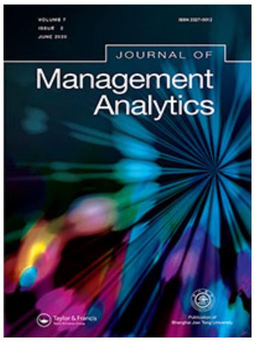

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An in-depth review of theory of the TOPSIS method: An experimental analysis

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Decision-making is an important part of daily and business life for both individuals and organizations. Although the multi-criteria decision-making methods provide decision makers the necessary tools, they have differences in terms of the assumptions and fundamental theory. Hence, selecting the right decision-making method is at least as important as making the decision. TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method, which is one of the most widely used multi-criteria decision-making methods, has gained attention of researchers and thus various improved versions of the method have been proposed. This study considers the conventional TOPSIS method and experimentally displays the underlying reasons of the lacks of the conventional TOPSIS method by using a simulation technique. Detailed experimental analysis based on simulation with an application is used to reveal theoretical fundamentals of the TOPSIS method to better understand it and contribute to its improvement.

Keywords: TOPSIS; Multiple criteria analysis; simulation analysis; Euclidean distance; n-dimensionality

1. Introduction

Multi-criteria decision-making (MCDM) methods provide the necessary tools for decision makers in selecting the most appropriate one among multiple alternatives in case of multiple conflicting criteria. One of the most important properties of MCDM methods is that they reflect the preferences of the decision makers rather than searching for an optimal solution (Wang, Zhu, & Huang, 2017). There are various developed MCDM methods with different characteristics and also different classifications of these methods in the literature, which can be found in Zardari, Ahmed, Shirazi, and Yusop (2015). MCDM methods distinguish from each other in terms of fundamental assumptions, calculation process and complexity. The common criticism for MCDM methods is that they may give different results when they are applied to the same problem (El Amine, Pailhes, & Perry, 2014; Guitouni& Martel, 1998). Since selecting the most appropriate MCDM method can be considered as an MCDM problem, it is important to know all the advantages and disadvantages of these methods quite well in order to make a good decision.

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The TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method which was developed by Hwang and Yoon (1981) is the second most widely used and popular MCDM method after Saaty's (1980) Analytic Hierarchy Process (AHP) (Zyoud& Fuchs-Hanusch, 2017). This is because it is simple and easy to use and can be applied to the problems that consist of a large number of criteria and alternatives (Byun& Lee, 2005; Chitsaz&Banihabib, 2015; Sureeyatanapas, Sriwattananusart, Niyamosoth, Sessomboon, &Arunyanart, 2018; Wang et al., 2017). The calculation steps of the TOPSIS can be summarized as normalizing the decision matrix by applying vector normalization, computing the weighted normalized decision matrix, determining the positive ideal (PIS) solution and negative ideal solution (NIS), calculating the separation or distance of each alternative from PIS and NIS, determining the relative closeness of each alternative to PIS by calculating the ranking index and finally ranking the preference order (Byun& Lee, 2005; Hwang & Yoon, 1981; Kuo, 2017).

Despite its ease of use and widespread application, there are also some criticisms to the TOPSIS method, which also led to the development of different versions of the method. One of the criticisms is the rank reversal phenomenon which is valid for AHP and TOPSIS methods. Rank reversal is related to the change in the ranking of alternatives when a criterion or an alternative is added or dropped (Shin, Lee, Chun, & Chung, 2013). Another important criticism is about the ranking index of the TOPSIS which was first mentioned by Opricovic and Tzeng (2004). The TOPSIS method aims at finding the compromise solution which has the closest distance from PIS and the farthest distance from NIS. The ranking index uses these two distances in the calculation and the relative importance or weights of these two distances are not considered (Kuo, 2017; Opricovic&Tzeng, 2004). Another criticism or disadvantage of the TOPSIS is related to the correlation between criteria (Wang & Wang, 2014; Xu, Zhang, Zhang, &Lv, 2015). Since the TOPSIS method uses Euclidean distance, which does not take into consideration the correlation, the results are affected due to the information overlap.

In this study, it is aimed at experimentally displaying the underlying reason of the lacks of the conventional TOPSIS method which were mentioned above. Especially in n-dimensional space with weights (for $n > 3$), Euclidean distance calculations to be used in ranking index calculation are misleading according to the visual dispersion of data set and thus cause unreasonable results to be obtained. This study presents a detailed analysis to reveal this case explicitly in order to make contribution to the development of both the TOPSIS method and MCDM literature. Organization of the paper is as follows; in section 2, literature review for the developed modified TOPSIS methods and a brief theoretical review of the TOPSIS method are given. Section 3 presents the experimental analysis by using simulation and section 4 presents the discussions on the analysis results. Finally, the conclusions are presented.

