vartheta = 0, \tag{4}$$

and

$$\left(2\rho\kappa + \nu\right)\vartheta' = 0. \tag{5}$$

From Equation (5), the constraint condition is acquired as:

$$\nu = -2\rho\kappa. \tag{6}$$

Taking into account the constraint condition in Equations (6), (4) symbolizes the NLODE form of Equation (2).

2.2. Ordinary differential equation structure of Equation (2)

In this part, employing the wave transformation given with Equation (3), the real and imaginary parts are derived as:

$$\left(2b_4\vartheta^5 + \rho\vartheta^3\right)\vartheta'' + 2b_4\vartheta^4\left(\vartheta'\right)^2 + b_3\vartheta^8 + b_2\vartheta^6 - \left(\kappa^2\rho + \omega\right)\vartheta^4 + b_1 = 0, \tag{7}$$

and

$$\left(2\kappa\rho + \nu\right)\vartheta^3\vartheta' = 0. \tag{8}$$

From Equation (8), the constraint condition is acquired as:

$$\nu = -2\rho\kappa. \tag{9}$$

To acquire closed-form solutions, we should define:

$$\vartheta = V^{\frac{1}{2}} \tag{10}$$

which reduces Equation (7) into the following ODE form of Equation (2):

$$2\left(2V^2b_4 + \rho V\right)V'' - \rho\left(V'\right)^2 + 4b_3V^4 + 4b_2V^3 - 4\left(\kappa^2\rho + \omega\right)V^2 + 4b_1 = 0. \tag{11}$$

3. Application

3.1. The new Kudryashov method (NKM)

The following factors constitute the basis of the selection of the NKM method in the study conducted within the scope of the article. The method does not require much mathematical processing, targets and presents certain types of solitons (bright, dark and kink), and is a widely used reliable method. It is also so easy to implement. The main stages of NKM [82] are stated as follows.

The following truncated series is considered as a solution of Equations (4) and (11):

$$V(\zeta)=\sum_{l=0}^{B}\Lambda_l\Phi^l(\zeta),\quad \Lambda_B\neq 0, \tag{12}$$

where Λ_l are real values. $\Phi^l(\zeta)$ ensures:

$$(\Phi'(\zeta))^2=\delta^2\Phi^2(\zeta)[1-\chi\Phi^2(\zeta)], \tag{13}$$

where χ, and δ are nonzero values to be figured out later. The Equation (13) serves the given solution as:

$$\Phi(\zeta)=\frac{4k}{4k^2\,e^{\delta\eta}+\chi\,e^{-\delta\eta}}, \tag{14}$$

where k is a real constant.

3.2. Application of the NKM to Equation (1)

In this section, we seek the soliton solutions of Equation (1) via NKM. Considering the terms $\vartheta^2\vartheta''$ and ϑ^5 in Equation (4) utilizing the homogeneous balance relation [83, 84], we get the balance term as $B=1$. Because of $B=1$, Equation (12) is expressed the following structure:

$$V(\zeta)=\Lambda_0+\Lambda_1\Phi(\zeta). \tag{15}$$

Unity of Equations (15), (13), (4) generates the following algebraic form:

$$\begin{aligned}
&\Phi^0(\zeta):\Lambda_0\left(\Lambda_0^4b_2+b_1\Lambda_0^2-\rho\kappa^2-\omega\right)=0,\\
&\Phi(\zeta):\Lambda_1\left(5\Lambda_0^4b_2+\left(2b_3\delta^2+3b_1\right)\Lambda_0^2\right.\\
&\quad\left.+\rho\delta^2-\rho\kappa^2-\omega\right)=0,\\
&\Phi^2(\zeta):\Lambda_0\left(10\Lambda_0^2b_2+6b_3\delta^2+3b_1\right)\Lambda_1^2=0,\\
&\Phi^3(\zeta):\Lambda_1\left(\left(10\Lambda_0^2b_2+4b_3\delta^2+b_1\right)\Lambda_1^2\right.\\
&\quad\left.-\chi\left(4\Lambda_0^2b_3+2\rho\right)\delta^2\right)=0,\\
&\Phi^4(\zeta):5\Lambda_0\Lambda_1^2\left(-2b_3\chi\delta^2+b_2\Lambda_1^2\right)=0,\\
&\Phi^5(\zeta):\Lambda_1^3\left(-6b_3\chi\delta^2+b_2\Lambda_1^2\right)=0.
\end{aligned}$$

The following solution functions for the derived solution sets from this algebraic system are obtained:

Set 1:

$$\left\{b_1=\frac{12{b_3}^2\delta^2\left(\kappa^2-\delta^2\right)+b_2\omega}{3b_3\left(\delta^2-\kappa^2\right)},\rho=\frac{\omega}{\delta^2-\kappa^2},\right.$$
$$\left.\Lambda_0=0,\Lambda_1=\frac{\sqrt{6b_2b_3\chi}\,\delta}{b_2}\right\} \tag{16}$$

