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# Multi-objective 3D bin packing problem with load balance and product family concerns

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ABSTRACT

The classical three-dimensional bin packing problem (3D-BPP) orthogonally packs a set of rectangular items with varying dimensions into the minimum number of three-dimensional rectangular bins. While ensuring the minimum number of bins used, the safety of the logistic operations is addressed with the complementary load-balancing objective for which concepts such as orientation and stability are used in the literature though not at the same time. In this study, we extend the load-balanced 3D-BPP by combining both orientation and stability, and introducing a new concept called family unity which encourages packing a family of products (e.g., from the same order and with the same destination) together. Although item related concerns are very common in practice, there are no multi-objective studies in the bin packing literature that includes family unity concept. Therefore, this is the first study that proposes a multi-objective mixed integer programming model for the extended problem to determine the optimal packing plan that minimizes the number of bins used and the deviation of balance from the ideal barycenter while maximizing family unity ratio via a weighted objective function. A numerical example is provided to analyze the performance of the proposed model. Furthermore, a real-life container loading problem is solved and outputs of the study implies the practical advantages of including family unity and load-balance considerations in solving 3D-BPP problems.

## 1. Introduction

The 3D-BPP is a real-world driven combinatorial optimization problem with a strong impact on the economy, on the environment, and safety (Ramos, Silva, & Oliveira, 2018). It is one of the prominent concerns in logistics where companies consolidate the goods into appropriate bins and deliver them to different locations along their supply chain. Inefficient planning methods for packing and shipping increase costs without adding value to the supply chain. To achieve high levels of economic efficiency, researchers have proposed a variety of mathematical models and solution methods for the NP-hard bin packing problem. Numerous publications in the literature attempt to lower transportation costs by orthogonally and completely packing multiple items into bins without overlapping (Moon & Nguyen, 2014).

In classical 3D-BPP, the objective is to pack rectangular items into the minimum number of three-dimensional rectangular bins. In addition to this objective, there are several factors to be considered in practice, such as load stability and load balance, besides other item and order-related concerns such as stacking requirements and packing the items within the same destination together. Load stability ensures that the minimum acceptable percentage of the base area of each item is supported, i.e., sits either on the floor or on top of another item. This maintains the stability of items for handling and transportation during internal and external logistics operations. The second factor, load-balance, is related to the weight of the items to be evenly distributed along the bin floor. Based on the mode of transportation, improper displacement from the ideal barycenter, which is defined as an ideal point such as the center of the bin, can increase the risks on cargo safety. Despite the apparent significance, load stability and balance factors are generally not considered explicitly in bin packing literature (Bortfeldt & Wäscher, 2013). Hence, the literature on the practical extension of 3D-BPP that takes these factors into account, namely load-balanced 3D-BPP, is relatively scarce.

In this paper, we first introduce a practical concept, namely family unity, into the bin packing problem. The concept of family has been defined as a new feature to represent related concerns (delivery,

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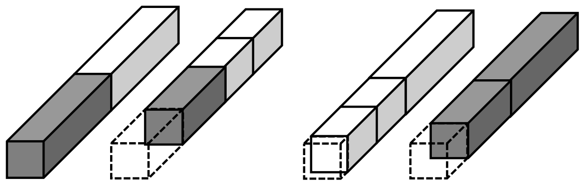


(a) Packing Plan without family unity (b) Packing Plan with family unity

**Fig. 1.** The effect of Family Unity packing plans.

product, or customer-specific) that can be attached as a characteristic to each item individually. We introduce the family concept to describe a more comprehensive set of features while the product family is used to represent items of the same size or the same characteristic in the literature. This new concept addresses the items that need to be packed into the same bin due to practical considerations such as having the same destination, handling requirements, storage conditions and so on. Improving family unity provides better solutions both in reducing the scheduling complexity of handling equipment and execution of handling operations. Fig. 1 provides an illustrative example which shows a simple packing plan with two bins. While more than one type of handling equipment may be required in order to load/unload both bins in Fig. 1 (a), each bin requires a single type of handling equipment in Fig. 1 (b). Furthermore, both bins can be sent to the final destination without any further handling at the transfer center where the first setup requires reassigning items to bins based on the destination.

Note that the concept of family unity differs from the compatibility concept readily available in the bin packing literature which allows or restricts the items to be packed into the same bin. The family concept can be considered as a specific feature that is assigned to each item to be packed into a number of bins. In the literature, item related concerns are considered under allocation constraints which can further be categorized as connectivity constraints and multi-drop constraints. These constraints, however, do not exactly match the concept of family unity that we introduce. Connectivity constraints are used in single bin packing problems to ensure that the items belonging to the same customer are located close to each other in the same bin. (Gimenez Palacios, Alonso, Alvarez Valdes, & Parreño, 2020; Liu, Yue, Dong, Maple, & Keech, 2011; Terno, Scheithauer, Sommerweiß, & Riehme, 2000). Another attempt for considering item-related constraints in single bin packing problem is presented by Egeblad, Garavelli, Lisi, and Pisinger (2010), and aim to pack irregular items of same shape close together. Furthermore, in routing problems, multi-drop constraints are encountered. Regarding the order of delivery to the customers and the space required for the delivery, items are positioned in the bins so that the customer's items become accessible without relocating another customer's items. The items are located on a customer basis using strategies such as LIFO, according to the distribution order. (Ceschia & Schaerf, 2013; Bortfeldt, 2012; Christensen & Rousøe, 2009; Tarantilis, Zachariadis, & Kiranoudis, 2009; Gendreau, Iori, Laporte, & Martello, 2006). In this study, we focus both on assigning the items in the same product family to the same bin and packing them within the bin taking balance constraints into account while the studies in literature generally focus on the close packaging of the products belonging to the same customer within the same bin.

Family unity, on the other hand, helps to take the additional distribution cost that is likely to emerge when the items with the same destination are packed within different bins into account implicitly. Thus, the overall cost is likely to decrease with the addition of this new practical concept though the total cost in the bin packing phase might increase.

Next, we introduce a multi-objective mathematical model for the load-balanced 3D-BPP where the orientation of the items is allowed and item-related concerns are considered. To the best of our knowledge, this is the first work that proposes a mixed integer programming (MIP) model with a comprehensive set of constraints addressing three-dimensional replacement, stability and orientation, and a set of objective functions composing minimizing the number of bins used and the deviation from the ideal barycenter, along with maximizing the family unity ratio by using a weighted objective function.

The remainder of this paper is organized as follows. We first provide a literature review on 3D-BPP in Section 2. In Section 3, we give a formal definition for the load-balanced 3D-BPP and family unity. Next, we propose a new mathematical model for 3D-BPP with orientation, stability, and family unity in Section 4. In the next section, we present a numerical example, comparative analysis, and discussion on the advantages of the proposed model. Also, the real case application, which led and inspired this work, is provided in Section 5. Finally, we present concluding remarks and suggestions for future research in Section 6.

## 2. Literature review

The packing problem has attracted research interest in the cutting and packing research community. Although the 3D-BPP can be modeled and solved as a mixed-integer program, there are few exact solution methodologies available (Elhedhli, Gzara, & Yildiz, 2019). In addition to exact methods, several studies have focused on developing heuristics for practical aspects of the problem (Chen, Lee, & Shen, 1995). Also, many heuristics, *meta*-heuristic and hybrid methods have been proposed to solve single and multi-bin packing problems (Elhedhli et al., 2019; Bortfeldt & Wäscher, 2013). We refer the reader to Zhao, Bennell, Bektaş, and Dowsland (2016) for a comparative review of recent literature on 3D-BPPs.

In this section, we review the studies that correspond to the solution method we have considered. Therefore, we will provide a comprehensive literature review on mathematical models in single and multiple bin packing problems.