2. Literature and theoretical review

Before presenting a theoretical review of the TOPSIS method, a detailed literature review for the different versions of the method will be provided. The mentioned flaws of the TOPSIS method have led many researchers to develop new modifications on the method. Deng, Yeh, and Willis (2000) proposed one of the earliest modified TOPSIS methods which is also the most widely used modified one. They propose to use weighted Euclidean distances in the aggregation rather than using the weighted decision matrix. Shyur (2006), Shyur and Shih (2006), Gangurde and Akarte (2013) and Kasirian and Yusuff (2013) presented different applications of Deng et al. (2000)'s modified approach. Wang and He (2007) indicated that the conventional TOPSIS method is not appropriate for the nominal-the-best type (NTB) problem and proposed a modified TOPSIS method for the mentioned case. Ren, Zhang, Wang, and Sun (2007) developed a modified synthetic evaluation method based on the TOPSIS method in which a different ranking index is used. They use optimized ideal reference point in calculating distance measures and the method overcomes the rank reversal problem. Huang, Wang, and Zhou (2009) modified the TOPSIS method by using Euler distance instead of Euclidean distance. Their global-optimization-based TOPSIS method requires to solve an optimization problem and thus can be too complex for the most applications. Wang (2009) modified the conventional TOPSIS method by proposing new distance measures which contains the calculation of entropy and difference coefficients. Dai and Wang (2011) presented the entropy improved TOPSIS method in which entropy is used to determine the weights which eliminates the lack of subjective weighting. Zhao, Liu, and Yu (2011) proposed an approach which replaces the general distance in conventional TOPSIS with gray relational analysis and also uses gray correlation to display the impact of the preference of the weighted matrix. Li and Ye (2014) developed a modified TOPSIS approach which uses vertical projection method and ranks the alternatives according to the calculated relative distances. Wang, Ji, and Chaudhry (2014) proposed a hybrid MCDM approach to evaluate supermarket food safety factors. The approach was designed under three steps. An index system was used in the first step. Then, in the second step, AHP was used to evaluate supermarket food safety factors. In the third and last step, the TOPSIS method was applied to determine the weaknesses and the strengths of food safety of the systems. Wang and Wang (2014) considered the correlation among the criteria and proposed an improved TOPSIS method which is based on the weighted Mahalanobis distance. Although their approach eliminates the linear correlation among criteria, it is not effective in overcoming nonlinear correlations. Xu et al. (2015) proposed an approach which overcomes the correlation between criteria by using an evaluation index system based on R cluster analysis. Their approach simplifies the similarity measurement and prevents the possibility of an alternative being close to the ideal and the nadir point concurrently by using vertical projection distance. Hu, Du, Mo, Wei, and Deng (2016) considered different centrality measures and develop a weighted TOPSIS method which consists of a dynamic weighting method. They also give an experimental comparison of the proposed method with the classical one and display the effectiveness of their approach. Wang et al. (2017) handled the rank reversal phenomenon and developed an improved TOPSIS method which is based on an experimental design to determine the criteria weights and Chebyshev orthogonal polynomial regression to analyze the attributes. As the authors mentioned that as the number of attributes increases, the complexity and computational efficiency of the method decreases. Kuo (2017) proposed a new ranking index which contains weights for each separation distance or measure determined by the decision maker and can be applied to other modified TOPSIS methods. Sureeyatanapas et al. (2018) presented an extension of the conventional TOPSIS method for the case of uncertain or unavailable criteria weights. They use rank order centroid (ROC) method for determining the criteria weights in which the assessment matrix is expressed as interval values. Huang, Shuai, Sun, Wang, and Antwi (2018) presented the entropy-TOPSIS method in which entropy weight method (EWM) that is based on Shannon entropy (Shannon & Weaver, 1947) is used to calculate the criteria weights. Chen, Shen, and Wang (2018) introduced an improved TOPSIS method in which the ideal solution is defined as the expected level. Their method improves the traditional TOPSIS method from the external set ideal solution which is included in calculations to improve the evaluation of lateral comparability. dos Santos, Godoy, and Campos (2019) proposed an integrated entropy-based fuzzy TOPSIS method. They integrated fuzzy approach, Shannon's entropy and the TOPSIS method. The authors emphasized that the integrated method is especially designed to overcome the uncertainty in the decision-making process. A spherical fuzzy TOPSIS method was proposed by KutluGündoğdu and Kahraman (2019). In the proposed approach, the spherical fuzzy TOPSIS method, by using three-dimensional spherical fuzzy sets, was used. The interval-valued hesitant Pythagorean fuzzy sets was used to deal with the uncertainty of decision-making process by Wang, Wang, Xu, and Ren (2019). TOPSIS and Choquet integral-based methods were also applied with the assistance of the interval-valued hesitant Pythagorean fuzzy sets. Yatsalo, Korobov, Öztayşi, Kahraman, and Martínez (2020) proposed a new fuzzy TOPSIS approach by using fuzzy criteria values and fuzzy weight coefficients in all calculation steps of the TOPSIS method with the use of fuzzy arithmetic operators. The proposed approach also provides confidence measures for the ranking of the alternatives. Garg and Kumar (2020) presented new exponential distance measures for the basis of TOPSIS. The interval-valued intuitionistic fuzzy sets and set pair analysis theory were also used within the solution process of the proposed approach.