Taking into account the Equation (16) with Equations (15), (3), we extract:

$$\vartheta_1(x,t)=\frac{4\sqrt{6b_2b_3\chi}\,\delta k}{b_2\left(4k^2\,e^{\delta\left(\frac{2\omega\kappa t}{\delta^2-\kappa^2}+x\right)}+\chi\,e^{-\delta\left(\frac{2\omega\kappa t}{\delta^2-\kappa^2}+x\right)}\right)}\times e^{i(-\kappa x+\omega t+\theta_0)}. \tag{17}$$

Set 2:

$$\left\{b_3=\frac{b_2\Pi}{6\left(2b_2\Lambda_1^2+3b1\chi\right)\kappa^2\chi},\right.$$
$$\delta=\frac{\sqrt{\Pi\left(2b_2\Lambda_1^2+3b_1\chi\right)}\kappa\Lambda_1}{\Pi},$$
$$\left.\rho=\frac{\Pi}{6\kappa^2\chi^2},\Lambda_0=0,\Lambda_1=\Lambda_1\right\}, \tag{18}$$

in which $\Pi=2b_2\Lambda_1^4+3b_1\chi\Lambda_1^2-6\chi^2\omega$. Considering the Equation (18) with Equations (15), (3), we construct:

$$\vartheta_2(x,t)=\frac{4\Lambda_1k}{4k^2e^{\frac{\sqrt{\Pi\left(2b2\Lambda_1^2+3b1\chi\right)}\kappa\Lambda_1\left(\frac{\Pi t}{3\kappa\chi^2}+x\right)}{\Pi}}+\chi\,e^{-\frac{\sqrt{\Pi\left(2b2\Lambda_1^2+3b1\chi\right)}\kappa\Lambda_1\left(\frac{\Pi t}{3\kappa\chi^2}+x\right)}{\Pi}}}\times e^{i(-\kappa x+\omega t+\theta_0)}. \tag{19}$$

3.3. Application of the NKM to Equation (2)

In this part, we search for the soliton solutions of Equation (2) via NKM. Taking into account the homogeneous balance relation [83, 84] between $\vartheta''\vartheta^2$ and ϑ^4 in Equation (11), we derive $B=2$. Therefore, Equation (12) can be written in the following format:

$$v(\zeta)=\Lambda_0+\Lambda_1\Phi(\zeta)+\Lambda_2\Phi(\zeta)^2,\quad \Lambda_2\neq 0. \tag{20}$$

Combination of Equations (15), (11), (13) yields:

$$\begin{aligned}
&\Phi^0(\zeta): \left(b_3\Lambda_0^2 + b_2\Lambda_0 - \left(\kappa^2\rho + \omega\right)\right)\Lambda_0^2 + b_1 = 0,\\
&\Phi(\zeta): \left(8\Lambda_0^2 b_3 + 2\left(b_4\delta^2 + 3b_2\right)\Lambda_0\right.\\
&\quad\left. + \rho\left(\delta^2 - 4\kappa^2\right) - 4\omega\right)\Lambda_0\Lambda_1 = 0,\\
&\Phi^2(\zeta): \left(24\Lambda_0^2 b_3 + \left(8b_4\delta^2 + 12b_2\right)\Lambda_0\right.\\
&\quad\left. + \rho\left(\delta^2 - 4\kappa^2\right) - 4\omega\right)\Lambda_1^2 + 16\Lambda_0^3\Lambda_2 b_3\\
&\quad + \left(4\left(4b_4\delta^2 + 3b_2\right)\Lambda_0\right.\\
&\quad\left. + 8\left(\rho\left(\delta^2 - \kappa^2\right) - \omega\right)\right)\Lambda_2\Lambda_0 = 0,\\
&\Phi^3(\zeta): \left(-2b_4\delta^2 - 4\Lambda_0 b_3 - 2b_2\right)\Lambda_1^3\\
&\quad + \left(-24\Lambda_0^2 b_3 - 4\left(5b_4\delta^2 + 3b_2\right)\Lambda_0\right.\\
&\quad\left. - \rho\left(3\delta^2 - 4\kappa^2\right) + 4\omega\right)\Lambda_2\Lambda_1\\
&\quad + \chi\Lambda_0\Lambda_1\delta^2\left(4b_4\Lambda_0 + 2\rho\right) = 0,\\
&\Phi^4(\zeta): 4\Lambda_1^4 b_3 + \left(\left(24b_4\delta^2 + 48\Lambda_0 b_3 + 12b_2\right)\Lambda_2\right.\\
&\quad\left. - \chi\delta^2\left(16b_4\Lambda_0 + 3\rho\right)\right)\Lambda_1^2\\
&\quad 4\Lambda_2\left(\left(6\Lambda_0^2 b_3 + \left(8b_4\delta^2 + 3b_2\right)\Lambda_0 - \rho\delta^2\right.\right.\\
&\quad\left.\left. + \kappa^2\rho + \omega\right)\Lambda_2 + \chi\Lambda_0\delta^2\left(6b_4\Lambda_0 + 3\rho\right)\right) = 0,\\
&\Phi^5(\zeta): \Lambda_1\left(2\left(\chi b_4\delta^2 - 2\Lambda_2 b_3\right)\Lambda_1^2\right.\\
&\quad - \left(3\left(3b_4\delta^2 + 4\Lambda_0 b_3 + b_2\right)\Lambda_2\right.\\
&\quad\left.\left. - \chi\delta^2\left(16b_4\Lambda_0 + 3\rho\right)\right)\Lambda_2\right) = 0,\\
&\Phi^6(\zeta): \Lambda_2\left(\left(10\chi b_4\delta^2 - 6\Lambda_2 b_3\right)\Lambda_1^2\right.\\
&\quad + \left(\left(-4b_4\delta^2 - 4\Lambda_0 b_3 - b_2\right)\Lambda_2\right.\\
&\quad\left.\left. + \chi\delta^2\left(12b_4\Lambda_0 + 2\rho\right)\right)\Lambda_2\right) = 0,\\
&\Phi^7(\zeta): \left(7\chi b_4\delta^2 - 2\Lambda_2 b_3 7\right)\Lambda_1\Lambda_2^2 = 0,\\
&\Phi^8(\zeta): 24\chi\delta^2\Lambda_2^3 b_4 + 4\Lambda_2^4 b_3 = 0.
\end{aligned}$$

