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Extraction of soliton waves from the longitudinal wave equation with local M-truncated derivatives

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Abstract
We have extracted some soliton solutions of the fractional longitudinal wave equation with the M-truncated derivative (M-LWE), which emerges in a magneto electro-elastic circular rod. To obtain new results of this model, the unified Riccati equation expansion and new Kudryashov methods have been utilized for the first time. The presented methods have been productively implemented to the considered model. With the help of these two methods, new soliton waves of the M-LWE have been obtained successfully. For a better understanding of the subject and analysis of the results, 3D, contour, and 2D graphs of some soliton solutions have been presented. The interesting part of our work is that both methods, named unified Riccati equation expansion and the new Kudryashov methods have successfully been applied for the first time to get new soliton solutions of M-LWE.

Keywords Unified Riccati equation expansion method · New Kudryashov method · The longitudinal wave equation with local M-derivative · Soliton solutions

1 Introduction

Soliton theory is of great significance as numerous mathematical physical models have various soliton solutions. The models that produce soliton solutions are being used in different sciences and engineering fields, especially in physical models that symbolize nonlinear partial differential equations (NLPDEs) such as the expansion of wave and heat flow phenomena, mathematical biology, chemical kinematics, electricity, optical fibers, plasma physics, and quantum mechanics. Therefore, many authors have attached

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great importance to studying soliton structures with various analytical approaches in nonlinear models. Such as, the bell-shaped perturbed dispersive optical solitons of the Biswas Arshed by the new Kudryashov's method (Ozisik et al. 2022), optical soliton to Fokas–Lenells model via the enhanced modified extended tanh-expansion technique (Esen et al 2022), perturbation of dispersive optical solitons with Schrödinger–Hirota equation with Kerr law and Spatio-temporal dispersion via the direct algebraic scheme of the enhanced modified extended tanh expansion method (Ozdemir et al. 2022), the F-expansion technique to the dimensionless form of the nonlinear Schrödinger model with Kerr law nonlinearity (Biswas et al. 2019), the solitary wave solutions of Chafee–Infante and (2 + 1)-dimensional breaking soliton models (Habiba et al. 2019), the bright and dark solitons of a weakly nonlocal Schrödinger model with the parabolic law nonlinearity (Hosseini et al. 2021), the Sine–Gordon and Kudryashov techniques (Akbar et al. 2021), the analytical solutions to Zoomeron model utilizing extended rational sin–cos and sinh–cosh approaches (Cinar et al. 2021), optical solitons to Chen–Lee–Liu equation (Ozdemir et al. 2021), convex-periodic, kink-periodic, peakon-soliton and kink bidirectional wave-solutions to Cahn–Allen model (Alquran and Alhami 2022), the exact traveling wave solutions and analysis of the dual-mode Hirota–Satsuma coupled KdV equations (Alquran et al. 2019), cubic-quartic optical solitons with Kudryashov's law of refractive index via Lie symmetry analysis (Kumar and Malik 2021), integrability, stability analysis, and soliton solutions for the (2 + 1)-dimensional combined KdV–mKdV model (Malik et al. 2022), optical soliton to generalized coupled nonlinear schrödinger KdV model (Akinyemi et al. 2021), soliton solutions of generalized (2 + 1) -dimensional Boussinesq–Kadomtsev–Petviashvili-like model (Akinyemi et al. 2022), the higher-order modified Korteweg-de Vries equation (Ntiamoah et al. 2022), the solitary wave solutions and stability analysis in liquid with gas bubbles (Zhao et al. 2022), the new Kudryashov method (Malik et al. 2023), the ansatz method (Akinyemi and Morazara 2023), and numerous kinds of research.

The M-LWE that we focused in this study is defined as: (Xue et al. 2011; Yépez-Martínez and Gómez-Aguilar 2018):

$$\mathbf{D}_{M,t}^{2\alpha,\gamma}\Theta(x,t)-\omega_0^2\mathbf{D}_{M,x}^{2\alpha,\gamma}\Theta(x,t)-\mathbf{D}_{M,x}^{2\alpha,\gamma}\left[\frac{\omega_0^2}{2}\Theta^2(x,t)+\rho\mathbf{D}_{M,t}^{2\alpha,\gamma}\Theta(x,t)\right]=0, \tag{1}$$

in which $\mathbf{D}_{M,t}^{2\alpha,\gamma}\Theta(x,t)$ expresses the M-truncated derivative of order α of Θ with respect to t. For $\alpha = 1$, Eq. (1) is defined as the longitudinal wave equation (LWE). ρ express the dispersion parameter, and ω_0 states the linear longitudinal wave velocity for a MEE circular rod; moreover, all of them hinge on geometry of the rod and the material characteristics. What motivates us for this study is that M-LWE and LWE have been studied few in the literature. Some of approaches utilized in the studies are the modified $\exp(\Omega(\eta))$-expansion function method in Yépez-Martínez and Gómez-Aguilar (2018), the Bernoulli sub-equation function method in Baskonus and Gómez-Aguilar (2019), the generalized exponential rational function method (Aljahdaly et al. 2021), the modified (G'/G)-expansion method (Ma et al. 2013), the modified Kudryashov method (Hosseini et al. 2018), the extended trial equation method (Seadawy and Manafian 2018).