In addition to these modified TOPSIS methods, other different fuzzy versions of the method were also developed. Since only the crisp method is considered in this study, the fuzzy versions are kept out of the scope of the study. The interested readers can look at the studies of Dalalah, Hayajneh, and Batieha (2011), Vahdani, Mousavi, and Tavakkoli-Moghaddam (2011), Chen and Lu (2015), Liu, You, Shan, and Shao (2015), Zhou, Liu, and Chang (2016), Büyüközkan and Güler (2017), Wu, Liu, and Liu (2018), and Shen, Ma, Li, Xu, and Cai (2018). A detailed survey study on fuzzy TOPSIS studies between 2007 and 2017 was published by Salih et al. in 2019.

TOPSIS is a utility-based method and its fundamental concept is related to the distance which is calculated from the NIS and the PIS. The method calculates the distances by using the n–dimensional Euclidean distance according to the number of the criteria of the problem. The calculation of the distance of alternative $i(D_i^+)$from the positive ideal solution (f_j^+) is shown in Equation (1).

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (f_{ij} - f_j^+)^2} \tag{1}$$

where f_{ij} is the decision matrix value for the jth criterion of the ith alternative. Similarly, the calculation of the distance of alternative $i(D_i^-)$from the negative ideal

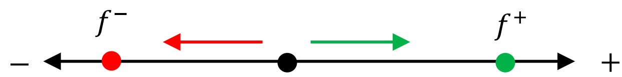

Figure 1. One-dimensional representation.

solution (f_j^-) is shown in Equation (2).

$$D_i^- = \sqrt{\sum_{j=1}^{n} (f_{ij} - f_j^-)^2} \tag{2}$$

Finally, the calculation for final ranking by using Equations (1) and (2) is shown in Equation (3).

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{3}$$

where C_i is between 0 and 1. According to the technique, the best result is the one with the highest C_i value and the worst result is the one with the lowest C_i value. This means that the alternative, which is distant from the negative ideal solution, becomes the best result. The main paradox of the technique starts from this proposition. Because, this is true only if the calculations are done in one dimension which is shown in Figure 1. If the calculations are done in more than one dimension, the alternative can become distant from the negative ideal solution and become close not only to the positive ideal solution but also to anywhere else. This can be seen a bit more clearly in two-dimensional calculations (Figure 2),

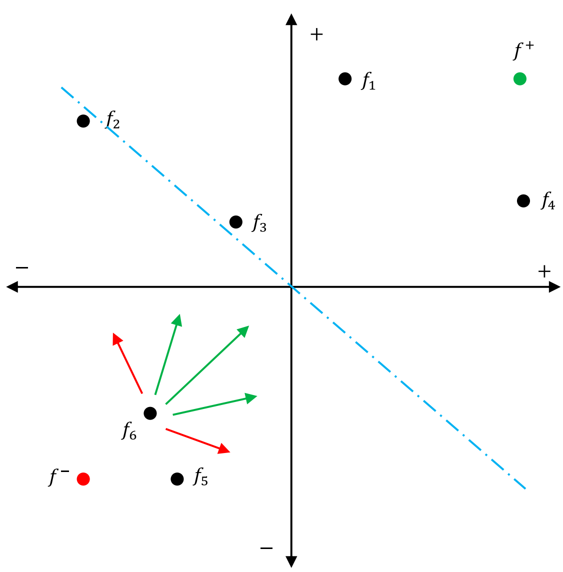

Figure 2. Two-dimensional Representation.

but when the calculations are done in more than three dimensions, it is almost impossible. Here, movement statements are used as representative to explain the paradox better. Normally, MCDM data are constant.

In Figure 1, it is clearly seen that the alternative will be the best alternative if it is distant from the negative ideal solution. Because, it has no alternative directions to head or move. In Figure 2, f_6 can move to any direction while becoming distant from the negative ideal solution. In this case, this movement does not mean that f_6 becomes adjacent to the positive ideal solution. And also, if f_6 becomes more distant from the negative ideal solution than the positive ideal solution, this shows it as if it is better than the previous situation due to the structure of the formula, Equation (3). In some cases, although it is a bad solution, its ranking can be better than most of the better solutions.

Blue line in Figure 2 is vertical to the line from the negative ideal solution to the positive ideal solution. The ranking of f_6 cannot change only if it moves parallel through the blue line due to Equation (3). But, if it moves vertically, its ranking is changed.

According to the Euclidean distance formulation, Equation (3) and the previous explanations, the TOPSIS technique gives the most correct ranking only if data spread linearly around the line from the negative ideal solution to the positive ideal solution, which is shown in Figure 3. The cause of this situation is that the spread of data approximates to one dimension. Consequently, Equations (1)–(3) work better in the ranking of data.

Contrariwise, if data spread around a vertical linear to the line from the negative ideal solution to the positive ideal solution, which is shown in Figure 4, the TOPSIS technique gives the least correct ranking. And also, the cause of this situation is that the spread of data moves away from one dimension to two dimension and thus Equations (1)–(3) do not work well in ranking.