By solving the above system, we generate the following sets and the corresponding solutions:

Set 3:

$$\left\{\begin{aligned}
b_1 &= \frac{\Xi^2\rho\left(2\chi\delta^2\rho + 8\delta^2\Lambda_2 b_4 - \Lambda_2 b_2\right)}{1728\chi^2\delta^4 b_4^3\Lambda_2},\\
b_3 &= \frac{6\chi b_4\delta^2}{\Lambda_2}, \Lambda_0 = \frac{\Xi}{12\chi b_4\delta^2}, \Lambda_1 = 0, \Lambda_2 = \Lambda_2\\
\omega &= \frac{\begin{array}{c}8\chi^2\delta^4\rho^2 - 24\chi\delta^2\kappa^2\rho\Lambda_2 b_4 + 16\delta^4\Lambda_2^2 b_4^2\\ -2\chi\delta^2\rho\Lambda_2 b_2 - \Lambda_2^2 b_2^2\end{array}}{24\chi b_4\delta^2\Lambda_2},
\end{aligned}\right\}, \tag{21}$$

where $\Xi = 2\chi\delta^2\rho - 4\delta^2\Lambda_2 b_4 - \Lambda_2 b_2$. Unity of Equations (21), (15), (3), (10), allows extracting solution of Equation (2):

$$\vartheta_3(x,t) = \left(\frac{\Xi}{12\chi b_4\delta^2} + \frac{16\Lambda_2 a^2}{\left(4a^2 e^{\delta(2\rho\kappa t + x)} + \chi\, e^{-\delta(2\rho\kappa t + x)}\right)^2}\right)^{\frac{1}{2}} \times e^{i\left(-\kappa x + \frac{\omega t}{24\chi b_4\delta^2\Lambda_2} + \theta_0\right)}. \tag{22}$$

Set 4:

$$\left\{\begin{aligned}
\omega &= \frac{\begin{array}{c}36\delta^4 b_4^2\left(b_4^2 - \kappa^2\rho b_3\right) + 2\rho^2 b_3^2\\ -3b_2 b_4(\rho b_3 + 3b_2 b_4\end{array}}{36 b_4^2 b_3},\\
b_1 &= \frac{\Upsilon^2\rho\left(24\delta^2 b_4^2 + \rho b_3 - 3b_2 b_4\right)}{1296 b_3^2 b_4^4},\\
\Lambda_0 &= -\frac{\Upsilon}{6b_3 b_4}, \Lambda_1 = 0, \Lambda_2 = \frac{6\chi b_4\delta^2}{b_3},
\end{aligned}\right\}, \tag{23}$$

where $\Upsilon = 12\delta^2 b_4^2 - \rho b_3 + 3b_2 b_4$. Combination of Equation (23) with Equations (15), (3), (10) serves the solution of Equation (2):