### 2.1. Single bin packing problem

The most common objective in single bin packing problem is to maximize the space utilization (Kang, Moon, & Wang, 2012; Junqueira, Morabito, & Yamashita, 2012; Moon & Nguyen, 2014; Araya, Guerrero, & Nuñez, 2017), or equivalently, to minimize the total unused space (Chen et al., 1995). Egeblad and Pisinger (2009) have proposed a MIP model that packs a maximum profit subset of items into a bin with fixed dimensions. Although the mathematical model does not include orientation, the authors have developed a well-performing packing heuristic which allows the orientation of the items for medium size instances. Baldi, Perboli, and Tadei (2012) have proposed a more efficient MIP formulation for the three-dimensional single bin packing problem with a profit-maximizing objective function and balancing constraints. Their model outperforms Egeblad and Pisinger (2009)'s using deriving both lower and upper bounds to the single bin packing problem. The results show that they obtain the optimal solution for small size instances. Moon and Nguyen (2014) have extended the work of Baldi et al. (2012) considering the maximization of volume utilization. The proposed algorithm takes the trade-off between volume utilization and weight balance into consideration and handles the balance constraints to increase the efficiency of oceanic transport while achieving superior volume utilization. Furthermore, Junqueira et al. (2012) present MIP models for the single bin packing problem with the issues of cargo stability and load-bearing.

### 2.2. Multiple bin packing problem

Bortfeldt and Wäscher (2013) have revealed that the practical considerations in multiple bin packing problems have almost been neglected completely compared to the vast single bin packing literature.

**Table 1**
Constraints in multiple bin packing mathematical models.

| Publication | Bins | | Balance Constraints | Stability Constraints | Orientation Constraints |
|---|---|---|---|---|---|
| | Identical | Variable | | | |
| Chen et al. (1995) | | + | | + | + |
| Eley (2003) | | + | | | |
| Trivella and Pisinger (2016) | + | | + | | |
| Pedruzzi et al. (2016) | | + | | + | + |
| Crainic, Gobbato, Perboli, and Rei (2016) | | + | | | |
| Alonso, Alvarez-Valdes, Iori, Parreño, and Tamarit (2017) | + | | + | | |

Although there is increasing momentum, the studies on the multiple bin case are still very scarce (Alonso, Alvarez-Valdes, Iori, & Parreño, 2019).

Prior work on optimal packing of items into the minimum number of bins is based on a two-dimensional bin packing model which is formulated by Martello and Vigo (1998). Liu, Tan, Huang, Goh, and Ho (2008) have further proposed a mathematical model for solving the two-dimensional bin packing problem regarding two objectives: minimizing the number of bins used and balancing the load in each bin. However, the orientations and the stability of items are not considered. While it is practical to solve the two-dimensional bin packing problem for some types of cargo, 3D-BPP models are particularly important for container loading problems. However, 3D-BPP is strongly NP-hard and extremely difficult to solve in practice (Martello, Pisinger, & Vigo, 2000).

To our knowledge, the first analytical model of 3D-BPP considering orientations, multiple item sizes, multiple bin sizes, avoidance of item overlapping, and space utilization constraints is presented by Chen et al. (1995). Pedruzzi, Nunes, Rosa, and Arpini (2016) extended Chen et al. (1995)'s model by defining additional support constraints and a novel objective function minimizing the sum of the coordinates in order to yield compact arrangement of the items in the bins. Alonso, Alvarez-Valdes, Iori, Parrẽno and Tamarit (2017) present mathematical models for 3D-BPP, starting from a simple model to more complex models by adapting several extensions such as the weight and volume of pallet bases, the position of the center of gravity and the minimization of the number of pallets.

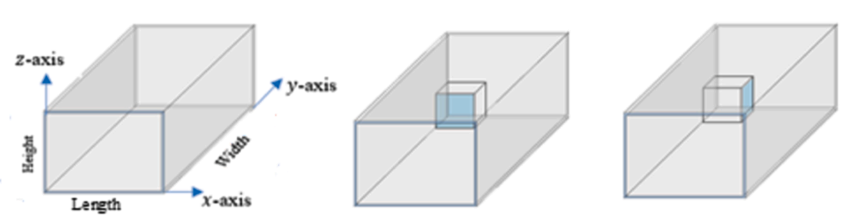
A comprehensive study of Bortfeldt and Wäscher (2013) have reviewed 163 publications considering practical constraints in bin packing literature Table 1 summarizes the supplementary constraints in the proposed MIP models for multiple bin packing problem in the literature.

Recent studies on 3D-BPP models reveal that the issue of satisfying several constraints simultaneously has not been satisfactorily addressed. Soft constraints have also not been substantially discussed, despite the fact that they are of considerable significance in practice (Bortfeldt & Wäscher, 2013). Even the smallest industry instances cannot be solved in reasonable times with current exact methodologies, and medium- and large-size 3D-BPP instances are only solved heuristically and remain out of reach of exact methods (Elhedhli et al., 2019). Elhedhli et al. (2019) have proposed an optimization model with layer-based column-generation approach that addresses practical constraints in order to distribute items in layers evenly for better spacing and increased support. Furthermore, Bortfeldt and Wäscher (2013) have stressed that multi-objective approaches appear to be a promising class of methods for solving these problems. Yet, bin packing problem with multiple objectives has been studied very rarely in the literature. To the best of our knowledge, the first multi-objective mathematical model for solving bin packing problems is proposed for two-dimensional problems by Liu et al. (2008). The only study that utilizes multiple objectives in 3D-BPP belongs to Trivella and Pisinger (2016). They propose a MILP model for a multi-dimensional bin packing problem where the sum of displacements from the desired barycenter over the smallest number of used bins is minimized.

## 3. Problem description

The multiple bin packing problem seeks for the minimum number of bins used to pack three-dimensional rectangular-prism-shaped items. There are $n$ items to be packed and each item $i$has its length$l_i$, width$w_i$, height $h_i$ and weight $a_i$ where$i = \{1, 2, \cdots, n\}$. We define the family of products to incorporate the delivery step of the items into the packing problem implicitly. Here, each family can be thought as a set of items with the same destination. An item can be a member of only one family of products $f$, which corresponds to the items that need to be packed together in the same bin. A binary parameter $\alpha_{if}$ expresses whether item$i$ belongs to family $f$ or not. A family can have only a single item, as well as all the items, can be in a single-family. In the best case, all the items in the product family will be fitted in a single bin and family unity, a measure of how good we perform to keep the items in the same family together, will be at its maximum. If this cannot be done, then the number of bins that the items in the family are scattered should be kept as minimum as possible to reduce the number of potential visits to the destination.

Moreover, for orthogonal packing, every face of the items is considered to be parallel to the faces of the bin. The items are loaded into the bins with length $L_j$, width $W_j$ and height$H_j$. The total weight of the items packed into a bin $j$ cannot exceed load bearing capacity$A_j$. For each bin$j$, we define a three-dimensional cartesian coordinate system with the left-bottom-back vertex in the coordinate origin. The directions in $x$-axis, $y$-axis and $z$-axis are considered parallel with the width, height and length of the bin, respectively. Regarding load balancing considerations, the ideal barycenter of a bin is assumed to be at its midpoint at




(a) Coordinate system of a bin (b) Allowed orientations of items in a bin

**Fig. 2.** Bin Coordinates and possible item orientations.

**Table 2**
Similarities and differences between the models for bin packing problems.

| Publication | Objective Function | | Bins Dimensions Identical/ Variable | Constraints | | | |
|---|---|---|---|---|---|---|---|
| | Single | Multi | | Orientation | Balance | Stabilty | Family Unity |
| Chen et al. (1995) | minimize the total unused space | | Variable 3D | *6 way* | | + | |
| Liu et al. (2008) | | **OBJ1:** minimize wasted space<br>**OBJ2:** minimize center of gravity deviation | Variable 2D | | + | | |
| Trivella and Pisinger (2016) | | **OBJ1:** minimize the total number of bins used<br>**OBJ2:** minimize center of gravity deviation | Identical 3D | | + | | |
| Proposed model | | **OBJ1:** minimize the total number of bins used<br>**OBJ2:** minimize deviation from the center of gravity<br>**OBJ3:** maximize the number of items from the same product family packed in any bin which addresses family unity | Variable 3D | *2 way* | + | + | + |

