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New optical solitons for Biswas–Arshed equation with higher order dispersions and full nonlinearity

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ABSTRACT

In this paper, the extended Jacobi's elliptic function approach is used to solve the Biswas–Arshed equation in two different types. This method reveals several optical solitons including traveling wave solutions. The found solutions are identified bright, dark, singular optical solitons and Jacobi elliptic function solutions. Reliability of the process is presented with graphical consequence of derived solutions.

1. Introduction

Recently, many researchers established various methods to construct optical solutions in the field of nonlinear optics because of optical solitons shape the fundamental component to transport data from side to side the earth for very wide distances. These methods are the functional variable method, the Kudryashov method, the trial solution method, the Jacobi elliptic function expansion method, the sine–cosine method, the Exp-function method, G'/G-expansion method and others [1–11]. There is a countless of style that effectively define this movement of soliton construction [12–28].

In this article, we investigate the Biswas–Arshed equation (BAE) [15] to construct optical solitons including traveling wave solutions using the extended Jacobi's elliptic function approach method. The found solutions are identified bright optical soliton, dark soliton, singular soliton and traveling wave solutions. Recently, a style of Jacobi elliptic function process, which is ordinary and effective, was analyzed to construct periodic wave solutions for certain nonlinear equations [19–24]. Biswas et al. [19] obtained singular and dark optical soliton solutions for the LPD equation with Kerr law nonlinearity via the method presented in this work and $\exp(-\Phi(\phi))$-expansion method, Yan et al. [20] obtained three classes of analytic periodic wave solutions expressed in terms of the Jacobi's elliptic functions including cn ; sn and dn functions, Inc [21], established exact special solutions for the nonlinearly dispersive Boussinesq equations by using extended $sn - cn$ method, Zayed and Elshater [22] analyzed the four higher-order nonlinear Schrodinger equations using two mathematical methods, Yomba [23] introduced the F-function method to solve the Zakharov-Kuznetsov equation with power law nonlinearity and nonlinear dispersion along with time-dependent coefficients, Guo et al. [24] derived the

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exact dark, bright soliton solutions, the traveling wave solutions and Gaussian solutions of the nonlinear Schrödinger examined model.

The BAE with full nonlinearity is presented by [15,16]:

$$\mathrm{i}q_t + \varepsilon_1 q_{xx} + \varepsilon_2 q_{xt} + i(\sigma_1 q_{xxx} + \sigma_2 q_{xxt} - \lambda(|q|^{2n}q)_x + \mu(|q|^{2n})_x q + \theta|q|^{2n}q_x) = 0, \tag{1.1}$$

where n is full nonlinearity criterion. When $n = 1$, Eq. (1.1) convert to BAE with higher order dispersals and lack of self-phase modulation (SPM) and this model is presented via [15,16]:

$$\mathrm{i}q_t + \varepsilon_1 q_{xx} + \varepsilon_2 q_{xt} + i(\sigma_1 q_{xxx} + \sigma_2 q_{xxt} - \lambda(|q|^2 q)_x + \mu(|q|^2)_x q + \theta|q|^2 q_x) = 0. \tag{1.2}$$

Recently, these equations are found out by Biswas and Arshed. These are new ideas for deal with certain problems. These models were offered in 2018 with two nonlinear types containing the Kerr and power law cases. In addition to they studied soliton actives for Biswas–Arshed type with the same forms by the assist of extended trial function method.

The first designate describes temporal development when ε_1 and ε_2 are the coefficients of GVD and spatio-temporal dispersal (STD) in Eq. (1.2). Besides, σ_1 and σ_2 constitute third order dispersal (3OD) and third order STD. These dispersal expressions will adapt the reduced number for GVD. Besides, with lack of SPM, the nonlinearity event stays from the values of λ, μ and θ that result from self-steepening influence and nonlinear dispersals respectively. Then mentioned compensating influences of dispersal and nonlinearity contribute the essential adjust to keep up soliton spread.

2. Traveling wave hypothesis

2.1. The BAE with full nonlinearity

We consider the given below traveling wave transformation [11]

$$q(x, t) = u(\phi)e^{i(-\kappa x + \vartheta t + \theta_0)}, \quad \phi = x - Qt, \tag{2.1}$$

where κ, ϑ, Q and θ_0, respectively, describe the frequency, wave number, the speed of the wave and phase constant.

By placing Eq. (2.1) into Eq. (1.1), are obtained real and imaginary sections [11].

The real section is as follows:

$$\begin{aligned}&(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2 - 2Q\kappa\sigma_2)u'' + (-\vartheta - \kappa^2\varepsilon_1 + \vartheta\kappa\varepsilon_2 - \kappa^3\sigma_1\\ &\quad + \vartheta\kappa^2\sigma_2)u - \kappa(\theta + \lambda)u^{2n+1} = 0,\end{aligned} \tag{2.2}$$

and by integrating with respect to ϕ once of imaginary sections and by integration constant neglect gives:

$$\begin{aligned}&(\sigma_1 - \sigma_2 Q)u'' + (\vartheta\varepsilon_2 + Q\kappa\varepsilon_2 - Q - 2\kappa\varepsilon_1 - 3\kappa^2\sigma_1 + 2\vartheta\kappa\sigma_2 + Q\kappa^2\sigma_2)u\\ &\quad - \left(\frac{\theta + (2n+1)(\lambda + \mu)}{2n+1}\right)u^{2n+1} = 0.\end{aligned} \tag{2.3}$$