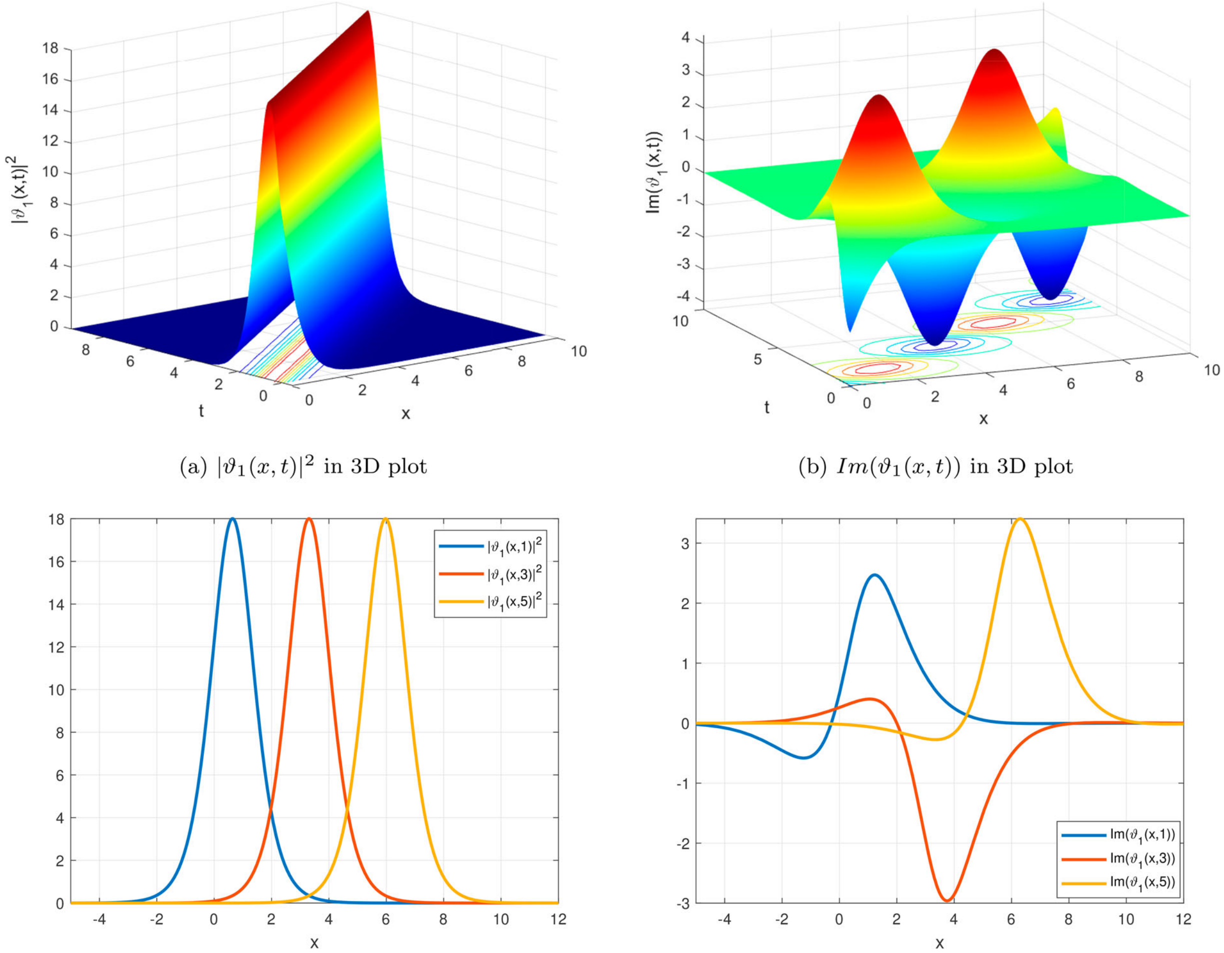
$$\vartheta_4(x,t) = \left(-\frac{\Upsilon}{6b_3 b_4} + \frac{96\chi b_4\delta^2 a^2}{b_3\left(4a^2 e^{\delta(2\rho\kappa t + x)} + \chi\, e^{-\delta(2\rho\kappa t + x)}\right)^2}\right)^{\frac{1}{2}} \times e^{i\left(-\kappa x + \frac{\omega t}{36 b_4^2 b_3} + \theta_0\right)}. \tag{24}$$

4. Results and discussion

This part comprises various graphical representations of Equations (17), (19), (22) and (24). Moreover, two-dimensional graphs are added showing the effects of some parameters in Equation (1) and Equation (2) on the soliton dynamics for each soliton.