The configuration of the article is presented as: In Sect. 2, we give the definition of the M-derivative. The nonlinear ordinary differential equation form of the M-LWE is acquired in Sect. 3. Sections 4 and 5 offer the main stages of the proposed methods and their applications to M-LWE. Physical interpretations of the graphs depicted in this study were fulfilled in Sect. 6. Section 7 is the conclusion part.

2 Definition of truncated M-fractional derivative

In this part, we provide descriptions of the local M-derivative.

Definition The truncated Mittag–Leffler function is expressed as follows (Sousa and de Oliveira 2017):

$$ {}_jE_\gamma(z) = \sum_{n=0}^{j} \frac{z^n}{\Gamma(\gamma n + 1)}, $$

where $\gamma > 0$ and $z \in C$.

Definition Assume that $P : [0, \infty) \to \mathbb{R}$ be a function, the local M-derivative of order $\alpha \in (0, 1)$, with respect to t is defined by (Sousa and de Oliveira 2017)

$$ \mathscr{D}_{M,t}^{\alpha,\gamma} P(t) = \lim_{\varepsilon \to 0} \frac{P\left(t\ {}_iE_\gamma(\varepsilon t^{-\alpha})\right) - P(t)}{\varepsilon}, $$

where $\gamma, t > 0$ and ${}_jE_\gamma(\cdot)$ states Mittag–Leffler function.

Theorem *Suppose that $P(t)$ be α order differentiable function at $t_0 > 0$ for $\alpha \in (0, 1]$ and $\gamma > 0$. Then, $P(t)$ is continuous at t_0 (Sousa and de Oliveira 2017).*

Theorem *Presume that P, Q is α-differentiable at any $t > 0$ where $0 < \alpha \le 1, \gamma > 0, p, q \in \mathbb{R}$. Then,*

1. $\mathscr{D}_{M,t}^{\alpha,\gamma}(pP + qQ)(t) = p\ \mathscr{D}_{M,t}^{\alpha,\gamma} P(t) + q\ \mathscr{D}_{M,t}^{\alpha,\gamma} Q(t)$,
2. $\mathscr{D}_{M,t}^{\alpha,\gamma}(PQ)(t) = P(t)\ \mathscr{D}_{M,t}^{\alpha,\gamma} Q(t) + Q(t)\ \mathscr{D}_{M,t}^{\alpha,\gamma} P(t)$,
3. $\mathscr{D}_{M,t}^{\alpha,\gamma}\left(\frac{P}{Q}\right)(t) = \frac{P(t)\ \mathscr{D}_{M,t}^{\alpha,\gamma} Q(t) - Q(t)\ \mathscr{D}_{M,t}^{\alpha,b-\gamma} P(t)}{P(t)^2}$,
4. *If P is differentiable, then* $\mathscr{D}_{M,t}^{\alpha,\gamma}(P)(t) = \frac{t^{1-\alpha}}{\Gamma(\gamma+1)} \frac{dP(t)}{dt}$.

3 Nonlinear ordinary differential form of the M-LWE

To acquire the more soliton solution of the nonlinear M-LWE utilizing the unified Riccati equation expansion and new Kudryashov methods. With the help of the following fractional wave transformation:

$$ \Theta = \Theta(x, t) = \Theta(\eta), \quad \eta = \frac{\lambda}{\alpha}\Gamma(1 + \gamma)(x^\alpha + \nu t^\alpha), \quad 0 < \alpha \le 1, \tag{2} $$

in which ν and λ are non-zero contants, Eq. (1) reduced to the following NLODE,

$$ \lambda^2 \nu^2 \rho \Theta'''' + (\omega_0^2 \Theta - \nu^2 + \omega_0^2)\Theta'' + \omega_0^2(\Theta')^2 = 0. \tag{3} $$

Integrating Eq. (3) twice and considering the integration constant as zero, we get the following equation:

$$2\lambda^2 v^2 \rho \Theta'' + (2\omega_0^2 - 2v^2)\Theta + \omega_0^2 \Theta^2 = 0. \tag{4}$$

4 Sketch and practice of new Kudryashov's method to M-LWE

In this part, the fundamental steps of the new Kudryashov method presented in Kudryashov (2020) and Rezazadeh et al. (2021) are explained as follows.