In Eqs. (2.2) and (2.3), by applying $u = r^{\frac{1}{2n}}$ transformation we obtain that the real section;

$$\begin{aligned}&2n(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2 - 2Q\kappa\sigma_2)rr'' - (-1 + 2n)(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2\\ &\quad - 2Q\kappa\sigma_2)(r')^2 + 4n^2(-\vartheta - \kappa^2\varepsilon_1 + \vartheta\kappa\varepsilon_2 - \kappa^3\sigma_1 + \vartheta\kappa^2\sigma_2)r^2 - 4n^2\kappa(\theta + \lambda)r^3 = 0,\end{aligned} \tag{2.4}$$

and by integrating with respect to ϕ once of imaginary sections and by integration constant neglect gives:

$$\begin{aligned}&2n(1 + 2n)(\sigma_1 - Q\sigma_2)rr'' + (-1 + 4n^2)(\sigma_1 - Q\sigma_2)(r')^2 + 4n^2(-Q - 2nQ - \theta\\ &\quad - 2\kappa(1 + 2n)\varepsilon_1 + (1 + 2n)((\vartheta + Q\kappa)\varepsilon_2 - 3\kappa^2\sigma_1 + 2\vartheta\kappa\sigma_2 + Q\kappa^2\sigma_2))r^2\\ &\quad - 4n^2(\theta + 2(n + 1)\lambda + (2n + 1)\mu)r^3 = 0.\end{aligned} \tag{2.5}$$

Eqs. (2.4) and (2.5) leads to:

$$
\begin{aligned}
&\frac{2n(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2 - 2Q\kappa\sigma_2)}{2n(1+2n)(\sigma_1 - Q\sigma_2)}\\
&= \frac{-(-1+2n)(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2 - 2Q\kappa\sigma_2)}{(-1+4n^2)(\sigma_1 - Q\sigma_2)}\\
&= \frac{4n^2(-\vartheta - \kappa^2\varepsilon_1 + \vartheta\kappa\varepsilon_2 - \kappa^3\sigma_1 + \vartheta\kappa^2\sigma_2)}{(1+2n)((\vartheta + Q\kappa)\varepsilon_2 - 3\kappa^2\sigma_1 + 2\vartheta\kappa\sigma_2 + Q\kappa^2\sigma_2))}\\
&= \frac{-4n^2\kappa(\theta+\lambda)}{-4n^2(\theta + 2(n+1)\lambda + (2n+1)\mu)}.
\end{aligned} \tag{2.6}
$$

From Eq. (2.6), we can write,

$$
\begin{aligned}
\mu &= \frac{1}{(1+2n)(-\vartheta - \kappa^2\varepsilon_1 + \vartheta\kappa\varepsilon_2 - \kappa^3\sigma_1 + \vartheta\kappa^2\sigma_2)}((\vartheta\theta - (2n+1)Q\theta\kappa\\
&\quad - \theta^2\kappa + 2(2n+1)\vartheta\lambda - (2n+1)Q\kappa\lambda - \theta\kappa\lambda - \kappa^2(2n\lambda + \theta(4n+1))\varepsilon_1\\
&\quad + \kappa(-\vartheta\lambda + (\theta+\lambda)Q\kappa + 2(\vartheta\theta + (\theta+\lambda)\mathrm{n}Q\kappa))\varepsilon_2 - 2\theta\kappa^3\sigma_1(1+3n)\\
&\quad - \kappa^3\lambda\sigma_1(1+4n) + \vartheta\theta\kappa^2\sigma_2(1+4n) + Q\theta\kappa^3\sigma_2(1+2n)\\
&\quad + \kappa^2\lambda\sigma_2(2n\vartheta + \kappa Q(1+2n)),
\end{aligned} \tag{2.7}
$$

$$
\begin{aligned}
\mu &= \frac{1}{(1+2n)(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2 - 2\sigma_2 Q\kappa)}((\theta + 2(1+n)\lambda)(-\varepsilon_1 + \varepsilon_2)\\
&\quad - 2\theta\kappa\sigma_1(1+2n) - \sigma_1\kappa\lambda(5+4n) + \vartheta\sigma_2(\theta + 2n\lambda) + Q\theta\kappa\sigma_2(1+2n)\\
&\quad + \sigma_2 Q\kappa\lambda(3+2n)).
\end{aligned}
$$

From Eqs. (2.7), we obtain that,

$$
\begin{aligned}
Q &= \frac{1}{2(1+2n)(\kappa\varepsilon_2^2 + 2\sigma_2\kappa(-1+\sigma_2\kappa^2) + \varepsilon_2(-1+3\sigma_2\kappa^2))}((\vartheta + 2n\vartheta)\varepsilon_2^2 - \varepsilon_2(\theta\\
&\quad + 6(2n+1)\kappa^2\sigma_1 - 4(2n+1)\vartheta\kappa\sigma_2) - (2n+1)\varepsilon_1(-1 + 3a\kappa + 4\kappa^2\sigma_2)\\
&\quad + \kappa(2\sigma_2(-\theta + 2(1+2n)\vartheta\kappa\sigma_2) - (1+2n)\sigma_1(-3 + 8\kappa^2\sigma_2))\\
&\quad + \left(\sqrt{\begin{array}{l}
-4(1+2n)(\kappa\varepsilon_2^2 + 2\kappa\sigma_2(-1+\kappa^2\sigma_2) + \varepsilon_2(-1+3\kappa^2\sigma_2))(2(\kappa + 2n\kappa)\varepsilon_1^2\\
+8(1+2n)\kappa^3\sigma_1^2 + \varepsilon_1(\theta - (\vartheta + 2n\vartheta)\varepsilon_2 + 8(1+2n)\kappa^2\sigma_1 - 4\vartheta\kappa\sigma_2\\
-8n\vartheta\kappa\sigma_2) + \vartheta\sigma_2(-\theta + (\vartheta + 2n\vartheta)\varepsilon_2 + 2(2n+1)\vartheta\kappa\sigma_2) - \sigma_1(\vartheta + 2n\vartheta\\
-3\theta\kappa + 2(2n+1)\vartheta\kappa\varepsilon_2 + 8(2n+1)\vartheta\kappa^2\sigma_2)) + ((\vartheta + 2n\vartheta)\varepsilon_2^2\\
-\varepsilon_2(\theta + 6(1+2n)\kappa^2\sigma_1 - 4(1+2n)\vartheta\kappa\sigma_2) - (1+2n)\varepsilon_1(-1 + 3\kappa\varepsilon_2\\
+4\kappa^2\sigma_2) + \kappa(2\sigma_2(-\theta + 2(1+2n)\vartheta\kappa\sigma_2) - (1+2n)\sigma_1(-3 + 8\kappa^2\sigma_2)))^2
\end{array}}\right).
\end{aligned} \tag{2.8}
$$