Figure 1 relates to the solution function in Equation (17) selecting the parameters as $a = 1, \omega = -1, b_2 = 1, b_3 = 3, \kappa = 0.5, \theta_0 = 4, \delta = 1, \chi = 1$. The 3D depictions of $|\vartheta_1(x,t)|^2$ and $Im(\vartheta_1(x,t))$ are illustrated in Figure 1(a,b), respectively. Figure 1(a,c) reflect a bright soliton. Figure 1(c) is a 2D chart that indicates the wave structure of $|\vartheta_1(x,t)|^2$ as it acts to the right at $t = 1, 3, 5$. The 2D illustration in 1(d) indicates the wave structures of $Im(\vartheta_1(x,t))$ at $t = 1, 3, 5$.

Figure 2(a) is the 2D projection that depicts the impact of the parameter of b_2 in Equation (1) on soliton dynamics. As seen in Figure 2(a), the amplitude of the soliton decreases if $b_1 > 0$ and b_1 increases. Figure 2(b) is the 2D portrayal that shows the effect of the parameter of b_3 in Equation (1) on soliton dynamics. As seen in Figure 2(b), the amplitude of the soliton increases when $b_3 > 0$ and the value of b_3 is raised. Thus, it is observed that b_2 and b_3 have the inverse effect on the amplitude of the soliton.