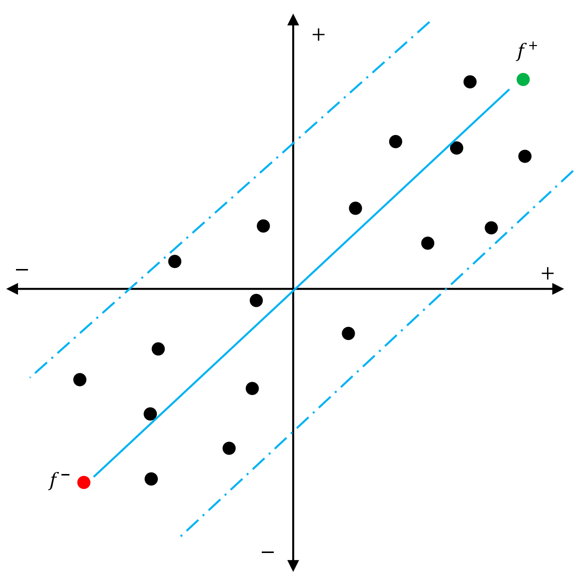

Figure 3. Data spread over the main diagonal horizontally.

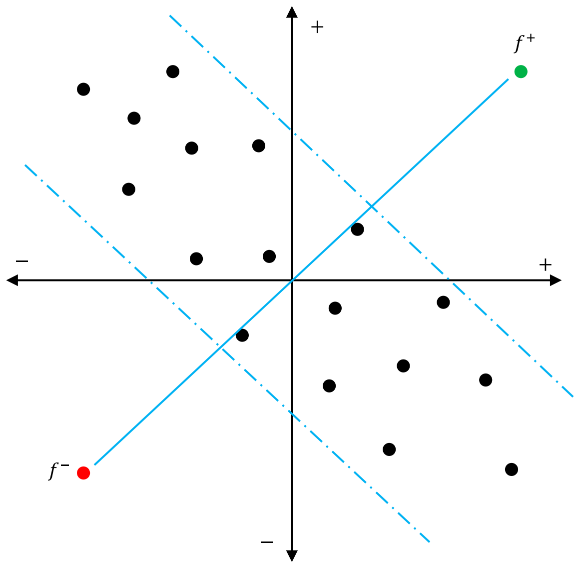

Figure 4. Data spread over the main diagonal vertically.

3. Simulation-based experimental analysis

This section is allocated to examine the TOPSIS technique with random data to observe the technique and its flaws in a better way. Grade Point Average (GPA) calculations were applied with random grades of 20 students for 10 courses and for 1000 simulation runs. In numerical analysis data sets, all of the grades of students and credits of courses were taken randomly. At the same time, in order to make comparisons among the other techniques, same random data sets were applied to AHP (Analytical Hierarchy Process) (Saaty, 1980), VIKOR (Serbian name: VlseKriterijumska Optimizacija I Kompromisno Resenje) (Opricovic, 1998) and MOORA (Multi-Objective Optimization on the Basis of Ratio Analysis) (Brauers & Zavadskas, 2006) techniques. Numerical simulations were conducted under the following three sub-sections and the results were also reported as tabular forms in detail.

In the tables of this section, "± 2" and "± 1" represent the difference between the real ranking and MCDM technique ranking. For example, "± 2" means that the ranking of techniques can be between 4 and 8 if the ranking of the real GPA calculation is 6. Percentages in the second columns represent the correct ranking rate under the conditions of these differences ("+2", "+1" or no difference). Percentage columns under the technique names represent the percentage of simulations which satisfy the least conditions on the line they are connected to. All of the results are given in detail as much as possible in various tables to observe the important points and deficiencies of the techniques.

3.1. *Random data*

In this sub-section, random grades of 20 students for 10 courses were generated without any limitations.

Table 1. Comparison of all of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±2	whose rate is more than 80%	99.9	99.7	99.9	99.3
	whose rate is more than 90%	99.3	91.3	98.7	91.4
	whose rate is 100%	77.5	53.4	81.8	49.2
The difference from the GPA ranking results is ±1	whose rate is more than 80%	95.0	79.1	96.0	79.7
	whose rate is more than 90%	74.6	48.5	82.1	49.3
	whose rate is 100%	31.9	12.1	38.8	11.5
No difference from the GPA ranking results	whose rate is more than 80%	21.3	5.7	25.3	5.9
	whose rate is more than 90%	5.4	1.1	7.1	1.3
	whose rate is 100%	0.7	0.1	0.1	0.3

Comparisons of all 20 students' ranking results are given in Table 1. If the ranking difference is kept flexible and the rate of correct ranking is kept low, numerical simulation results are the highest for all techniques and almost equal to 100%. However, when the ranking difference is kept lower and the rate of correct ranking is kept higher, numerical simulation results decrease faster for TOPSIS and VIKOR. Under this case, there is no difference from the GPA ranking results and the rate of correct ranking is 80%, and AHP and MOORA results are 4 times better than the TOPSIS and VIKOR results. This means that at least 80% percent of the students' (16 of 20 students) ranking results are exactly same with the real GPS calculation ranking results. Same with these cases, all of the techniques' correct ranking success percentages given under the technique names in all tables decrease inversely proportional to the rate of correct ranking under the second column.