2.2. The BAE with higher order dispersions and lack of SPM

By placing the same (2.1) wave transformation for Eq. (1.2) and are obtained real and imaginary sections. The real section is as follows:

$$
\begin{aligned}
&(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2 - 2Q\kappa\sigma_2)u'' + (-\vartheta - \kappa^2\varepsilon_1 + \vartheta\kappa\varepsilon_2 - \kappa^3\sigma_1\\
&\quad + \vartheta\kappa^2\sigma_2)u - \kappa(\theta + \lambda)u^3 = 0,
\end{aligned} \tag{2.9}
$$

and by integrating with respect to ϕ once of imaginary sections and by integration constant neglect gives:

$$
\begin{aligned}
&(\sigma_1 - \sigma_2 Q)u'' + (\vartheta\varepsilon_2 + Q\kappa\varepsilon_2 - Q - 2\kappa\varepsilon_1 - 3\kappa^2\sigma_1 + 2\vartheta\kappa\sigma_2 + \kappa^2 Q\sigma_2)u\\
&\quad - \left(\frac{1}{3}\theta + \lambda + \frac{2}{3}\mu\right)u^3 = 0.
\end{aligned} \tag{2.10}
$$

Eqs. (2.9) and (2.10) leads to:

$$
\begin{aligned}
&\frac{(\varepsilon_1 - Q\varepsilon_2 + 3\kappa\sigma_1 - \vartheta\sigma_2 - 2Q\kappa\sigma_2)}{(\sigma_1 - Q\sigma_2)} = \frac{-\kappa(\theta+\lambda)}{-(\frac{1}{3}\theta + \lambda + \frac{2}{3}\mu)}\\
&= \frac{(-\vartheta - \kappa^2\varepsilon_1 + \vartheta\kappa\varepsilon_2 - \kappa^3\sigma_1 + \vartheta\kappa^2\sigma_2)}{(+\vartheta\varepsilon_2 + Q\kappa\varepsilon_2 - Q - 2\kappa\varepsilon_1 + 2\vartheta\kappa\sigma_2 + Q\kappa^2\sigma_2 - 3\kappa^2\sigma_1)}.
\end{aligned} \tag{2.11}
$$

From Eq. (2.11), we obtain that,

$$
\begin{aligned}
\mu &= \frac{1}{2(-\vartheta - \kappa^2\varepsilon_1 + \vartheta\kappa\varepsilon_2 - \kappa^3\sigma_1 + \vartheta\kappa^2\sigma_2)}((\vartheta\theta + 3\vartheta\lambda - \kappa^2(5\theta + 3\lambda)\varepsilon_1 \\
&+ 2\kappa\vartheta\theta\varepsilon_2 - 8\theta\kappa^3\sigma_1 - 6\kappa^3\lambda\sigma_1 + 5\vartheta\theta\kappa^2\sigma_2 + 3\vartheta\kappa^2\lambda\sigma_2) - 3Q\theta\kappa \\
&- 3Q\kappa\lambda + 3Q\kappa^2(\theta + \lambda)\varepsilon_2 + 3Q\theta\kappa^3\sigma_2 + 3Q\kappa^3\lambda\sigma_2, \\
\mu &= -\frac{1}{2(2\sigma_2 Q\kappa - \varepsilon_1 + Q\varepsilon_2 - 3\kappa\sigma_1 + \vartheta\sigma_2)}(\sigma_2 Q\theta\kappa + 3\sigma_2 Q\kappa\lambda - (\theta + 3\lambda)\varepsilon_1 \\
&+ Q(\theta + 3\lambda)\varepsilon_2 - 6\kappa\lambda\sigma_1 + \vartheta\theta\sigma_2 + 3\vartheta\lambda\sigma_2 - 3Q\kappa\lambda\sigma_2).
\end{aligned}
\tag{2.12}
$$

From Eqs. (2.12), we obtain that,

$$
\begin{aligned}
Q &= \frac{1}{2(2\sigma_2\kappa + \varepsilon_2)(-1 + \kappa(\varepsilon_2 + \kappa\sigma_2))}(\varepsilon_1(-1 + 4\sigma_2\kappa^2 + 3\kappa\varepsilon_2) - 3\kappa\sigma_1 \\
&+ 6\sigma_2\kappa^3\sigma_1 + 6\kappa^2\varepsilon_2\sigma_1 + 4\varepsilon_1\vartheta\kappa^2\sigma_2 - \vartheta(\varepsilon_2 + 2\kappa\sigma_2)^2 \\
&+ \left.\sqrt{\begin{array}{c}(-4(2\sigma_2\kappa + \varepsilon_2)(-1 + \kappa(\varepsilon_2 + \kappa\sigma_2))(2\kappa\varepsilon_1^2 + 8\kappa^3\sigma_1^2 + \vartheta^2\sigma_2(\varepsilon_2 + 2\kappa\sigma_2) \\ -\varepsilon_1\vartheta\varepsilon_2 + 4\kappa(-2\kappa\sigma_1 + \vartheta\sigma_2) - \vartheta\sigma_1(1 + 2\kappa(\varepsilon_2 + 4\kappa\sigma_2))) \\ +(\vartheta\varepsilon_2^2 + \varepsilon_1(1 - 4\sigma_2\kappa^2 - 3\kappa\varepsilon_2) + 2\kappa\varepsilon_2(-3\kappa\sigma_1 + 2\vartheta\sigma_2) \\ +\kappa(4\vartheta\kappa\sigma_2^2 + \sigma_1(3 - 8\kappa^2\sigma_2)))^2)\end{array}}\right).
\end{aligned}
\tag{2.13}
$$

3. Applications of Jacobi's cn elliptic function expansion for presented equations

3.1. The BAE with full nonlinearity

We can give the extended Jacobi's cn elliptic function method [19–24] to solve the BAE with full nonlinearity.