**Table 3**
Decision variables in the formulation of 3D-BPP.

| Symbol | Definition |
|---|---|
| $p_{ij}$ | 1 if item $i$ is packed into bin $j$, 0 otherwise. |
| $u_j$ | 1 if bin $j$ is utilized, 0 otherwise. |
| $x_{ik}^{+}$ | 1 if item $i$ is placed to the left of item $k$ in the same bin, 0 otherwise. |
| $x_{ik}^{-}$ | 1 if item $i$ is placed to the right of item $k$ in the same bin, 0 otherwise. |
| $y_{ik}^{+}$ | 1 if item $i$ is placed behind of item $k$ in the same bin, 0 otherwise. |
| $y_{ik}^{-}$ | 1 if item $i$ is placed in front of item $k$ in the same bin, 0 otherwise. |
| $z_{ik}^{+}$ | 1 if item $i$ is placed below of item $k$ in the same bin, 0 otherwise. |
| $z_{ik}^{-}$ | 1 if item $i$ is placed above of item $k$ in the same bin, 0 otherwise. |
| $l_i^x$ | 1 if the length of item $i$ is parallel to X-axis, 0 otherwise. |
| $l_i^y$ | 1, if the length of item $i$ is parallel to Y-axis, 0 otherwise. |
| $w_i^x$ | 1, if the width of item $i$ is parallel to X-axis , 0 otherwise. |
| $w_i^y$ | 1, if the width of item $i$ is parallel to Y-axis, 0 otherwise. |
| $\beta_{jf}$ | 1 if at least one item in family $f$ is packed in bin $j$ , 0 otherwise. |
| $x_i, y_i, z_i$ | continuous nonnegative variables denoting the left-bottom-back coordinate of item $i$ in a bin |
| $g_j^x, g_j^y, g_j^z$ | continuous variables denoting barycenter of bin $j$ on the $x$-, $y$- and $z$- axes respectively |

the bottom which corresponds to $\left(\frac{L_j}{2}, \frac{W_j}{2}, 0\right)$ on the $x$-axis, $y$-axis and $z$-axis, respectively. The coordinate system for each bin is defined in Fig. 2(a).

In 3D-BPP problems, the items can be packed into bins with 6 possible orientations. However, in the majority of loading operations items are only allowed to be rotated on their base and overhead rotation is not allowed. In accordance with, common practice, we allow items to be loaded into bins in only two possible orientations as depicted in Fig. 2 (b).

In this study, we introduce a comprehensive extension of 3D-BPP including packing, stacking, orientation, capacity, and stability constraints with load-balancing and item-related concerns. The objective of the problem is to pack a set of items into the smallest number of bins while ensuring the load balance and family unity for families of items. Besides the additional complexity of load balance consideration on the already NP-hard packing problems (Trivella & Pisinger, 2016), we incorporate item-related constraints and a new conflicting objective on the problem which will further be explained in detail in the following chapter. The problem was already NP-hard, and by incorporating additional constraints and implementing new objective functions, we make it even more complicated. The main focus of this study is to examine the behavior of the mathematical model when family unity is included, rather than exhibit the model's ability to solve large-scale problems.

## 4. Proposed mathematical model-formulation

In this section, we present a multi-objective MIP model for 3D-BPP with item-related and load-related constraints. We propose a comprehensive extension of the mathematical models in the literature that deal with multiple objectives as in Chen et al. (1995), Liu et al. (2008), Trivella and Pisinger (2016). Table 2 provides a brief overview of the distinguishing features of these studies.

The proposed model addresses the fundamental constraints inherent in 3D-BPP, as presented by Chen et al. (1995), but differs in terms of stability, balance, orientation and family unity respects. The proposed model also differs from the work of Liu et al. (2008), which proposes two-dimensional bin packing model with multiple objectives: minimizing wasted space and minimizing center of gravity deviation of all bins. Trivella and Pisinger (2016) expand their work to solve 3D-BPPs. In this study, we present a multi-objective mathematical model, inspired by Trivella and Pisinger (2016), by defining supplementary stability constraints and family unity constraints. We also introduce a novel objective function that maximizes the number of products from the same product family packed in every bin, addressing family unity.

### 4.1. Decision variables

The decision variables of the comprehensive 3D-BPP model are summarized in Table 3.

The binary variables can be classified as assignment variables ($p_{ij}$,$u_j$), position variables ($x_{ik}^{+}$, $x_{ik}^{-}$, $y_{ik}^{+}$, $y_{ik}^{-}$, $z_{ik}^{+}$, $z_{ik}^{-}$), orientation variables ($l_i^x$,$l_i^y$, $w_i^x$, $w_i^y$) and others ($\beta_{jf}$). The continuous variables indicate the coordinates of items $(x_i, y_i, z_i)$ and barycenter of bins $\left(g_j^x, g_j^y, g_j^z\right)$. The position variables denote the position of items relative to each other where$i < k$.

### 4.2. The objectives

The proposed 3D-BPP with multiple objectives aims to pack all items into bins with three goals:

- Use fewest bins possible,
- Keep items from the same product family together while allocating to bins,
- Maintain load balance in bins.

Therefore, we define three objective functions that minimizes the total number of bins used, maximizes family unity and minimizes total deviation from the ideal barycenter of each bins.

The first objective function addresses the most common objective function of minimizing the total number of bins in 3D-BPP as in (1).

$$\textbf{Minimize} \quad \sum_{j}^{|J|} u_j \tag{1}$$

The second objective aims to maximize the number of items from the same product family packed in any bin which addresses family unity.

$$\textbf{Minimize} \quad \sum_{f=1}^{|F|} \sum_{j=1}^{|J|} \beta_{jf} \tag{2}$$

Thus, the assignments of items in the same family of products to the same bin are encouraged by (2). Keeping all the items from the same family of products in the same bin may not be achievable for the majority of the cases. However, the model promotes such solutions to ensure that the items assigned to the bins to be from the same family of products as much as possible. In the best-case scenario, each bin may have items only from the same product family, where $\sum_f \beta_{jf} = 1 (\forall j \in J)$ and$\sum_j \beta_{jf} \le \sum_j u_j (\forall f \in F)$. In the worst-case scenario, each bin has at least one item from each product family, where $\sum_j \sum_f \beta_{jf} = |F|^*|J|$.

The third objective is the load-balance objective function that minimizes the total deviation of the actual barycenter from the ideal center of gravity for all bins, as in (3).

$$\textbf{Minimize} \quad \sum_{j=1}^{|J|} \left| g_j^x - \frac{L_j}{2} \right| + \left| g_j^y - \frac{W_j}{2} \right| + \left| g_j^z - 0 \right| \tag{3}$$

Consequently, we formulate the weighted objective function of (1), (2) and (3) by defining parameters$\theta_1, \theta_2, \theta_3 \ge 0$ respectively for relative importance interests. For scaling, we employ a typical proportion-based normalization procedure. In order to normalize each objective in the entire range of the pareto-optimal region, the nadir and ideal objective vectors can be used (Deb, 2001). Each objective function is normalized to be in the interval of [0,1] by computing differences of optimal function values in the nadir and utopian objective function values that gives the length of the intervals where the optimal objective functions vary within the Pareto optimal set. Let $o_i^U$ is the utopian objective value and $o_i^N$is the nadir objective value, then objective function $f_i(x)$ is normalized as $\frac{f_i(x) - o_i^U}{o_i^N - o_i^U}$.