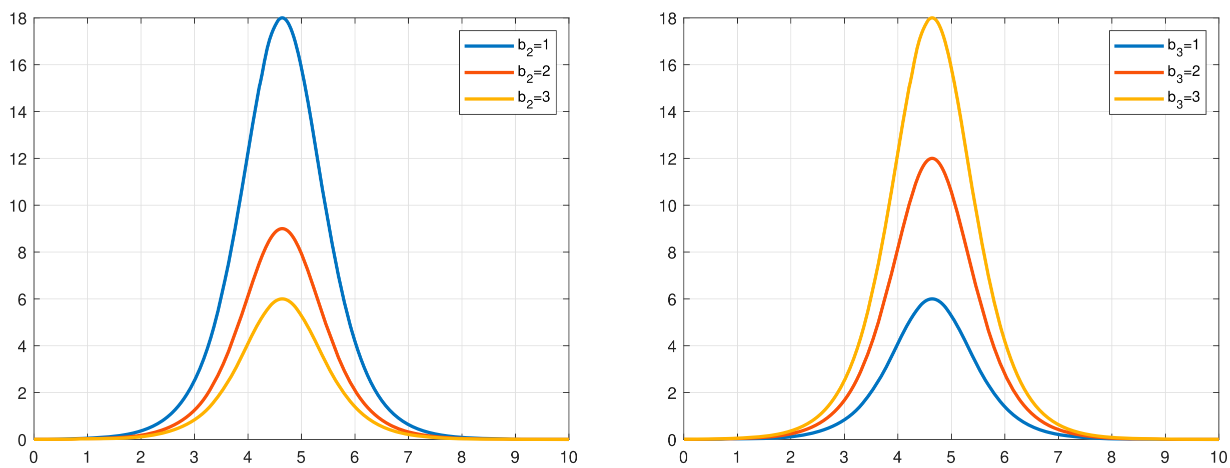
(a) $|\vartheta_1(x,t)|^2$ in 3D plot

(b) $Im(\vartheta_1(x,t))$ in 3D plot

(c) 2D views of $|\vartheta_1(x,t)|^2$

(d) 2D views of $Im(\vartheta_1(x,t))$

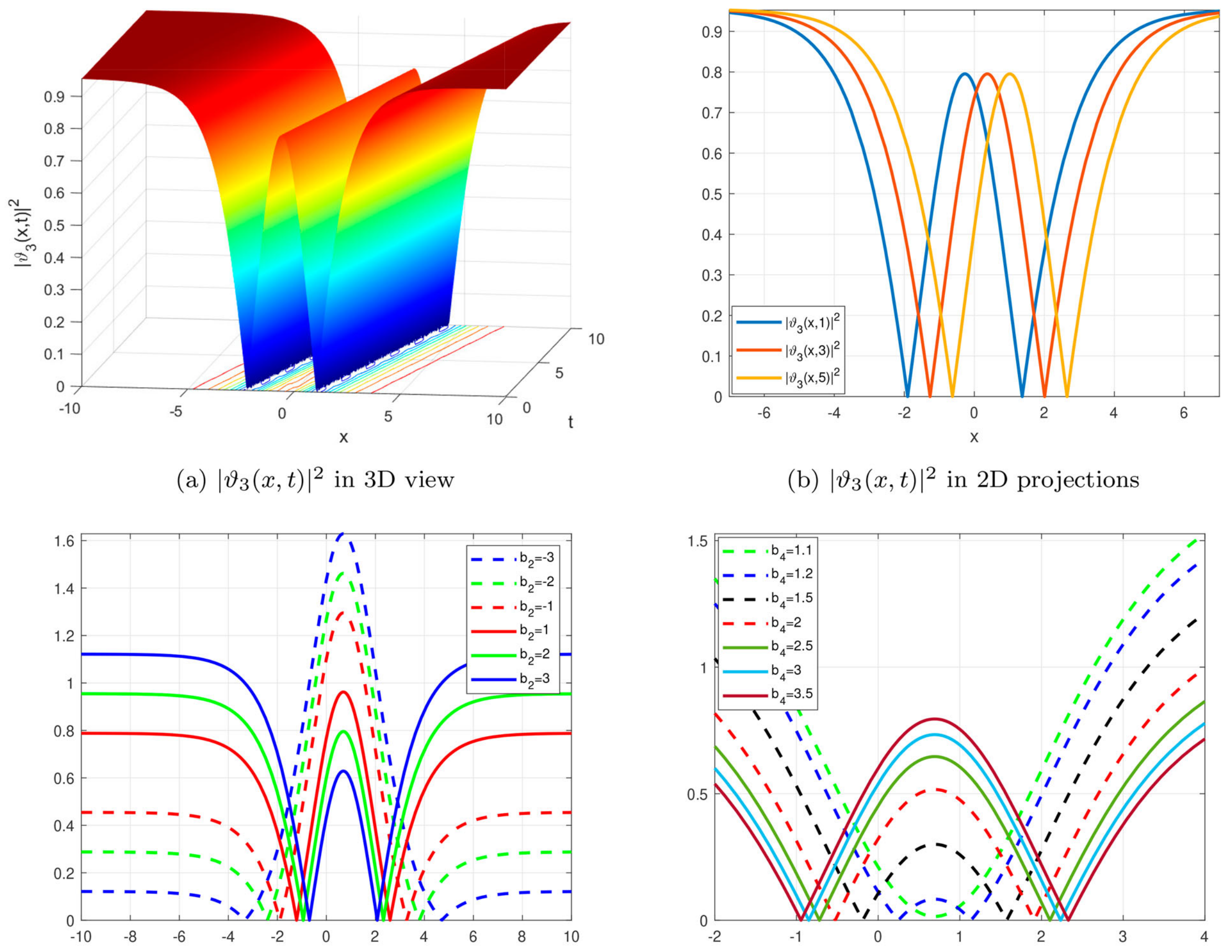
Figure 1. The graphical simulations of $\vartheta_1(x,t)$ in Equation (17) for $a=1, \omega=-1, b_2=1, b_3=3, \kappa=0.5, \theta_0=4, \delta=1, \chi=1$. (a) $|\vartheta_1(x,t)|^2$ in 3D plot. (b) $Im(\vartheta_1(x,t))$ in 3D plot. (c) 2D views of $|\vartheta_1(x,t)|^2$ and (d) 2D views of $Im(\vartheta_1(x,t))$.



(a) 2D views of $|\vartheta_1(x,t)|^2$ for b_2 at $t=4$

(b) 2D views of $|\vartheta_1(x,t)|^2$ for b_3 at $t=4$

Figure 2. The graphics in 2D for $\vartheta_1(x,t)$ in the Equation (17) for $a=1, \omega=-1, b_2=1, b_3=3, \kappa=0.5, \theta_0=4, \delta=1, \chi=1$. (a)2D views of $|\vartheta_1(x,t)|^2$ for b_2 at $t=4$ and (b) 2D views of $|\vartheta_1(x,t)|^2$ for b_3 at $t=4$