Assumed the solution of Eq. (2.4) is demonstrable as a finite series as follows:

$$
r(x, t) = r(\phi) = \sum_{j=0}^{N} \alpha_j \,\mathrm{cn}^j[\phi; m] + \sum_{j=1}^{N} \beta_j \,\mathrm{cn}^{-j}[\phi; m],
$$

where $cn[\phi\,; m]$ is the Jacobi elliptic cn function with the parameter m $(0 < m < 1)$, $\phi = x - Qt$ and α_0, α_j, β_j, θ_0, θ_j, ξ_j for $j = 1, \bar{}N$ are values to be definited.

By balancing rr'' with r^3 in Eq. (2.4) or (2.5), we obtain $N = 2$. The solution of Eq. (2.4) is of the shape:

$$
r(\phi) = \alpha_0 + \alpha_1 \,\mathrm{cn}\,[\phi; m] + \alpha_2 \,\mathrm{cn}^2\,[\phi; m] + \beta_1 \,\mathrm{cn}^{-1}[\phi; m] + \beta_2 \,\mathrm{cn}^{-2}[\phi; m].
\tag{3.1}
$$

When $m \to 0$, we obtain new triangular periodic wave solutions of the BAE and when $m \to 1$, we obtain new hyperbolic soliton wave solutions of the BAE.

Substituting (3.1) into (2.4), collecting the coefficients of $cn[\phi\,; m]$, and solving the obtaining system, the following groups of some solutions are found.

The group of values are as follows:

$$
\begin{aligned}
\alpha_0 &= \frac{(1 + n)}{3n^2\kappa(\theta + \lambda)}(-n^2 w + (-1 + 2m^2 - n^2\kappa^2)\varepsilon_1 + (v - 2m^2 v + n^2 w\kappa)\varepsilon_2 \\
&- 3\kappa\sigma_1 + 6m^2\kappa\sigma_1 - n^2\kappa^3\sigma_1 + w\sigma_2 - 2m^2 w\sigma_2 + 2v\kappa\sigma_2 - 4m^2 v\kappa\sigma_2 + n^2 w\kappa^2\sigma_2), \\
\alpha_1 &= 0, \\
\alpha_2 &= ((1 + n)(n^4 w^2 + (-1 + m^2 - m^4 + n^4\kappa^4)\varepsilon_1^2 + ((-1 + m^2 - m^4)v^2 + n4w^2\kappa^2)\sigma_2^2 \\
&+ 2n^4 w\kappa^3\sigma_1 - 9\kappa^2\sigma_1^2 + 9m^2\kappa^2\sigma_1^2 - 9m^4\kappa\sigma_1^2 + n^4\kappa^6\sigma_1^2 - 2n^4 w^2\kappa^2\sigma_2 \\
&+ 6w\kappa\sigma_1\sigma_2 - 6m^2 w\kappa\sigma_1\sigma_2 + 6m^4 w\kappa\sigma_1\sigma_2 + 12v\kappa^2\sigma_1\sigma_2 - 12m^2 v\kappa^2\sigma_1\sigma_2 \\
&+ 12m^4 v\kappa^2\sigma_1\sigma_2 - 2n^4 w\kappa^5\sigma_1\sigma_2 - w^2\sigma_2^2 + m^2 w^2\sigma_2^2 - m^4 w^2\sigma_2^2 - 4\mathrm{vw}\kappa\sigma_2^2 \\
&+ 4m^2\mathrm{vw}\kappa\sigma_2^2 - 4m^4\mathrm{vw}\kappa\sigma_2^2 - 4v^2\kappa^2\sigma_2^2 + 4m^2 v^2\kappa^2\sigma_2^2 - 4m^4 v^2\kappa^2\sigma_2^2 \\
&+ n^4 w^2\kappa^4\sigma_2^2 - 2\varepsilon_1(-n^4 w\kappa^2 + ((-1 + m^2 - m^4)v + n^4 w\kappa^3)\varepsilon_2 + \kappa(3 - 3m^2 \\
&+ 3m^4 - n^4\kappa^4)\sigma_1 - w\sigma_2 + m^2 w\sigma_2 - m^4 w\sigma_2 - 2v\kappa\sigma_2 + 2m^2 v\kappa\sigma_2 - 2m^4 v\kappa\sigma_2 \\
&+ n^4 w\kappa^4\sigma_2) + 2\varepsilon_2(-n^4 w^2\kappa + \kappa(3(1 - m^2 + m^4)v - n^4 w\kappa^3)\sigma_1 \\
&+ (-(1 - m^2 + m^4)\mathrm{vw} - 2(1 - m^2 + m^4)v^2\kappa + n^4 w^2\kappa^3)\sigma_2)))/(15(-1 \\
&+ m^2)n^2\kappa(\theta + \lambda)(-\varepsilon_1 + v\varepsilon_2 - 3\kappa\sigma_1 + w\sigma_2 + 2v\kappa\sigma_2)), \\
\beta_1 &= 0, \\
\beta_2 &= \frac{1}{n^2\kappa(\theta + \lambda)}(-1 + m^2)(1 + n)(-\varepsilon_1 + v\varepsilon_2 - 3\kappa\sigma_1 + w\sigma_2 + 2v\kappa\sigma_2).
\end{aligned}
\tag{3.2}
$$