In Table 2, comparisons of only the first three ranking results for the techniques are given. According to the results in Table 2, success percentage of the TOPSIS for the first three ranking is lower than that of the others, even with the higher ranking difference and lower rate of correct ranking. Although success percentage of VIKOR is lower, it is almost 2 times better than that of the TOPSIS. The other techniques, AHP and MOORA are also almost 3 times higher than the TOPSIS for the best condition, which means no difference from the GPA ranking results and the rate of correct ranking is 100%.

In Table 3, comparisons of only the last three ranking results of the techniques are given. Results of Table 3 are similar to those of Table 2. Success percentage of the TOPSIS for the last three rankings is lower than that of the others, even with the higher ranking difference and lower correct ranking. Although success percentage of VIKOR is lower, it is 2 times better than the success percentage of the TOPSIS.

Table 2. Comparison of the first three of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±1	whose rate is more than 66.6%	99.9	98.6	99.8	91.3
	whose rate is 100%	87.6	79.4	88.6	59.3
No difference from the GPA ranking results	whose rate is more than 66.6%	85.4	72.9	88.5	50.1
	whose rate is 100%	64.6	46.4	69.9	23.7

AHP and MOORA have also almost 3 times higher success percentage than that of the TOPSIS for no difference from the GPA ranking results and rate of 100%.

Comparisons of the middle 50% (between 5 and 16) of the ranking results are given in Table 4 where the TOPSIS has more errors than the others. All of the techniques' correct ranking success percentages have decreased in Table 4, but the correct ranking success percentages of the TOPSIS have decreased much more than those of the others. Success percentages of AHP, VIKOR and MOORA techniques which are more than 80% in all simulations are 3.5 times better than success percentage of the TOPSIS. If the results are observed for 100%, AHP is 18 times better, VIKOR is 10 times better and MOORA is 9 times better than the success percentage of the TOPSIS. For the middle 50% of data, it also means that the ranking results of the TOPSIS can be wrong with 75.4%, 89.9% and 97.7%, respectively.

If these techniques are used in order to select the best alternative, important results are only the first and the last alternatives. Because of this, comparisons of the first and the last alternatives of ranking results are given in Table 5. According to the aim of these solutions, only comparison results of rankings with no difference from the real

Table 3. Comparison of the last three of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±1	whose rate is more than 66.6%	99.7	99.2	98.6	92.3
	whose rate is 100%	84.7	82.1	86.1	58.9
No difference from the GPA ranking results	whose rate is more than 66.6%	82.3	79.1	86.1	53.6
	whose rate is 100%	57.9	55.4	60.4	21.2

Table 4. Comparison of the middle 50% of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±1	whose rate is more than 80%	88.8	83.3	90.1	24.6
	whose rate is more than 90%	70.3	47.1	44.7	10.1
	whose rate is 100%	41.0	22.5	19.4	2.3

GPA ranking are given. Success percentages of AHP, VIKOR, MOORA and TOPSIS are 91.1%, 87.9%, 92.7% and 78.8%, respectively. Table 5 indicates that the best alternative results are correct with these percentages. At the same time, these mean that the best alternative results of the techniques are wrong with 8.9%, 12.1%, 7.3% and 21.2%, respectively. If this selection and decision have serious and high impact result, 21.1% is a really high error rate.

3.2. *Horizontally random data*

In this sub-section, random grades of 20 students for 10 courses were generated around the line which is from the negative ideal solution to the positive ideal solution.

In Table 6, comparisons of all 20 students' ranking results are given. In these cases, all of the techniques' correct ranking success percentages have increased even they are equal to 80% with ±2 in Table 6 with respect to Table 1, but the correct ranking success percentages of the TOPSIS have decreased. In all cases in Table 6, the correct ranking success percentages of AHP and MOORA results are still better than the TOPSIS and VIKOR results. Same with these cases, all of the techniques' correct ranking success percentages under the technique names in all tables decrease below inversely proportional to the rate of correct ranking under the second column.

In Table 7, comparisons of only the first three ranking results of the techniques with horizontally random data are given. In Table 7, success percentage of the TOPSIS for the first three ranking is higher than random data results in Table 2, but still lower than the others in Table 7, even with the higher ranking difference and lower correct ranking. According to Table 7, all percentages of techniques have increased. The

Table 5. Comparison of the first and last alternatives of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
No difference from the GPA ranking results	the First alternative	91.1	87.9	92.7	78.8
	the Last alternative	90.7	87.3	92.3	74.5

Table 6. Comparison of all of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±2	whose rate is more than 80%	100.0	100.0	100.0	97.1
	whose rate is more than 90%	100.0	98.7	100.0	87.0
	whose rate is 100%	96.7	62.5	95.2	19.1
The difference from the GPA ranking results is ±1	whose rate is more than 80%	100.0	95.4	99.9	66.7
	whose rate is more than 90%	97.9	69.7	97.4	25.1
	whose rate is 100%	68.7	19.3	65.4	10.9
No difference from the GPA ranking results	whose rate is more than 80%	68.5	12.7	70.0	0.1
	whose rate is more than 90%	30.1	3.1	28.8	0
	whose rate is 100%	4.3	0.9	2.5	0

reason of this increase is horizontal data which spread around the line from the negative ideal solution to the positive ideal solution.