### 4.3. Constraints

The 3D-BPP seeks the optimal packing plan for a given set of items assuming that the number of bins is not limited. Constraints (4) ensure that an item $i$ can be assigned to a bin $j$ only if bin $j$ is used.

$$p_{ij} \le u_j \qquad \forall i \in I, \forall j \in J \tag{4}$$

Constraints (5) ensure that each item $i$ is packed.

$$\sum_{j=1}^{|J|} p_{ij} = 1 \qquad \forall i \in I \tag{5}$$

Constraints (6) and (7) ensure that items are placed into bin without exceeding the length/width of bin where $M$ is a large scalar. Note again that we consider only two possible orientations. Therefore, rotation in the $z$-axis is not taken into account in constraints (8) which ensures that the height of the bin is not exceeded.

$$x_i + l_i l_i^x + w_i w_i^x \le L_j u_j + (1 - p_{ij})M \qquad \forall i \in I, \forall j \in J \tag{6}$$

$$y_i + l_i l_i^y + w_i w_i^y \le W_j u_j + (1 - p_{ij})M \qquad \forall i \in I, \forall j \in J \tag{7}$$

$$z_i + h_i \le H_j u_j + \left(1 - p_{ij}\right)M \qquad \forall i \in I, \forall j \in J \tag{8}$$

Constraints (9)–(14) prevents overlapping of items $i$ and $k$, if they are packed in the same bin.

$$x_i + l_i l_i^x + w_i w_i^x \le x_k + (1 - x_{ik}^+)M \qquad \forall i, k \in I, i < k \tag{9}$$

$$x_k + l_k l_k^x + w_k w_k^x \le x_i + (1 - x_{ik}^-)M \qquad \forall i, k \in I, i < k \tag{10}$$

$$y_i + l_i l_i^y + w_i w_i^y \le y_k + (1 - y_{ik}^+)M \qquad \forall i, k \in I, i < k \tag{11}$$

$$y_k + l_k l_k^y + w_k w_k^y \le y_i + (1 - y_{ik}^-)M \qquad \forall i, k \in I, i < k \tag{12}$$

$$z_i + h_i \le z_k + \left(1 - z_{ik}^+\right)M \qquad \forall i, k \in I, i < k \tag{13}$$

$$z_k + h_k \le z_i + \left(1 - z_{ik}^-\right)M \qquad \forall i, k \in I, i < k \tag{14}$$

Constraints (15) ensure that if item $i$ and $k$ are in the same bin, then at least one of the relative positions (left, right, above, under, behind or in-front-of) should hold.

$$x_{ik}^+ + x_{ik}^- + y_{ik}^+ + y_{ik}^- + z_{ik}^+ + z_{ik}^- \ge p_{ij} + p_{kj} - 1 \qquad \forall i, k \in I, i < k \tag{15}$$

Constraints (16)–(19) define the proper orientations for each item.

$$l_i^x + l_i^y = 1 \qquad \forall i \in I \tag{16}$$

$$l_i^x + w_i^x = 1 \qquad \forall i \in I \tag{17}$$

$$w_i^x + w_i^y = 1 \qquad \forall i \in I \tag{18}$$

$$l_i^y + w_i^y = 1 \qquad \forall i \in I \tag{19}$$

Constraint (20) ensures that the total weight of items in bin $j$ does not exceed the capacity of the bin.

$$\sum_{i=1}^{|I|} a_i p_{ij} \le A_j u_j \qquad \forall j \in J \tag{20}$$

Constraints (21)–(23) computes the barycenter of each bin in $x$-axis, $y$-axis and $z$-axis.

$$g_j^x \sum_{i=1}^{|I|} a_i p_{ij} = \sum_{i=1}^{|I|} a_i p_{ij} \left( x_i + \frac{1}{2} l_i l_i^x + \frac{1}{2} w_i w_i^x \right) \qquad \forall j \in J \tag{21}$$

$$g_j^y \sum_{i=1}^{|I|} a_i p_{ij} = \sum_{i}^{|I|} a_i p_{ij} \left( y_i + \frac{1}{2} l_i l_i^y + \frac{1}{2} w_i w_i^y \right) \qquad \forall j \in J \tag{22}$$

$$g_j^z \sum_{i=1}^{|I|} a_i p_{ij} = \sum_{i}^{|I|} a_i p_{ij} \left( z_i + \frac{1}{2} h_i \right) \qquad \forall j \in J \tag{23}$$

Since $\beta_{jf}$ represents the existence of a family in a bin $j$, constraints (24) connect family existence and used bins. Constraints (25) determine if at least one item exists in a bin $j$ from family $f$, i.e. $\beta_{jf} = 1$, for each bin considering the families of the items, $\alpha_{if}$.

$$\beta_{jf} \le u_j \qquad \forall j \in J, \forall f \in F \tag{24}$$

$$\sum_{i=1}^{|I|} \alpha_{if} p_{ij} \le \beta_{jf} M \qquad \forall j \in J, \forall f \in F \tag{25}$$

Finally, the binary variables and non-negativity constraints are given in constraints (26) and (27), respectively.

$$p_{ij}, u_j, x_{ik}^+, x_{ik}^-, y_{ik}^+, y_{ik}^-, z_{ik}^+, z_{ik}^-, l_i^x, l_i^y, w_i^x, w_i^y, \beta_{jf} \in \{0, 1\} \tag{26}$$

$$x_i, y_i, z_i, g_j^x, g_j^y, g_j^z \ge 0 \tag{27}$$

### 4.4. Linearization for load-balance formulations

In this section, we linearize the model by defining new auxiliary variables and additional constraints in the model. The load-balance objective function (3) uses a rectilinear metric to calculate the distance between the actual barycenter and the ideal barycenter of a bin. We introduce additional nonnegative variables for each absolute value term in (3) and replace the absolute term with the summation of new

**Table 4**
3D-BPP instance with 16 items (Trivella & Pisinger, 2016).

| Item | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $w_i$ | 9 | 3 | 6 | 6 | 5 | 2 | 7 | 4 | 2 | 7 | 8 | 1 | 2 | 5 | 6 | 3 |
| $l_i$ | 7 | 5 | 6 | 1 | 3 | 3 | 3 | 8 | 7 | 10 | 6 | 2 | 4 | 4 | 9 | 7 |
| $h_i$ | 2 | 8 | 5 | 8 | 9 | 10 | 8 | 3 | 6 | 3 | 9 | 2 | 1 | 5 | 3 | 3 |

**Table 5**
Comparative results for the mathematical models.

| | (Trivella & Pisinger, 2016) | | | The proposed model (with no orientation) | | | The proposed model (with orientation) | | |
|---|---|---|---|---|---|---|---|---|---|
| Bin | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 1st | 2nd | 3rd |
| $g_j^x$ | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5.071 |
| $g_j^y$ | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5.031 |
| $g_j^z$ | 3.716 | 3.341 | 4.265 | 3.716 | 3.341 | 4.265 | 3.933 | 2.613 | 4.244 |
| **Total deviation** | **11.322** | | | **11.322** | | | **10.892** | | |

nonnegative variables. We employ Eq. (28) for load balance objective function as in:

$$\textbf{Minimize} \quad \sum_{j}^{|J|}\left(e_j^{x+}+e_j^{x-}+e_j^{y+}+e_j^{y-}+e_j^{z+}+e_j^{z-}\right) \tag{28}$$

where $\left|g_j^x - \frac{L_j}{2}\right| = e_j^{x+} - e_j^{x-}, e_j^{x+} \geq 0, e_j^{x-} \geq 0,$

$\left|g_j^y - \frac{W_j}{2}\right| = e_j^{y+} - e_j^{y-} e_j^{y+} \geq 0 e_j^{y-} \geq 0,$

$\left|g_j^z\right| = e_j^{z+} - e_j^{z-} e_j^{z+} \geq 0 e_j^{z-} \geq 0.$

Consequently, the linearized objective function (29) can be defined as;

$$\textbf{Minimize} \quad \theta_1 \sum_{j}^{|J|} u_j + \theta_2 \sum_{f=1}^{|F|}\sum_{j=1}^{|J|}\beta_{jf} + \theta_3 \sum_{j=1}^{|J|}\left(e_j^{x+}+e_j^{x-}+e_j^{y+}+e_j^{y-}+e_j^{z+}+e_j^{z-}\right) \tag{29}$$

In the mathematical model, also constraints (21)–(23) computing the barycenter of each bin $j$ are nonlinear equations. The terms in Eq. (21) are rearranged as in Eq. (30).

$$\sum_i a_i p_{ij} g_j^x - \sum_i a_i p_{ij} x_i - \sum_i a_i p_{ij} \frac{1}{2} l_i l_i^x - \sum_i a_i p_{ij} \frac{1}{2} w_i w_i^x = 0 \tag{30}$$

There are two types of nonlinear terms in Eq. (30), where two terms appear to be the multiplication of one binary variable and one continuous variable, and the remaining two terms are depicted as the multiplication of two binary variables.