From these outcomes, Jacobi elliptic cn function solution of Eq. (1.1) is obtained as,

$$q(x,t)=r(x,t)^{1/2n}e^{i(-\kappa x+\vartheta t+\theta_0)}, \tag{3.3}$$

$$\begin{aligned}
r(x,t) =\ & \frac{(1+n)}{3n^2\kappa(\theta+\lambda)}(-n^2w+(-1+2m^2-n^2\kappa^2)\varepsilon_1+(v-2m^2v+n^2w\kappa)\varepsilon_2-3\kappa\sigma_1\\
&+6m^2\kappa\sigma_1-n^2\kappa^3\sigma_1+w\sigma_2-2m^2w\sigma_2+2v\kappa\sigma_2-4m^2v\kappa\sigma_2+n^2w\kappa^2\sigma_2)\\
&+((1+n)(n^4w^2+(-1+m^2-m^4+n^4\kappa^4)\varepsilon_1^2+((-1+m^2-m^4)v^2+n^4w^2\kappa^2)\sigma_2^2\\
&+2n^4w\kappa^3\sigma_1-9\kappa^2\sigma_1^2+9m^2\kappa^2\sigma_1^2-9m^4\kappa^2\sigma_1^2+n^4\kappa^6\sigma_1^2-2n^4w^2\kappa^2\sigma_2\\
&+6w\kappa\sigma_1\sigma_2-6m^2w\kappa\sigma_1\sigma_2+6m^4w\kappa\sigma_1\sigma_2+12v\kappa^2\sigma_1\sigma_2-12m^2v\kappa^2\sigma_1\sigma_2\\
&+12m^4v\kappa^2\sigma_1\sigma_2-2n^4w\kappa^5\sigma_1\sigma_2-w^2\sigma_2^2+m^2w^2\sigma_2^2-m^4w^2\sigma_2^2-4\mathrm{vw}\kappa\sigma_2^2\\
&+4m^2\mathrm{vw}\kappa\sigma_2^2-4m^4\mathrm{vw}\kappa\sigma_2^2-4v^2\kappa^2\sigma_2^2+4m^2v^2\kappa^2\sigma_2^2-4m^4v^2\kappa^2\sigma_2^2\\
&+n^4w^2\kappa^4\sigma_2^2-2\varepsilon_1(-n^4w\kappa^2+((-1+m^2-m^4)v+n^4w\kappa^3)\varepsilon_2+\kappa(3-3m^2\\
&+3m^4-n^4\kappa^4)\sigma_1-w\sigma_2+m^2w\sigma_2-m^4w\sigma_2-2v\kappa\sigma_2+2m^2v\kappa\sigma_2-2m^4v\kappa\sigma_2\\
&+n^4w\kappa^4\sigma_2)+2\varepsilon_2(-n^4w^2\kappa+\kappa(3(1-m^2+m^4)v-n^4w\kappa^3)\sigma_1\\
&+(-(1-m^2+m^4)\mathrm{vw}-2(1-m^2+m^4)v^2\kappa+n^4w^2\kappa^3)\sigma_2)))/(15(-1)\\
&+m^2)n^2\kappa(\theta+\lambda)(-\varepsilon_1+v\varepsilon_2-3\kappa\sigma_1+w\sigma_2+2v\kappa\sigma_2))\mathrm{cn}^2\,[(x-\mathrm{Q}t);m]\\
&+\frac{1}{n^2\kappa(\theta+\lambda)}(-1+m^2)(1+n)(-\varepsilon_1+v\varepsilon_2-3\kappa\sigma_1+w\sigma_2+2v\kappa\sigma_2)\,\mathrm{cn}^{-2}[(x-\mathrm{Q}t);m].
\end{aligned} \tag{3.4}$$

When the parameter $m\to 0$ in Eq. (3.3), singular soliton solution as follows:

$$\begin{aligned}
q(x,t) =\ & \Bigg(\frac{(1+n)}{3n^2\kappa(\theta+\lambda)}(-n^2w+(-1-n^2\kappa^2)\varepsilon_1+(v+n^2w\kappa)\varepsilon_2-3\kappa\sigma_1-n^2\kappa^3\sigma_1+w\sigma_2\\
&+2v\kappa\sigma_2+n^2w\kappa^2\sigma_2)+((1+n)(n^4w^2+(-1+n^4\kappa^4)\varepsilon_1^2+(-v^2+n^4w^2\kappa^2)\sigma_2^2\\
&+2n^4w\kappa^3\sigma_1-9\kappa^2\sigma_1^2+n^4\kappa^6\sigma_1^2-2n^4w^2\kappa^2\sigma_2+6w\kappa\sigma_1\sigma_2+12v\kappa^2\sigma_1\sigma_2\\
&-2n^4w\kappa^5\sigma_1\sigma_2-w^2\sigma_2^2-4\mathrm{vw}\kappa\sigma_2^2-4v^2\kappa^2\sigma_2^2+n^4w^2\kappa^4\sigma_2^2-2\varepsilon_1(-n^4w\kappa^2\\
&+(-v+n^4w\kappa^3)\varepsilon_2+\kappa(3-n^4\kappa^4)\sigma_1-w\sigma_2-2v\kappa\sigma_2+n^4w\kappa^4\sigma_2)\\
&+2\varepsilon_2(-n^4w^2\kappa+\kappa(3v-n^4w\kappa^3)\sigma_1+(-\mathrm{vw}-2v^2\kappa+n^4w^2\kappa^3)\sigma_2)))/(15\\
&\times(-n^2\kappa(\theta+\lambda)(-\varepsilon_1+v\varepsilon_2-3\kappa\sigma_1+w\sigma_2+2v\kappa\sigma_2))\cos^2[(x-\mathrm{Q}t)]\\
&+\frac{-1}{n^2\kappa(\theta+\lambda)}(1+n)(-\varepsilon_1+v\varepsilon_2-3\kappa\sigma_1+w\sigma_2+2v\kappa\sigma_2)\\
&\times\sec^2[(x-\mathrm{Q}t)]\Bigg)^{1/2n}\exp[i(-\kappa x+\vartheta t+\theta_0)].
\end{aligned} \tag{3.5}$$

3.2. The BAE with higher order dispersions and lack of SPM

We can give the extended Jacobi's cn elliptic function method [19–24] to solve the BAE with higher order dispersions and lack of SPM.