In Table 8, comparisons of only the last three ranking results of the techniques with horizontally random data are given. Results of Table 8 are similar to those of Tables 3

Table 7. Comparison of the first three of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±1	whose rate is more than 66.6%	100	99.5	100.0	94.7
	whose rate is 100%	85.8	75.2	83.3	39.8
No difference from the GPA ranking results	whose rate is more than 66.6%	97.4	72.1	94.9	50.6
	whose rate is 100%	81.5	32.6	79.6	15.2

Table 8. Comparison of the last three of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±1	whose rate is more than 66.6%	99.8	99.1	99.3	90.8
	whose rate is 100%	87.8	83.2	87.5	54.0
No difference from the GPA ranking results	whose rate is more than 66.6%	89.6	84.3	86.0	41.1
	whose rate is 100%	59.7	52.7	53.1	13.4

and 7. Success percentage of the TOPSIS for the last three rankings is lower than that of the others, even with the higher ranking difference and lower correct ranking. Although the success percentage of VIKOR is lower, it is almost equal to 4 times of success percentage of the TOPSIS. The other techniques, AHP and MOORA, are 3 times better than the success percentage of the TOPSIS for no difference from the GPA ranking results and rate of 100%.

Comparisons of the middle 50% (between 5 and 16) of the ranking results of horizontally random data are given in Table 9 where the TOPSIS has more errors than the others. According to Table 4, all of the techniques' correct ranking success percentages increased and the correct ranking success percentages of the TOPSIS increased 4, 5 and 10 times from top to the bottom of the TOPSIS column of the table, respectively. This comparison means that the TOPSIS results in Table 4 can be wrong with 75.4%, 89.9% and 97.7%, respectively, but the TOPSIS results in Table 9 can be correct with 82.9%, 59.4% and 27.6%, respectively. Therefore, we can say that correct ranking results of the TOPSIS are too flexible according to the spread of data. Because of this, we can never say something definite about the results and correctness of the TOPSIS.

As it was mentioned before, if these techniques are used for the selection of the best alternative, important results are only the first and the last alternatives. Because of this,

Table 9. Comparison of the middle 50% of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±1	whose rate is more than 80%	99.9	91.5	98.9	82.9
	whose rate is more than 90%	96.5	69.4	93.5	59.4
	whose rate is 100%	73.1	36.5	69.7	27.6

Table 10. Comparison of the first and last alternatives of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
No difference from the GPA ranking results	the First alternative	99.4	95.1	99.5	91.8
	the Last alternative	88.1	87.0	87.2	65.6

comparison of the first and the last alternatives of ranking results of horizontally random data is given in Table 10. According to the aim of this kind of solutions, only comparison results of rankings with no difference from the real GPA ranking are given. Success percentages of AHP, VIKOR, MOORA and TOPSIS are 99.4%, 95.1%, 99.5% and 91.8%, respectively. It is clear with Table 10 that the best alternative results are correct with these percentages. At the same time, this means that the best alternative results of techniques are wrong with 0.6%, 4.9%, 0.5% and 8.2%, respectively. 8.2% may not be a high error rate; however, it is really high according to the results of other techniques, especially if this selection and decision have serious and high impact as a result.

3.3. *Vertically random data*

In this sub-section, random grades of 20 students for 10 courses were generated around the vertical linear line to the line from the negative ideal solution to the positive ideal solution.

In Table 11, comparisons of all 20 students' ranking results are given. In this case, all of the techniques' correct ranking success percentages have decreased even they are equal to 80% with ± 2 in Table 11, and the correct ranking success percentages of the TOPSIS have decreased much more than the others. Most of the correct ranking success percentages of the TOPSIS are equal to 0% which means that there is no correct ranking among the 1000 simulation runs. Same with these cases, all of the techniques' correct ranking success percentages under the technique names in all tables are decreasing downwards of the columns inversely proportional to the rate of correct ranking under the second column, like the first two sub-sections.

In Table 12, comparisons of only the first three ranking results of the techniques with vertically random data are given. In Table 12, all of the techniques' correct ranking success percentages have decreased even they are equal to 66.6% with ± 1 in Table 12, and the correct ranking success percentages of the TOPSIS have decreased much more than the others. Decreases of correct ranking success percentages of AHP, VIKOR and MOORA are lower than 2% which are not important differences according to the spread of data set. The decrease of correct ranking success percentages of the TOPSIS is about 30% which is a really high difference. When the data spread change is considered, the decreases of the correct ranking success percentages are normal, excluding the TOPSIS. The reason of this decrease of the correct ranking success percentages of the TOPSIS is related to the theory of the TOPSIS, as it is explained in Section 2.