For linearizing the term including multiplication of one binary variable and one continuous variable, we introduce a new nonnegative variable $d_{ij}$ with $d_{ij} = p_{ij} g_j^x$ and add the inequalities given in Eqs. (31)–(33) to the mathematical model.

$$d_{ij} \geq g_j^x - M\left(1 - p_{ij}\right) \tag{31}$$

$$d_{ij} \leq M p_{ij} \tag{32}$$

$$d_{ij} \leq g_j^x \tag{33}$$

By setting the same linearization method, we define a nonnegative variable $e_{ij}$, where $e_{ij} = p_{ij} x_i$, and add the following inequalities, in Eqs. (34)–(36), to the mathematical model.

$$e_{ij} \geq x_i - M\left(1 - p_{ij}\right) \tag{34}$$

$$e_{ij} \leq M p_{ij} \tag{35}$$

$$e_{ij} \leq x_i \tag{36}$$

For linearizing the multiplication of two binary variables in the proposed model, we introduce binary variable $b_{ij}$ with $b_{ij} = p_{ij} l_i^x$, and add Eqs. (37)–(39) to the mathematical model.

$$b_{ij} \leq l_i^x \tag{37}$$

$$b_{ij} \leq p_{ij} \tag{38}$$

$$b_{ij} \geq l_i^x + p_{ij} - 1 \tag{39}$$

Similarly, we employed the same procedure to linearize $\left(p_{ij} w_i^x\right)$ multiplication and introduce nonnegative variable $c_{ij}$ with $c_{ij} = p_{ij} w_i^x$ and add Eqs. (40)–(42) to the mathematical model.

$$c_{ij} \leq w_i^x \tag{40}$$

$$c_{ij} \leq p_{ij} \tag{41}$$

$$c_{ij} \geq w_i^x + p_{ij} - 1 \tag{42}$$

Consequently, we replace Eq. (21) with Eq. (43) in the proposed model and add the constraints in Eqs. (31)–(42) to mathematical model.

$$\sum_i a_i d_{ij} - \sum_i a_i e_{ij} - \sum_i \frac{1}{2} a_i l_i b_{ij} - \sum_i \frac{1}{2} a_i w_i c_{ij} = 0 \tag{43}$$

Similarly, we utilize the same transformation procedures to linearize constraint (22) and replace it with the constraints (44)–(56) in the mathematical model.

$$\sum_i a_i t_{ij} - \sum_i a_i v_{ij} - \sum_i \frac{1}{2} a_i l_i r_{ij} - \sum_i \frac{1}{2} a_i w_i s_{ij} = 0 \tag{44}$$

$$t_{ij} \geq g_j^y - M\left(1 - p_{ij}\right) \tag{45}$$

**Table 6**
Comparative Results for Different Family Settings.

| | Single Family Case $|F = 1$ | | | 2-Family Case $|F| = 2$ | | | 3-Family Case (a) $|F| = 3$ | | | 3-Family Case (b) $|F| = 3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bin | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 4th |
| $g_j^x$ | 5 | 5 | 5.071 | 5 | 5 | 4.797 | 5 | 5 | 5 | 5 | 4.621 | 5 | 5 |
| $g_j^y$ | 5 | 5 | 5.031 | 5 | 5.226 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| $g_j^z$ | 3.933 | 2.613 | 4.244 | 2.603 | 4.149 | 4.418 | 4.193 | 3.795 | 4.119 | 2.806 | 3.262 | 4.309 | 4.401 |
| Items | 3,6,7,10,13,14 | 1,8,12,15,16 | 2, 4, 5, 9, 11 | 10, 12, 15, 16 | 1, 2, 3, 4, 5, 7 | 6,8,9, 11, 13, 14 | 7, 9, 11 | 8,10,12,13,14,15,16 | 1,2,3, 4,5,6 | 10,15 | 1,5,6,8,12 | 2,4,11,13 | 3,7,9,14,16 |
| Family (%) | | | | 100% B | 100 %A | 33% A67% B | 100% B | 29% B71% C | 100 %A | 100 %C | 100 %A | 100 %C | 100 %B |
| FUR | None | None | None | 100% | 100% | 67% | 100% | 71% | 100% | 100% | 100% | 100% | 100% |
| Total deviation | 10.892 | | | 11.599 | | | 12.107 | | | 15.157 | | | |

$$t_{ij} \leq Mp_{ij} \tag{46}$$

$$t_{ij} \leq g_j^y \tag{47}$$

$$v_{ij} \geq y_i - M(1 - p_{ij}) \tag{48}$$

$$v_{ij} \leq Mp_{ij} \tag{49}$$

$$v_{ij} \leq y_i \tag{50}$$

$$r_{ij} \leq l_i^y \tag{51}$$

$$r_{ij} \leq p_{ij} \tag{52}$$

$$r_{ij} \geq l_i^y + p_{ij} - 1 \tag{53}$$

$$s_{ij} \leq w_i^y \tag{54}$$

$$s_{ij} \leq p_{ij} \tag{55}$$

$$s_{ij} \geq w_i^y + p_{ij} - 1 \tag{56}$$

where $t_{ij} = p_{ij}g_j^y$ , $v_{ij} = p_{ij}y_i$, $r_{ij} = p_{ij}l_i^y$ and $s_{ij} = p_{ij}w_i^y$ hold.

Finally, we introduce new variables as $m_{ij} = p_{ij}g_j^z$ and $n_{ij} = p_{ij}z_i$ and linearize constraints (23) by substituting constraints (57)–(63) into the mathematical model.

$$\sum_i a_i m_{ij} - \sum_i a_i n_{ij} - \sum_i \frac{1}{2} a_i h_i p_{ij} = 0 \tag{57}$$

$$m_{ij} \geq g_j^z - M(1 - p_{ij}) \tag{58}$$

$$m_{ij} \leq Mp_{ij} \tag{59}$$

$$m_{ij} \leq g_j^z \tag{60}$$

$$n_{ij} \geq z_i - M(1 - p_{ij}) \tag{61}$$

$$n_{ij} \leq Mp_{ij} \tag{62}$$

$$n_{ij} \leq z_i \tag{63}$$

Summarizing the previous concerns, the resulting multi-objective MILP model aims to minimize functions in Eq. (29). The proposed model contains the constraints in Eqs. (4)–(20), (24)–(27) and (31)–(63). Therefore, by eliminating the nonlinear expressions, we highlight the advantages of MILP formulation due to the increasing size of the mathematical model.

## 5. Application

In this section, we present a numerical study by using the instances used in Trivella and Pisinger (2016) to verify our model and a real-case study that stimulates this work to include item-related concerns in solving 3D-BPPs.