Assumed the solution of Eq. (2.9) is demonstrable as a finite series as follows:

$$u(x,t)=u(\phi)=\sum_{j=0}^{N}\alpha_j\,\mathrm{cn}^j[\phi;m]+\sum_{j=1}^{N}\beta_j\,\mathrm{cn}^{-j}[\phi;m],$$

where $cn[\phi;m]$ is the Jacobi elliptic cn function with the parameter m $(0<m<1)$, $\phi=x-Qt$ and $\alpha_0,\alpha_j,\beta_j,\theta_0,\theta_j,\xi_j$ for $j=1,\bar{}N$ are values to be definited.

By balancing u'' with u^3 in Eq. (2.9) or (2.10), we obtain $N=1$. The solution of Eq. (2.9) is of the shape:

$$u(\phi)=\alpha_0+\alpha_1\,\mathrm{cn}\,[\phi;m]+\beta_1\,\mathrm{cn}^{-1}[\phi;m]. \tag{3.6}$$

When $m\to 0$, we obtain new triangular periodic wave solutions of the BAE and when $m\to 1$, we obtain new hyperbolic soliton wave solutions of the BAE.

Substituting (3.6) into (2.9), collecting the coefficients of $cn[\phi;m]$, and solving the obtaining system, the following groups of some solutions are found.

The group of values are as follows:

$$\begin{aligned}\alpha_0 &= 0,\\ \alpha_1 &= \pm\,(i(w + (1 - 2m^2 + \kappa^2)\varepsilon_1 + (-\nu + 2m^2\nu - w\kappa)\varepsilon_2 + 3\kappa\sigma_1 - 6m^2\kappa\sigma_1\\ &\quad + \kappa^3\sigma_1 - w\sigma_2 + 2m^2 w\sigma_2 - 2\nu\kappa\sigma_2 + 4m^2\nu\kappa\sigma_2 - w\kappa^2\sigma_2))/(3\sqrt{2}\sqrt{-1 + m^2}\\ &\quad \times \sqrt{\kappa(\theta + \lambda)}\sqrt{\varepsilon_1 - \nu\varepsilon_2 + 3\kappa\sigma_1 - w\sigma_2 - 2\nu\kappa\sigma_2}),\\ \beta_1 &= \pm\,\frac{i\sqrt{2}\sqrt{-1 + m^2}\sqrt{\varepsilon_1 - \nu\varepsilon_2 + 3\kappa\sigma_1 - w\sigma_2 - 2\nu\kappa\sigma_2}}{\sqrt{\kappa(\theta + \lambda)}}.\end{aligned} \tag{3.7}$$

From these outcomes, Jacobi elliptic *cn* function solution of Eq. (1.2) is obtained as,

$$q(x,\, t) = u(x,\, t)e^{i(-\kappa x + \vartheta t + \theta_0)}, \tag{3.8}$$

$$\begin{aligned}u(x,\, t) &= \pm\,((i(w + (1 - 2m^2 + \kappa^2)\varepsilon_1 + (-\nu - w\kappa + 2m^2\nu)\varepsilon_2 + 3\kappa\sigma_1 - 6m^2\kappa\sigma_1\\ &\quad + \kappa^3\sigma_1 - w\sigma_2 + 2m^2 w\sigma_2 - 2\nu\kappa\sigma_2 + 4m^2\nu\kappa\sigma_2 - \sigma_2 w\kappa^2))/(3\sqrt{2}\sqrt{-1 + m^2}\\ &\quad \times \sqrt{\kappa(\theta + \lambda)}\sqrt{\varepsilon_1 - \nu\varepsilon_2 + 3\kappa\sigma_1 - w\sigma_2 - 2\nu\kappa\sigma_2}))\mathrm{cn}[(x - \mathrm{Q}t);\, m]\\ &\quad \pm\,\frac{i\sqrt{2}\sqrt{-1 + m^2}\sqrt{\varepsilon_1 - \nu\varepsilon_2 + 3\kappa\sigma_1 - w\sigma_2 - 2\nu\kappa\sigma_2}}{\sqrt{(\theta + \lambda)\kappa}}\,\mathrm{cn}^{-1}[(x - \mathrm{Q}t);\, m].\end{aligned} \tag{3.9}$$

When the parameter $m \to 0$ in Eq. (3.9), then we acquire the exact solution:

$$\begin{aligned}q(x,\, t) &= (\pm((i(w + (1 + \kappa^2)\varepsilon_1 + (-\nu - w\kappa)\varepsilon_2 + 3\kappa\sigma_1\\ &\quad + \kappa^3\sigma_1 - w\sigma_2 - 2\nu\kappa\sigma_2 - w\kappa^2\sigma_2))/(3\sqrt{2}i\\ &\quad \times \sqrt{\kappa(\theta + \lambda)}\sqrt{\varepsilon_1 - \nu\varepsilon_2 + 3\kappa\sigma_1 - w\sigma_2 - 2\nu\kappa\sigma_2}))\cos[(x - \mathrm{Q}t)]\\ &\quad \mp\,\frac{\sqrt{2}\sqrt{\varepsilon_1 - \nu\varepsilon_2 + 3\kappa\sigma_1 - w\sigma_2 - 2\nu\kappa\sigma_2}}{\sqrt{\kappa(\theta + \lambda)}}\,\sec[(x - \mathrm{Q}t)])\exp[i(-\kappa x + \vartheta t + \theta_0)].\end{aligned} \tag{3.10}$$

We wrote some of the solutions found for BAE with full nonlinearity and with higher order dispersions and lack of SPM. Besides we showed 3D and 2D graphics for some of solutions in Figs. 1 and 2 .