Presume that a solution of the form of Eq. (4) is taken as follow:

$$\Theta(\eta) = \sum_{l=0}^{m} A_l \theta^l(\eta),\ A_l \neq 0, \tag{5}$$

in which $A_l, 0 \leq l \leq m$ are real constants to be evaluated. The function $\theta^l(\eta)$ accomplishes the following first order differential equation,

$$(\theta'(\eta))^2 = \chi^2 \theta^2(\eta)[1 - \kappa \theta^2(\zeta)], \tag{6}$$

where χ, and κ are nonzero values to be figured out later. The solution of Eq. (6) is expressed as follows:

$$\theta(\eta) = \frac{4\sigma}{4\sigma^2 e^{\chi\eta} + \kappa e^{-\chi\eta}}, \tag{7}$$

where σ is a real constant. Utilizing form the homogeneous balance rule and the terms Θ'' and Θ^2 in Eq. (4), $m = 2$ is derived. So, Eq. (5) converts into the following expression:

$$\Theta(\eta) = A_0 + A_1\theta^1(\eta) + A_2\theta^2(\eta),\ A_2 \neq 0. \tag{8}$$

If the Eq. (8) and its derivatives according to Eq. (6) are substituted into Eq. (4), then the polynomial of $\theta(\eta)$ is acquired. Gathering all the θ^l coefficients and equalizing the coefficients to zero, then the following system is generated:

$$\begin{aligned}
\theta^0(\eta) &: -2A_0\left(\left(-\frac{A_0}{2} - 1\right)\omega_0^2 + v^2\right) = 0,\\
\theta^1(\eta) &: 2A_1\left(\left(A_0 + 1\right)\omega_0^2 + v^2\left(\chi^2\rho\,\lambda^2 - 1\right)\right) = 0,\\
\theta^2(\eta) &: \left(A_1^2 + 2A_2\left(A_0 + 1\right)\right)\omega_0^2 + 8v^2A_2\left(\chi^2\rho\,\lambda^2 - \frac{1}{4}\right) = 0,\\
\theta^3(\eta) &: -4A_1\left(v^2\chi^2\rho\kappa\,\lambda^2 - \frac{A_2\omega_0^2}{2}\right) = 0,\\
\theta^4(\eta) &: -12\chi^2\kappa\,\lambda^2\rho\,v^2A_2 + \omega^2A_2^2 = 0.
\end{aligned}$$

Solving this algebraic system with the aid of a Computer Algebraic Software, the following parameters families are derived:

Case 1:

$$\chi = \pm\frac{\sqrt{\rho\left(\omega_0^2 - \nu^2\right)}}{2\rho\nu\lambda}, A_0 = -\frac{2\left(\omega_0^2 - \nu^2\right)}{\omega_0^2}, A_1 = 0, A_2 = \frac{3\left(\omega_0^2 - \nu^2\right)\kappa}{\omega_0^2}. \tag{9}$$

Utilizing the parameters in Case 1, we acquire the following solution functions:

$$\Theta_1(x,t) = -\frac{2\left(\omega_0^2 - \nu^2\right)}{\omega_0^2} + \frac{48\left(\omega_0^2 - \nu^2\right)\kappa\,\sigma^2}{\omega_0^2\left(4\sigma^2 \mathrm{e}^{\frac{\sqrt{\rho\left(\omega_0^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}} + \kappa\,\mathrm{e}^{-\frac{\sqrt{\rho\left(\omega_0^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}}\right)^2}, \tag{10}$$

$$\Theta_2(x,t) = -\frac{2\left(\omega_0^2 - \nu^2\right)}{\omega_0^2} + \frac{48\left(\omega_0^2 - \nu^2\right)\kappa\,\sigma^2}{\omega_0^2\left(4\sigma^2 \mathrm{e}^{-\frac{\sqrt{\rho\left(\omega_0^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}} + \kappa\,\mathrm{e}^{\frac{\sqrt{\rho\left(\omega_0^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}}\right)^2}. \tag{11}$$

Case 2:

$$\chi = \pm\frac{\sqrt{-\rho\left(\omega_0^2 - \nu^2\right)}}{2\rho\nu\lambda}, A_0 = 0, A_1 = 0, A_2 = -\frac{3\left(\omega_0^2 - \nu^2\right)\kappa}{\omega_0^2}. \tag{12}$$