### 5.1. Numerical study

For the verification and validation of the comprehensive 3D-BPP model, we present a comparative analysis with Trivella and Pisinger (2016), as this is the first and only study that discusses the outputs of the proposed load-balanced multi-dimensional bin packing MIP formulation in the literature. Martello et al. (2000) present 8 data classes for three-dimensional multiple bin packing problems which have been widely accepted in the literature. Trivella and Pisinger (2016) further incorporate a new dimension, item weight, to the datasets. The data instance, in Table 4, is employed for presenting an explanatory example by

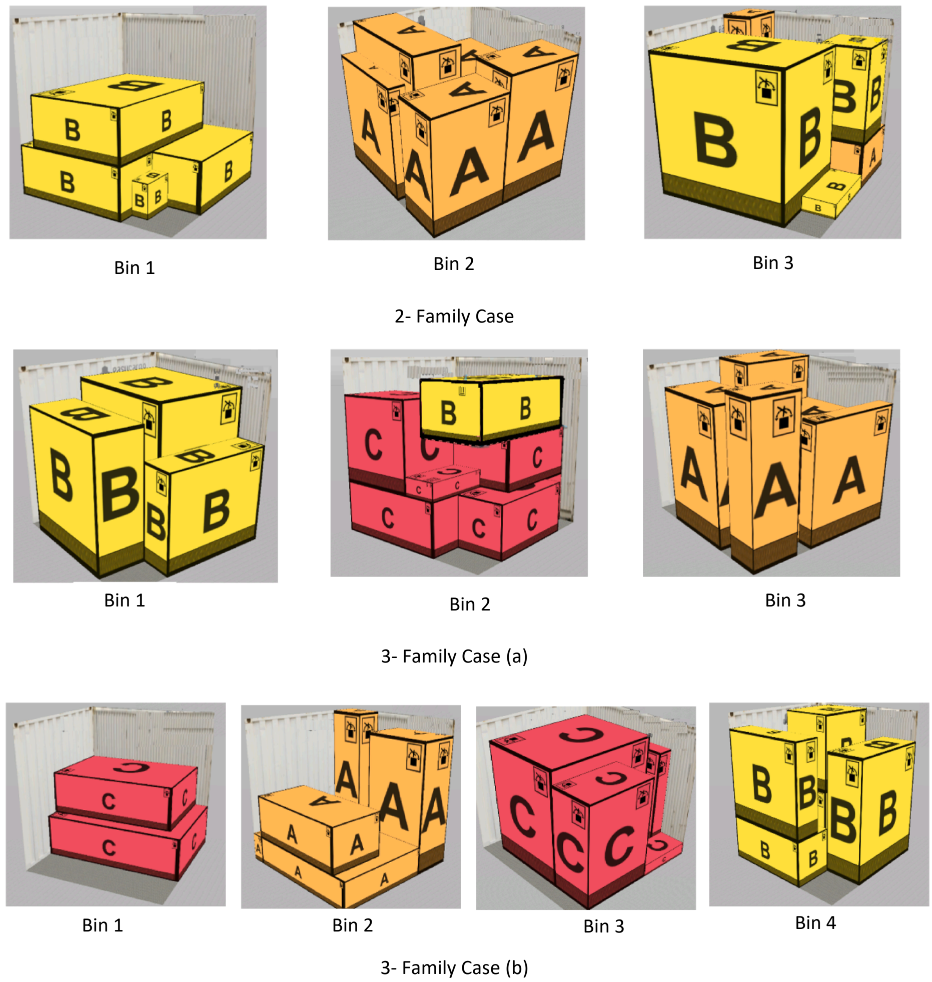



**Fig. 3.** Optimal Packing Plans.

**Table 7**
Comparative Results for Different Weight Combinations.

| Weights | Case 1<br>$\theta_1 = 1, \theta_2 = 0, \theta_3 = 0$ | | | Case 2<br>$\theta_1 = 0, \theta_2 = 0, \theta_3 = 1$ | | | Case 3<br>$\theta_1 = 0, \theta_2 = 1, \theta_3 = 0$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Bin | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 4th |
| $g_j^x$ | 5.164 | 4.299 | 5.261 | 5 | 5 | 5 | 3.977 | 4.18 | 4.266 | 4.863 |
| $g_j^y$ | 5.483 | 5.103 | 4.691 | 5 | 5 | 5 | 3.919 | 4.269 | 4.968 | 5.011 |
| $g_j^z$ | 5.021 | 5.591 | 4.22 | 4.282 | 2.976 | 4.309 | 4.256 | 6.838 | 4.247 | 4.262 |
| Family (%) | 80 %A<br>20 %C | 20% A<br>40% B<br>40% C | 12% A<br>50% B<br>33% C | 20 %A<br>60 %B<br>20 %C | 40 %B<br>60 %C | 83 %A<br>17 %C | 100 %B | 100 %C | 100 %B | 100 %A |
| FUR | 80% | 40% | 50% | 60% | 60% | 83% | 100% | 100% | 100% | 100% |
| Total deviation | 16.853 | | | 11.567 | | | 24.172 | | | |

| Weights | Case 4<br>$\theta_1 = 0.5, \theta_2 = 0.5, \theta_3 = 0$ | | | Case 5<br>$\theta_1 = 0.5, \theta_2 = 0, \theta_3 = 0.5$ | | | Case 6<br>$\theta_1 = 0, \theta_2 = 0.5, \theta_3 = 0.5$ | | | Case 7<br>$\theta_1 = 0.33, \theta_2 = 0.33, \theta_3 = 0.33$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bin | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 1st | 2nd | 3rd | 1st | 2nd | 3rd |
| $g_j^x$ | 5.195 | 5.276 | 5.219 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| $g_j^y$ | 5.558 | 4.939 | 4.491 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| $g_j^z$ | 5.729 | 4.929 | 4.687 | 2.280 | 4.710 | 4.242 | 4.193 | 3.851 | 4.119 | 4.193 | 3.795 | 4.119 |
| Family (%) | 67 %B<br>33 %C | 100 %A | 100 %C | 67 %B<br>33 %C | 50% A<br>50% C | 33% A<br>50% B | 100 %B | 29 %B<br>71 %C | 100 %A | 100 %B | 29 %B<br>71 %C | 100 %A |
| FUR | 67% | 100% | 100% | 67% | 50% | 50% | 100% | 71 %C | 100 %A | 100% | 71% | 100% |
| Total deviation | 17.163 | | | 11.232 | | | 12.163 | | | 12.107 | | |

**Table 8**
Filter Types.

| Filter Type | Number | Width (cm) | Length (cm) | Height (cm) |
|---|---|---|---|---|
| Air filter | 8 | 145 | 145 | 175 |
| Oil filter | 8 | 145 | 145 | 175 |
| Fuel filter | 12 | 138 | 138 | 158 |

Trivella and Pisinger (2016). This data instance containing 16 items is generated according to the 6th class of Martello et al. (2000)'s datasets and item densities are assumed to be 1. The bins are identical to each other and each in the shape of a cube with a side of 10 units.

To be consistent with Trivella and Pisinger (2016), we restricted our model with two assumptions: (i) orientation of items is not allowed and (ii) family unity related components are omitted. The model is coded in GAMS 22.8 on an Intel i5 2.50 GHz computer with 8.0 GB RAM and 64-bit operating system and solved for optimality. Then, we allowed the orientation of items in x- and y-axis as in our proposed model and obtained better results as expected. The coordinate of the ideal barycenter is (5, 5, 0) and the total deviation is computed by using the formulation as in (2). The results are summarized in Table 5 where the total deviation is calculated using (2).

It is not a coincidence that the objective function is improved when we allow the orientation of items. For this dataset, about a 4% improvement is achieved on the objective function, even when rotation is allowed only on its base.