In Table 13, comparisons of only the last three ranking results of the techniques with vertically random data are given. The results of Table 13 are similar to those of Tables 3, 8 and 12. Success percentage of the TOPSIS for the last three rankings

Table 11. Comparison of all of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±2	whose rate is more than 80%	88.7	74.2	94.9	26.8
	whose rate is more than 90%	58.0	42.1	74.6	6.0
	whose rate is 100%	13.9	7.4	23.2	0.1
The difference from the GPA ranking results is ±1	whose rate is more than 80%	31.4	23.1	48.7	0.9
	whose rate is more than 90%	7.7	6.5	17.2	0.1
	whose rate is 100%	0.6	0.2	1.9	0
No difference from the GPA ranking results	whose rate is more than 80%	0	0.1	0.8	0
	whose rate is more than 90%	0	0	0	0
	whose rate is 100%	0	0	0	0

is lower than that of the others, even with the higher ranking difference and lower correct ranking. Although the success percentage of VIKOR is lower, but it is almost equal to 2 times of success percentage of the TOPSIS. AHP and MOORA

Table 12. Comparison of the first three of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ±1	whose rate is more than 66.6%	98.7	95.9	98.4	68.2
	whose rate is 100%	77.9	69.4	78.5	29.8
No difference from the GPA ranking results	whose rate is more than 66.6%	68.3	58.4	75.4	30.9
	whose rate is 100%	42.2	30.1	48.8	7.7

Table 13. Comparison of the last three of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ± 1	whose rate is more than 66.6%	94.5	91.2	95.4	85.6
	whose rate is 100%	63.5	60.2	69.5	46.8
No difference from the GPA ranking results	whose rate is more than 66.6%	67.0	56.4	74.2	47.4
	whose rate is 100%	35.9	25.4	42.3	15.0

are much better than 2 times of success percentage of the TOPSIS for no difference from the GPA ranking results and rate of 100%.

Comparisons of the middle 50% (between 5 and 16) of the ranking results of vertically random data are given in Table 14 where the TOPSIS has more error than the others, as in Tables 4 and 9. In Table 14, all of the techniques' correct ranking success percentages have decreased according to Tables 4 and 9, and the correct ranking success percentages of the TOPSIS has decreased much more than others, which is almost 0%. This comparison means that the TOPSIS results in Table 14 can be wrong with 97.3%, 99.5% and 100%, respectively; the TOPSIS results in Table 4 can be wrong with 75.4%, 89.9% and 97.7%, respectively; and the TOPSIS results in Table 9 can be correct with 82.9%, 59.4% and 27.6%, respectively. Therefore, we can say that correct ranking results of the TOPSIS are too flexible according to the spread of data. Because of this, we can never say something definite about the results and correctness of the TOPSIS. Especially, if the data spread around the vertical linear line to the line from the negative ideal solution to the positive ideal solution, middle 50% data of ranking results of the TOPSIS are almost certainly wrong.

As it was mentioned before, if these techniques are chosen for the selection of the best alternative, only the first and the last alternatives have important results. Because

Table 14. Comparison of the middle 50% of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
The difference from the GPA ranking results is ± 1	whose rate is more than 80%	20.7	19.7	34.2	2.7
	whose rate is more than 90%	9.0	8.3	16.4	0.5
	whose rate is 100%	2.4	2.1	5.9	0

Table 15. Comparison of the first and last alternatives of the ranking results.

Result comparison description		AHP (%)	VIKOR (%)	MOORA (%)	TOPSIS (%)
No difference from the GPA ranking results	the First alternative	86.1	83.2	88.9	61.0
	the Last alternative	78.8	76.1	81.7	71.7

of this, comparison of the first and the last alternatives of ranking results of vertically random data is given in Table 15. According to the aim of this kind of solutions, only comparison results of rankings with no difference from the real GPA ranking are given. Success percentages of AHP, VIKOR, MOORA and TOPSIS are 86.1%, 83.2%, 88.9% and 61.0%, respectively. It is clear with Table 15 that the best alternative results are highly correct with these percentages excluding the TOPSIS. At the same time, these results mean that the best alternative results of techniques are wrong with 13.9%, 16.8%, 11.1% and 39.0%, respectively. 39.0% is a really bad selection rate, especially, if this selection and decision have serious and high impact as a result.

4. Discussion

This study aims at demonstrating the basis of the criticisms about the theory of the TOPSIS method in the literature. According to the criticisms in the literature, there are three main flaws of the TOPSIS method as listed below.