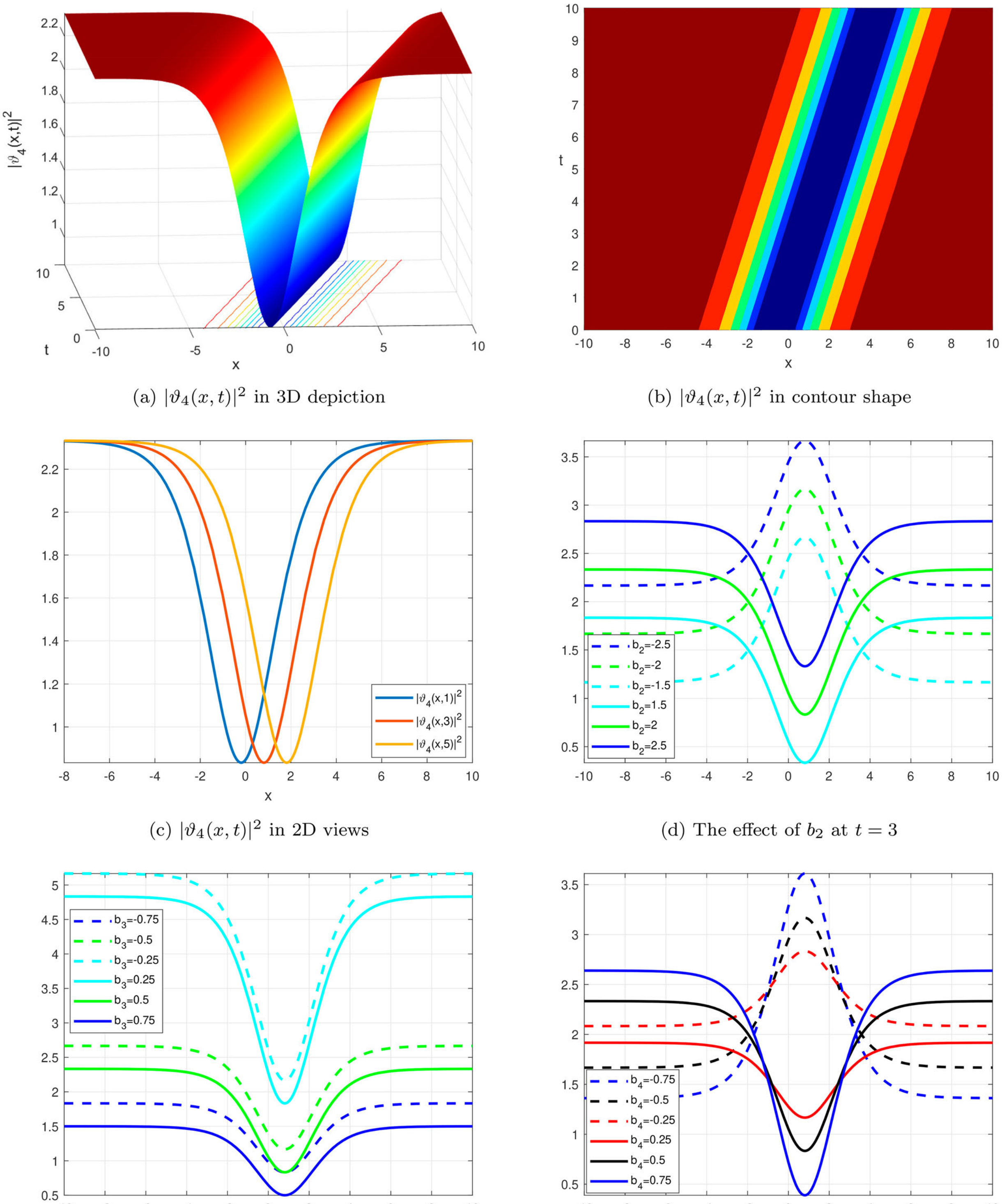
(a) $|\vartheta_3(x,t)|^2$ in 3D view (b) $|\vartheta_3(x,t)|^2$ in 2D projections

(c) The impact of b_2 (d) The impact of b_4

Figure 3. Diverse graphs for $\vartheta_3(x,t)$ in the Equation (24) for $\Lambda_2 = 0.35, a = 0.3, b_2 = 2, b_4 = 3.5, \rho = -0.8, \delta = 0.5, \theta_0 = 0.5, \chi = 0.2, \kappa = 0.2$. (a) $|\vartheta_3(x,t)|^2$ in 3D view. (b) $|\vartheta_3(x,t)|^2$ in 2D projections. (c) The impact of b_2 and (d) The impact of b_4.

Figure 3 belongs to diverse graphical simulations of $\vartheta_3(x,t)$ in Equation (22). Figure 3(a) is the 3D depiction. 3D graph indicates the W-like soliton for $\Lambda_2 = 0.35, a = 0.3, b_2 = 2, b_4 = 3.5, \rho = -0.8, \delta = 0.5, \theta_0 = 0.5, \chi = 0.2, \kappa = 0$. Figure 3(b) expresses 2D soliton profile for $t = 1,\ 3,\ 5$. It is observed that the amplitude and the W-like soliton stay during the propagation. As the value od t is raised, the soliton also moves towards the right. Figure 3(c) is the 2D portraiture to depict the impact of the b_2 considering the values as $-3, -2, -2, 1, 2, 3$, respectively. Soliton maintains its W-like axis, it decreases in amplitude due to the increasing values of b_2 in the middle part of the soliton, which gives the appearance of the bright soliton, while there is an increase in the wing parts as opening to both sides. Figure 3(d) is the 2D graphical projection to indicate the impact of the b_4 considering the values as 1, 1.5, 2, 2.5, 3, 3.5, respectively. Soliton remains its W-like soliton structure. While the soliton has the dark soliton structure at $b_4 = 1.1$, it degenerates into the W-like soliton view for $b_4 > 1.1$. In this context, the value of $b_4 = 1.1$ is a critical value according to the investigated situation and the specified parameter selection. In particular, we need to add a few more sentences about the results acquired in this section and the findings that can be considered as an additional contribution to the study. The graphs given in Figure 3, which basically reflect the W-like soliton type, are unique to this form of the equation. In other words, it is not a type of soliton directly called W-like soliton in some studies. Because when the descriptions given in Figure 3 are examined more carefully, it is observed that this is specific to the anti-cubic law with nonlocal form and depending on the values of the parameters b_2 and b_4 coefficients (the coefficients of the cubic and nonlocal nonlinearity terms). Again, this formation does not occur directly as a W-like waveform, but by degenerating from the dark soliton to W-like (dark-bright-dark) soliton.