Subsequently, we incorporate the family unity concept into load-balanced 3D-BPP. According to our knowledge, this study is the first work in the literature that introduces the product family concept for 3D-BPP. Therefore, we use the numerical example of Trivella and Pisinger (2016) as a testbed to better emphasize the significance of the concept. We generate cases by assigning items to families to represent the results for three different cases in addition to Single Family Case, as in Table 6. In 2-Family Case, for ease of control, we define two types of product family where the first 8 items (1–8) are assumed to be in family A and the remaining items (9–16) are in family B. In order to examine the implications of higher the family variety, we briefly represent two different 3-Family Cases. The difference between these two cases is only related the item-family assignments. The items (1–6) belong to Family A, the items (7–11) belong to Family B and the remaining items belong to Family C in the 3-Family Case (a) where 5 items (1,5,6,8,12) belong to Family A, 5 items (3,7,9,14,16) belong to Family B and remaining items (2,4,10,11,13,15) belong to Family C in the 3-Family Case (a).

We introduce the concept of family unity ratio (FUR) to evaluate the packing of items from the same family into the same bin. Let $\mathscr{A}$ be the set of items packed in a bin, $\mathscr{F}$ be the set of product families and $\mathscr{e}$ is an injective function that maps distinct subsets of $\mathscr{A}$ to distinct elements of its codomain set $\mathscr{F}$. Suppose $\mathscr{A}_1, \mathscr{A}_2 \cdots, \mathscr{A}_f$ denote the collection of distinct subsets of $\mathscr{A}$ where $\mathscr{A} = \mathscr{A}_1 \cup \mathscr{A}_2 \cdots \cup \mathscr{A}_f$ holds. Then, $\mathscr{A}_f$ is one of the subsets of $\mathscr{A}$ and is associated with a product family $f$ in set $\mathscr{F}$ as in (64)

$$\mathscr{A}_f = \{ i | i \in \mathscr{A} \wedge \mathscr{e}(i) = f \} \qquad \forall f \in \mathscr{F} \tag{64}$$

The family unity ratio is the ratio of the cardinality of the subset with the highest cardinality to the cardinality of $\mathscr{A}$.

$$FUR = \frac{\max_f |\mathscr{A}_f|}{|\mathscr{A}|} \tag{65}$$

FUR values are calculated using (65). Under single family case, FUR values will not give any insight as they all will be calculated as 100%.

The result of the numerical study shows that three objectives in the mathematical model are conflicting objectives and ignoring the family unity objective may prevent to determine superior solutions without disrupting the objectives of minimizing the number of bins used but not a deviation from the ideal barycenter. For $|F| = 2$ and $|F|_{(b)} = 3$ cases, all items are packed in 3 bins, as in Single Family Case. Despite the increase in item variety, the number of bins used has not changed, but the family unity ratio of each bin cannot be achieved at 100%. Additionally, for three family cases, 3-Family Case (b) is better than 3-Family Case (a) in terms of family unity objectives, but sacrifices from the load-balance objective, considerably. The load-balance objective may result in inferior values as family diversity increases. By presenting these cases, we show possible consequences of including/ignoring one of these objective functions. Therefore, we emphasize the importance of evaluating solutions in terms of more than one objective function. Fig. 3 illustrates the optimal packing plans for 2-Family case and 3-Family case which are visualized by using EasyCargo software.

When the packing plans in the Fig. 3 are examined, it is clearly seen that in 2- Family Case and 3- Family Case (a), the two bins are completely homogeneous in terms of product family, and all bins used in 3- Family Case (b) are homogeneous.

We present the outputs for various combinations of $\theta$ values in order to analyze the potential consequences of changes in the importance weights of objective functions. Table 7 shows seven cases, each of which accounts for a single objective function in the first three cases, the outcomes of a two-combination of the three objectives in Cases 4–6, and the final case (Case 7) demonstrates the simultaneous consideration of the three objective functions.

While the number of bins used in Case 1 and 2 is 3, when the weight of objective 2 is taken as 1, the number of bins increased to 4, and the FUR value is found to be 100% in all bins. However, the deviation from the center of gravity was dramatically high. In other combinations, uniform changes observed according to the weights.

The optimal packing decision is affected regarding the importance weight of family unity, the number of bins used and the total deviation from the ideal barycenter. The proposed model can increase the awareness of decision makers on the tradeoffs of these objective functions. This study anticipates the disadvantages that may arise from ignoring the product family concept.

**Table 9**
Case study results.

| | Solution for the real problem without product family (Case I) | | | | | | Solution for the real problem with product family (Case II) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Bin | 1st | 2nd | 3rd | 4th | 5th | 6th | 1st | 2nd | 3rd | 4th | 5th | 6th |
| $g_j^x$ | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 |
| $g_j^y$ | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 | 152.00 |
| $g_j^z$ | 81.46 | 85.68 | 87.50 | 81.46 | 83.68 | 165.79 | 87.50 | 145.83 | 87.50 | 87.50 | 79.00 | 158.00 |
| Air filter | | 3 | 2 | 1 | 1 | 1 | 2 | 6 | | | | |
| Oil filter | 1 | | 2 | | 1 | 4 | | | 4 | 4 | | |
| Fuel filter | 3 | 1 | | 3 | 2 | 3 | | | | | 4 | 8 |
| FUR | 75% | 75% | 50% | 75% | 50% | 50% | 100% | 100% | 100% | 100% | 100% | 100% |
| Total Deviation | 585.57 | | | | | | 645.33 | | | | | |

### 5.2. Real case study

This work is motivated by a real problem from a Turkish filter company that produces several types of filters for local and international markets, especially in Europe, Russia and the Middle East regions. The company outsources logistics operations for overseas regions and uses its trucks for road transportation. The company makes three main classes of filters: air filters, oil filters and fuel filters. Regarding the specific requirements of handling, loading and storing, the company realizes the advantages of packing similar items together and reducing the item variety in bins. In Table 8, we summarize the dimensions and the quantities for three filter types. The filters are packed into identical bins with dimensions 305 × 305 × 365 cm.

We note that the data has been slightly modified in accordance with the confidentiality agreement. As depicted in Table 8, the air filters and the oil filters are the same in size and quantity but they have different handling requirements. When the family unity aspect of the problem is ignored, the mathematical models and methods generate layouts regardless of family unity requirements. Therefore, in order to observe the effect on results, we introduce Case I which neglects product family aspect and Case II includes real data of product families. We summarize the results for both cases in Table 9.

For the real problem, Case I provides the perfect value of the total deviation and the number of bins because the product family aspect is neglected. When the results of Case I is compared to Case II, the items from the same family can be packed into same bin without increasing the number of bins used. Regarding the outputs of the mathematical model for this problem, the minimum number of used bins is 6 for both Case I and Case II, and the product family unity in each bin in Case II is better than the bins in Case I. However, the deviation from the ideal barycenter has increased when the product family unity is encouraged. The results show that there exists a tradeoff between product family unity and load-balance objectives.

Lastly, the company's feedbacks on the quality of problem outputs can be addressed. Their business strategy is based primarily on on-time delivery and accurate order fulfillment. The use of analytical approach to solve 3D-BPP mainly improve their operations strategy and the family unity aspect allow them to create better loading plans and better handling operations in terms of time and cost. Using the proposed approach to ensure bin homogeneity reduces the need for various handling equipment at the same time simplifies delivery operations planning and reduces transportation costs as multiple visits to a single customer is minimized.

## 6. Conclusions and future research

Nowadays, as the competitive conditions between businesses increase, businesses improve themselves to reduce the costs of their processes and to provide more efficient service to customers. One of the most important processes in the order and delivery phase of the customers is the logistics process. In the logistics process, businesses are primarily concerned with loading boxes into bins more efficiently. Furthermore, it is known that effective packaging plans not only significantly reduce logistics and transportation costs but also increase customer satisfaction. In the literature, mostly, boxes are placed in bins, while the constraints used in real-life problems are ignored. Therefore, considering the constraints of real-life problems, it has been introduced a mathematical model for the load-balanced 3D-BPP where the orientation of the items is allowed and item-related concerns are considered via product family vision.