- The rank reversal phenomenon
- The ranking index
- The correlation between criteria

Actually, all of these criticisms are based on the same reason. The TOPSIS method supposes that all of the MCDM problems satisfy the conditions of Euclidean space and the conditions of Euclidean distance indirectly. There are two well-known instances of Euclidean space related to the properties of triangles: “the sum of the interior angles of a triangle is 180^{o}” and according to Pythagoras’ theorem “the squared length of the hypotenuse of a right-angled triangle is the sum of the squared lengths of the other two sides” (Campbell et al., 2009). Considering these definitions, MCDM problems where the TOPSIS method is applied should satisfy these conditions. Otherwise, these applications may give results with incorrect ranking and incorrect best alternative. Besides, MCDM problems never consist of only decision matrices. There should be weights that need to be assigned to the criteria according to their importance. However, as Brander and Sinclair (1996) emphasize for *k*–shortest path algorithms, the weights, which are applied to the edges, are not necessarily related to the distance only, so may not follow these rules. These explanations and theories about the Euclidean space and Euclidean distance are directly related to all criticisms about the TOPSIS and are also good expressions of the basis of the criticisms.

Moreover, with the basis of these theories, as it is explained in Section 2 with Figures 1 and 2, an alternative can move farther and faster from PIS while moving farther from the NIS. This explains the first and the second criticisms. The TOPSIS method assumes that if an alternative is far from NIS, it is also close to PIS. However, there is no two-sided relation in this case, especially for the distances in n-dimensional space. An alternative can be far from both NIS and PIS. Even, an outlier alternative can be resulted as better than an alternative which is much closer to the PIS, especially because of the combination of Equation (3) and Euclidean distance in n-dimensional space. Besides, "real" NIS (or the worst alternative) can be resulted as the best alternative if NIS is selected from the opposite direction of the "real" NIS. This is caused by the use of Equation (3) and Euclidean distance in n-dimensional space and can be observed in Section 3 with performed simulations, especially by assessing the differences among the results given in Sections 3.2 and 3.3. This explains how and why the rank reversal phenomenon and the ranking index flaw occur wherefore using Euclidean distances.

If all these details of the TOPSIS method are interpreted together with the theoretical review in Section 2 and numerical simulation results in Section 3, misinterpretation of the TOPSIS method as an MCDM method can be seen clearly. According to the theoretical background and the hypothesis of the method, the TOPSIS method can never guarantee to obtain higher accuracy rates. Because, in order to obtain higher accuracy rates, before applying the TOPSIS method, it has to be checked if it satisfies the necessary conditions of the underlying assumptions. This can be observed especially with cross-checks of the tables in Section 3.2 and Section 3.3. With random data, the accuracy rate of the TOPSIS method for the best alternative is 78.8%. As in Section 3.2, when the data set is chosen randomly and horizontally around the line from NIS to PIS, the accuracy rate of the TOPSIS method for the best alternative increases to 91.8% which is still lower than that of the other methods. Contrariwise, as in Section 3.3, when the data set is chosen randomly around the vertical line from NIS to PIS, the accuracy rate of the TOPSIS method for the best alternative decreases to 61.0% which is much lower than that of the other methods. In a simple way, the main reason of these differences can be seen from visual dispersion of the data set. The data set spreads around the vertical line from NIS to PIS approximates to one-dimensional space more than the other data sets. Because of these, it can be applied with Euclidean distances which can result in high accuracy rates. On the contrary, the data set that is spread over a large space or around the horizontal line from NIS to PIS (this data set also spreads over a large space because of NIS and PIS as shown in Figure 4) becomes distant from one-dimensional space. Thus, it cannot be applied with Euclidean distance calculations and assumptions. Eventually, it results in much lower accuracy rates.

5. Conclusion

This study considers the conventional TOPSIS method and experimentally displays the underlying reasons of the lacks of the conventional TOPSIS method by using a simulation technique. Firstly, a detailed literature review about the criticisms about the TOPSIS method is given. Then, a brief theoretical review of the TOPSIS method is explained and analyzed with graphical representations. Detailed experimental analysis based on simulation with an application to GPA calculations is used to clarify the criticisms about the TOPSIS method. The simulation experiments are applied with three different sets of data in different ways to observe and demonstrate the flaws clearly.

First of all, results of this study show that results of the TOPSIS method are significantly affected by the spread of the data set. The difference in the accuracy rate percentages for the best alternative with the TOPSIS method between Tables 10 and 15 is 30.8%. This percentage means that the accuracy rate of the best result decreases by 30.8% when the spread of the data set is changed in another way. Statistically and numerically, it is a really high inaccuracy rate for the possible wrong decisions. On the other hand, the basis of the high inaccuracy rates of the TOPSIS method and criticisms about its theory should not be underestimated. The main findings about the basis of the criticisms about the TOPSIS method identified in this study with the literature review and numerical simulations can be listed as below.

- Euclidean space assumptions
- Euclidean distance calculations
- Ranking index

Without fixing these problems as mentioned in the relevant sections, using the TOPSIS method for MCDM problems may cause quite wrong decisions. At this point, it should also be expressed that it is not intended to just criticize or discredit the TOPSIS method. This study should be assessed as a different approach for making contribution to both the improvement of the TOPSIS method and MCDM literature. It can also be concluded that the experimental detailed analysis approach presented in this study should be applied to both the improved TOPSIS methods and the other MCDM methods in order to justify them. In this way, it will be possible to both understand the MCDM methods better and to strengthen their theoretical fundamentals, which can be considered as promising future research alternatives.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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