Also, we obtained many new Jacobi elliptic *sn* function solutions of BAE and we showed in Figs. 3 and 4 .

In the graphics above, some of the Jacobi elliptic *sn* function solutions and Jacobi elliptic *cn* function solutions of BAE are derived from $m \to 0$ and $m \to 1$.

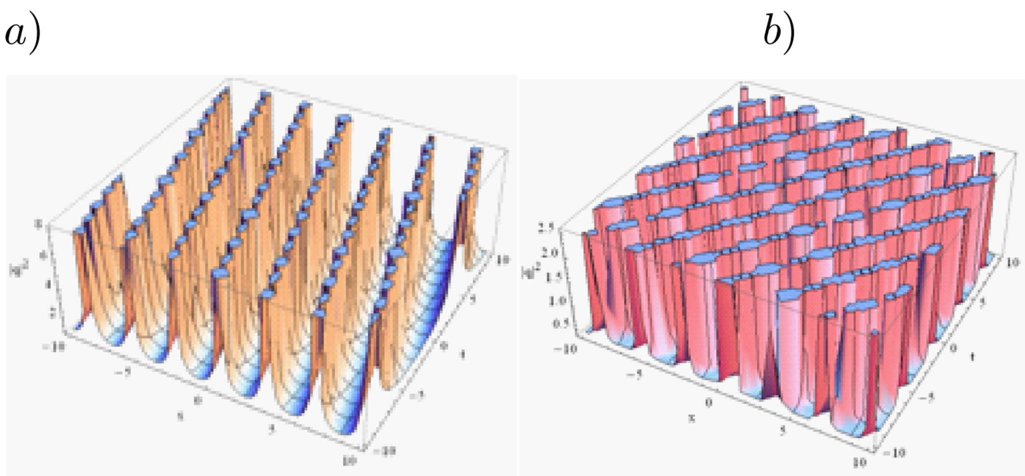

Fig. 1. The 3D graphic for the $|q(x,\, t)|^2$ analytical solution of the BAE for $\kappa = 1$, $\vartheta = \lambda = 2$, $\theta_0 = \theta = 1$, $\varepsilon_1 = \varepsilon_2 = 2$, $\sigma_1 = \sigma_2 = 1$ (a) Eq. (3.5) ($n = 2$), (b) Eq. (3.10).

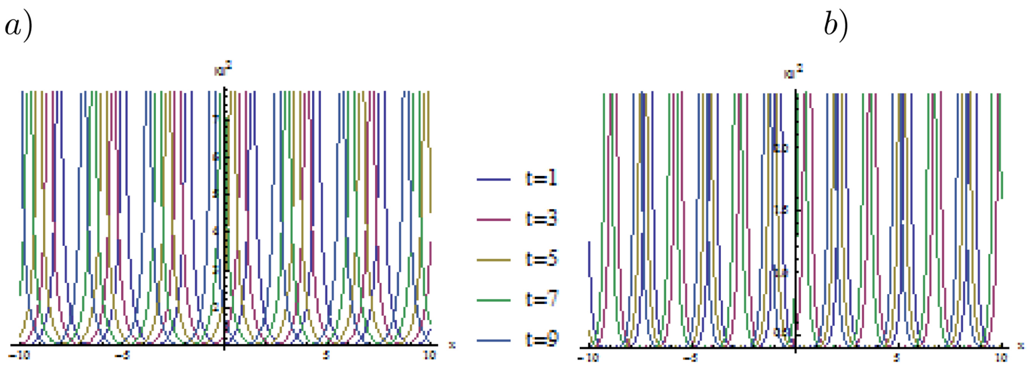

Fig. 2. The 2D graphic for the $|q(x, t)|^2$ analytical solution of the BAE for different value of t. ($\kappa = 1$, $\vartheta = \lambda = 2$, $\theta_0 = \theta = 1$, $\varepsilon_1 = \varepsilon_2 = 2$, $\sigma_1 = \sigma_2 = 1$) (a) Eq. (3.5) ($n = 2$), (b) Eq. (3.10).