Firstly, in the proposed mathematical model, constraints (20)–(22) and objective function (3) are linearized and solved. For validating the model, Trivella and Pisinger (2016)'s problem is solved with the proposed model without allowing rotation of items and assuming the items are from the same family. Next, the comparative results are examined when we allow orientation of items in both x- and y-axis as in our proposed model. It is not a coincidence that the objective function is improved when we allow the 3D orientation of items. For the used dataset, about a 4% improvement has been achieved on the objective function, even when only rotation is allowed only on its base. Lastly, by changing the number of product families, the cases are created and it is observed that items belonging to the same family are assigned to the same bin as much as possible. However, as the number of families increased, deviation from the ideal barycenter increased.

Furthermore, a real 3D-BPP, which arises as a container loading problem with specific requirements, is presented and solved with the proposed mathematical model. The company has three types of product family and operates its packing activities by neglecting the specific requirements of these families. Ignoring the differences between product families increases complexity in loading operations and causes implicit logistic costs. Therefore, the packing decision is made by considering the product family in order to take the advantage of similar items being packed together in handling, loading and storage operations. The outputs of the real case study shows that the number of bins obtained in both cases remain same. However, the deviation from the ideal barycenter has increased when the product family unity is encouraged. The results show that there exists a tradeoff between product family unity and load-balance objectives.

Consequently, the purpose of this study is, we introduce a mathematical model for the load balanced 3D-BPP where orientation of the items is allowed and item-related concerns are considered via product family vision. According to our knowledge, we first propose a MIP model with a comprehensive set of constraints addressing three-dimensional replacement, stability and orientation, and a set of objective functions minimizing the number of bins used, the product family unity and the deviation from the ideal barycenter. The proposed mathematical model is promising, showing that effective load balancing and efficient family unity can be obtained. For solving large scale problems, the mathematical modelling approach may be considered unsatisfactory. Future work may seek to develop an efficient metaheuristic algorithm that addresses all the constraints and concepts proposed in this study.

## References

Alonso, M. T., Alvarez-Valdes, R., Iori, M., & Parreño, F. (2019). Mathematical models for multi container loading problems with practical constraints. *Computers & Industrial Engineering, 127*, 722–733.
Alonso, M. T., Alvarez-Valdes, R., Iori, M., Parreño, F., & Tamarit, J. M. (2017). Mathematical models for multicontainer loading problems. *Omega, 66*, 106–117.
Araya, I., Guerrero, K., & Nuñez, E. (2017). VCS: A new heuristic function for selecting boxes in the single container loading problem. *Computers & Operations Research, 82*, 27–35.
Baldi, M. M., Perboli, G., & Tadei, R. (2012). The three-dimensional knapsack problem with balancing constraints. *Applied Mathematics and Computation, 218*(19), 9802–9818.
Bortfeldt, A. (2012). A hybrid algorithm for the capacitated vehicle routing problem with three-dimensional loading constraints. *Computers & Operations Research, 39*(9), 2248–2257.
Bortfeldt, A., & Wäscher, G. (2013). Constraints in container loading–A state-of-the-art review. *European Journal of Operational Research, 229*(1), 1–20.
Ceschia, S., & Schaerf, A. (2013). Local search for a multi-drop multi-container loading problem. *Journal of Heuristics, 19*(2), 275–294.
Chen, C. S., Lee, S. M., & Shen, Q. S. (1995). An analytical model for the container loading problem. *European Journal of Operational Research, 80*(1), 68–76.
Christensen, S. G., & Rousøe, D. M. (2009). Container loading with multi-drop constraints. *International Transactions in Operational Research, 16*(6), 727–743.
Crainic, T. G., Gobbato, L., Perboli, G., & Rei, W. (2016). Logistics capacity planning: A stochastic bin packing formulation and a progressive hedging meta-heuristic. *European Journal of Operational Research, 253*(2), 404–417.
Deb, K. (2001). Multi-Objective Optimization Using Evolutionary Algorithms. USA: John Wiley & Sons, Inc. (ISBN: 978-0-471-87339-6).
Gendreau, M., Iori, M., Laporte, G., & Martello, S. (2006). A tabu search algorithm for a routing and container loading problem. *Transportation Science, 40*(3), 342–350.
EasyCargo [Computer Software]. (2021). Retrieved from https://www.easycargo3d.com/.
Egeblad, J., & Pisinger, D. (2009). Heuristic approaches for the two-and three-dimensional knapsack packing problem. *Computers & Operations Research, 36*(4), 1026–1049.
Egeblad, J., Garavelli, C., Lisi, S., & Pisinger, D. (2010). Heuristics for container loading of furniture. *European Journal of Operational Research, 200*(3), 881–892.

Eley, M. (2003). A bottleneck assignment approach to the multiple container loading problem. *OR Spectrum, 25*(1), 45–60.
Elhedhli, S., Gzara, F., & Yildiz, B. (2019). Three-dimensional bin packing and mixed-case palletization. *Informs Journal on Optimization, 1*(4), 323–352.
Gimenez Palacios, I., Alonso, M. T., Alvarez Valdes, R., & Parreño, F. (2020). Logistic constraints in container loading problems: the impact of complete shipment conditions. *Top, 29*, 177–203.
Junqueira, L., Morabito, R., & Yamashita, D. S. (2012). Three-dimensional container loading models with cargo stability and load bearing constraints. *Computers & Operations Research, 39*(1), 74–85.
Kang, K., Moon, I., & Wang, H. (2012). A hybrid genetic algorithm with a new packing strategy for the three-dimensional bin packing problem. *Applied Mathematics and Computation, 219*(3), 1287–1299.
Liu, D. S., Tan, K. C., Huang, S. Y., Goh, C. K., & Ho, W. K. (2008). On solving multiobjective bin packing problems using evolutionary particle swarm optimization. *European Journal of Operational Research, 190*(2), 357–382.
Liu, J., Yue, Y., Dong, Z., Maple, C., & Keech, M. (2011). A novel hybrid tabu search approach to container loading. *Computers & Operations Research, 38*(4), 797–807.
Martello, S., & Vigo, D. (1998). Exact solution of the two-dimensional finite bin packing problem. *Management science, 44*(3), 388–399.
Martello, S., Pisinger, D., & Vigo, D. (2000). The three-dimensional bin packing problem. *Operations Research, 48*(2), 256–267.
Moon, I., & Nguyen, T. V. L. (2014). Container packing problem with balance constraints. *OR spectrum, 36*(4), 837–878.
Pedruzzi, S., Nunes, L. P. A., Rosa, R., & Arpini, B. (2016). A mathematical model to optimize the volumetric capacity of trucks utilized in the transport of food products. *Gest. Prod, 23*, 350–364.
Ramos, A. G., Silva, E., & Oliveira, J. F. (2018). A new load balance methodology for container loading problem in road transportation. *European Journal of Operational Research, 266*(3), 1140–1152.
Tarantilis, C. D., Zachariadis, E. E., & Kiranoudis, C. T. (2009). A hybrid metaheuristic algorithm for the integrated vehicle routing and three-dimensional container-loading problem. *IEEE Transactions on Intelligent Transportation Systems, 10*(2), 255–271.
Terno, J., Scheithauer, G., Sommerweiß, U., & Riehme, J. (2000). An efficient approach for the multi-pallet loading problem. *European Journal of Operational Research, 123*(2), 372–381.
Trivella, A., & Pisinger, D. (2016). The load-balanced multi-dimensional bin-packing problem. *Computers & Operations Research, 74*, 152–164.
Zhao, X., Bennell, J. A., Bektaş, T., & Dowsland, K. (2016). A comparative review of 3D container loading algorithms. *International Transactions in Operational Research, 23*(1–2), 287–320.