Figure 4 presents the varied simulations of $\vartheta_4(x,t)$ in Equation (24). 3D and contour projections are given in Figure 4(a), Figure 4(b), respectively. 3D graph indicates the dark soliton for $a = 1, b_2 = 2, b_3 = 0.5, b_4 = 0.5, \rho = 0.5, \delta = 0.5, \theta_0 = 5, \chi = 2, \kappa = -0.5$. Figure 4(c) is 2D soliton form for $t = 1,\ 3,\ 5$. When the wave propagation of the soliton is observed, it is seen that both the amplitude and the dark form remain



(a) $|\vartheta_4(x,t)|^2$ in 3D depiction

(b) $|\vartheta_4(x,t)|^2$ in contour shape

(c) $|\vartheta_4(x,t)|^2$ in 2D views

(d) The effect of b_2 at $t = 3$

(e) The effect of b_3 at $t = 3$

(f) The effect of b_4 at $t = 3$

Figure 4. Various graphs for $\vartheta_4(x,t)$ in the Equation (24) for $a = 1, b_2 = 2, b_3 = 0.5, b_4 = 0.5, \rho = 0.5, \delta = 0.5, \theta_0 = 5, \chi = 2, \kappa = -0.5$. (a) $|\vartheta_4(x,t)|^2$ in 3D depiction. (b) $|\vartheta_4(x,t)|^2$ in contour shape. (c) $|\vartheta_4(x,t)|^2$ in 2D views. (d) The effect of b_2 at $t = 3$. (e) The effect of b_3 at $t = 3$ and (f) The effect of b_4 at $t = 3$.

the same. But, as the value of t is raised, the soliton acts to the right. Figure 4(d) shows impact of the b_2 considering the values as $-2.5, -2, -1.5, 1.5, 2, 2.5$, respectively. Soliton keeps the dark soliton structure for the values $b_2 > 0$ but the bright soliton is obtained for the values $b_2 < 0$. Figure 4(e) is the 2D graphical projection to indicate the impact of the b_3 regarding the values as $-0.75, -0.5, -0.25, 0.25, 0.5$ and 0.75, respectively. Soliton remains its dark soliton structure for $-0.75, -0.5, -0.25, 0.25, 0.5$ and 0.75. Moreover, the soliton amplitude increases if $|b_3|$ increases. When b_3 receives the negative minimum value, the soliton has the original dark soliton form, while b_3 gradually approaches the horizontal axis depending on its increasing values (the dark soliton image degenerates) and when b_3 gets its maximum value ($b_3 = 0.75$), it has both the peak on the horizontal axis and the minimum amplitude. Figure 4(f) express the 2D graphical representations indicating the effect of the b_4 taking the values as $-0.75, -0.5, -0.25, 0.25, 0.5$ and 0.75 respectively. Soliton keeps the dark soliton structure for the values $b_4 > 0$ but the bright soliton is obtained for the values $b_4 < 0$. In Figure 4(f), the soliton amplitude increases as b_4 increases. But, the soliton amplitude increases as b_4 decreases. In this respect, negative or positive values of b_4 result in the bright-dark transition of the soliton.

It should be noted here that the main factors in the selection of the above parameter are as follows. First of all, attention was paid to ensure that there is no conflict with the definitions and limitations of the model and method in the selection of parameters. One of them was to note that the $\vartheta(\zeta)$ expression, which determines the amplitude of the soliton in the transformation given by Equation (3), must be real. In addition, various attempts were carried out to obtain a meaningful soliton type, and the parameter values that occurred when the presented soliton types were obtained are selected

5. Conclusion

In this work, a set of optical soliton solutions by investigating the (1+1)-dimensional NLSE having parabolic and anti-cubic law with a weakly nonlocal nonlinearity have been successfully generated via the new Kudryashov scheme. To our knowledge, the models examined in the article have not been carried out before. The gained results have not been reported in the literature. In addition, unlike the studies in the literature, the effects of the parameters, which are generally included as coefficients in the model, on the soliton dynamics were investigated and reported. For the models utilizing NKM, diverse optical solitons have been gained, such as bright, W-like and dark soliton structures. We observed that NKM is an advantageous and effective tool in deriving solitons that have a main impact on mathematical physics. Moreover, we rely on the results will contribute to the literature in all these aspects. In the future, the generation of fractional, stochastic forms of the presented models and obtaining other types of solitons through various procedures may be the focus of researchers in this field.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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