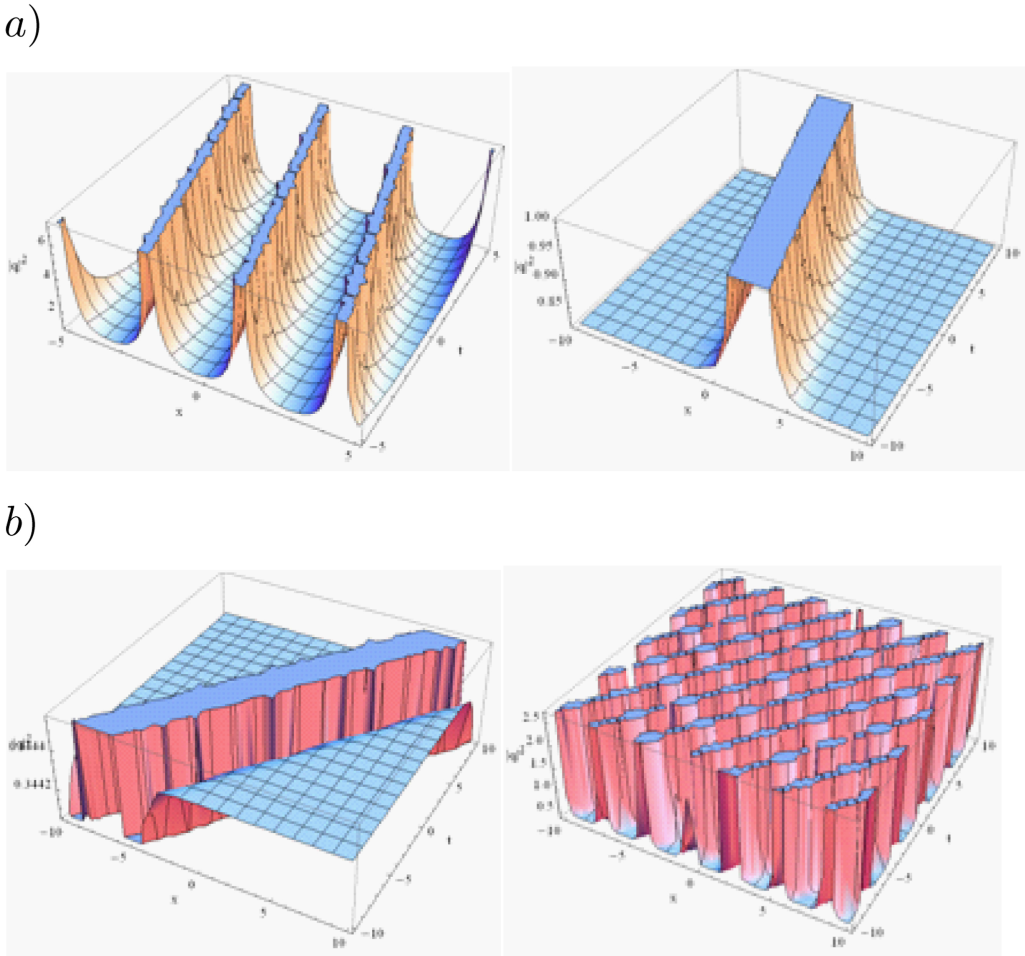

Fig. 3. The 3D graphic for the $|q(x, t)|^2$ analytical solution of the BAE for $\kappa = 1$, $\vartheta = \lambda = 2$, $\theta_0 = \theta = 1$, $\varepsilon_1 = \varepsilon_2 = 2$, $\sigma_1 = \sigma_2 = 1$ (a) Eq. (1.2) ($n = 2$), (b) Eq. (1.1).

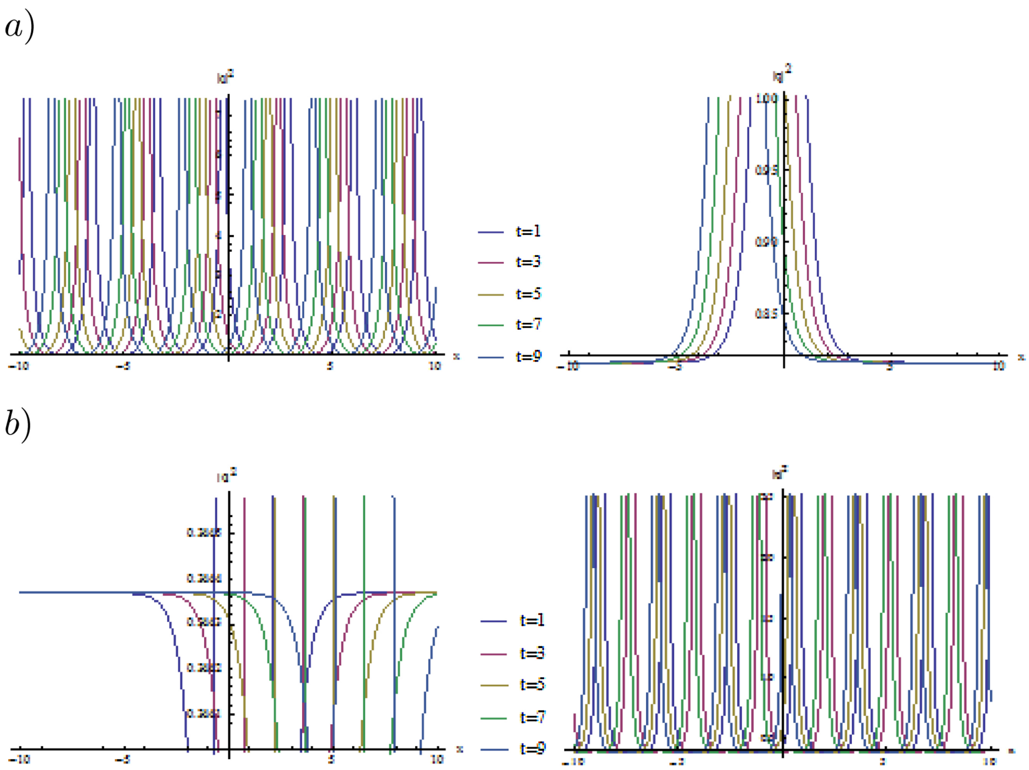

Fig. 4. The 2D graphic for the $|q(x,\ t)|^2$ analytical solution of the BAE for different value of . . ($\kappa = 1,\ \vartheta = \lambda = 2,\ \theta_0 = \theta = 1,\ \varepsilon_1 = \varepsilon_2 = 2,\ \sigma_1 = \sigma_2 = 1$) (a) Eq. (1.2) ($n = 2$), (b) Eq. (1.1).

4. Conclusion

In this paper, the extended Jacobi's elliptic function approach is used to find new exact traveling wave solutions of the BAE. Two nonlinear shapes are analyzed. These BAE with higher order dispersions and lack of SPM and BAE with full nonlinearity. When modulus $m \to 1$ or 0, the Jacobian elliptic functions degenerate as hyperbolic functions and trigonometric functions. The existences of solutions derived from these functions are all guaranteed through constraint conditions that are also listed besides the solutions.

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