Using the parameters in Case 2, we obtain the following solution functions:

$$\Theta_3(x,t) = -\frac{48\left(\omega_0^2 - \nu^2\right)\kappa\,\sigma^2}{\omega_0^2\left(4\sigma^2 \mathrm{e}^{\frac{\sqrt{-\rho\left(\omega_0^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}} + \kappa\,\mathrm{e}^{-\frac{\sqrt{-\rho\left(\omega^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}}\right)^2}, \tag{13}$$

$$\Theta_4(x,t) = -\frac{48\left(\omega_0^2 - \nu^2\right)\kappa\,\sigma^2}{\omega_0^2\left(4\sigma^2 \mathrm{e}^{-\frac{\sqrt{-\rho\left(\omega_0^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}} + \kappa\,\mathrm{e}^{\frac{\sqrt{-\rho\left(\omega^2-\nu^2\right)}\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\rho\nu\alpha}}\right)^2}. \tag{14}$$

5 Sketch and practice of unified Riccati equation expansion method to M-LWE

In this part, the fundamental steps of the unified Riccati equation expansion method defined in Sirendaoreji (2017) are explained as follows.

Presume that the solution form of Eq. (4) as follow:

$$\Theta(\eta) = \sum_{l=0}^{m} B_l \theta^l(\eta),\ B_l \neq 0, \tag{15}$$

in which $B_l, 0 \leq l \leq m$ are constants to be evaluated. The function $\theta^l(\eta)$ satisfies the following first order differential equation,

$$\theta'(\eta) = \varrho_0 + \varrho_1 \theta(\eta) + \varrho_2 \theta^2(\eta). \tag{16}$$

The special solutions of Eq. (16) are proposed as follows:

Set 1: If $\Delta > 0$, then

$$\theta_1 = -\frac{\varrho_1}{2\varrho_2} - \frac{\sqrt{\Delta}}{2\varrho_2} \tanh\left(\frac{\sqrt{\Delta}}{2}\eta\right),$$
$$\theta_2 = -\frac{\varrho_1}{2\varrho_2} - \frac{\sqrt{\Delta}}{2\varrho_2} \coth\left(\frac{\sqrt{\Delta}}{2}\eta\right).$$

Set 2: If $\Delta = 0$, then

$$\theta_3 = -\frac{\varrho_1}{2\varrho_2} - \frac{1}{\varrho_2 \eta + k}.$$

Set 3: If $\Delta < 0$, then

$$\theta_4 = -\frac{\varrho_1}{2\varrho_2} - \frac{\sqrt{-\Delta}}{2\varrho_2} \tan\left(\frac{\sqrt{-\Delta}}{2}\eta\right),$$
$$\theta_5 = -\frac{\varrho_1}{2\varrho_2} - \frac{\sqrt{-\Delta}}{2\varrho_2} \cot\left(\frac{\sqrt{-\Delta}}{2}\eta\right).$$

in which $\Delta = \varrho_1^2 - 4\varrho_0\varrho_2$ and k is an arbitrary real constant. In spite of Eq. (16) having twenty-seven solutions expressed in many types of research (Zhu 2008; Guo et al. 2011; Li and Dai 2010), these solutions correspond to the solutions expressed above.

Taking into account the terms Θ'' and Θ^2 in Eq. (4), and utilizing the homogeneous balance rule, $m = 2$ is achieved. So, Eq. (15) convert into the following expression:

$$\Theta(\eta) = B_0 + B_1\theta(\eta) + B_2\theta^2(\eta),\ A_2 \neq 0. \tag{17}$$

When the Eq. (17) and its derivatives according to Eq. (16) are inserted into Eq. (4), then acquire a polynomial of $\theta(\eta)$. When we collect all the θ^l coefficients and equalize the coefficients to zero, then the following system is achieved:

$$\theta^0(\eta):\ \left(4\lambda^2\rho\,\varrho_0^2 A_2 + 2\lambda^2 \rho\varrho_0\varrho_1 A_1 - 2A_0\right)v^2 + \omega_0^2 A_0\left(A_0 + 2\right) = 0,$$
$$\theta^1(\eta):\ \left(12A_2\rho\,\lambda^2\varrho_0\varrho_1 + 4\left(-\frac{1}{2} + \left(\varrho_0\varrho_2 + \frac{\varrho_1^2}{2}\right)\rho\,\lambda^2\right)A_1\right)v^2 + 2A_1\omega_0^2\left(A_0 + 1\right) = 0,$$
$$\theta^2(\eta):\ \left(\left(-2 + \left(16\varrho_0\varrho_2 + 8\varrho_1^2\right)\rho\,\lambda^2\right)A_2 + 6A_1\rho\,\lambda^2\varrho_1\varrho_2\right)v^2 + \omega_0^2\left(\left(2A_0 + 2\right)A_2 + A_1^2\right) = 0,$$
$$\theta^3(\eta):\ 4\rho\,\lambda^2\varrho_2\left(5\varrho_1 A_2 + \varrho_2 A_1\right)v^2 + 2A_1A_2\omega_0^2 = 0,$$
$$\theta^4(\eta):\ 12\lambda^2v^2\rho\,\varrho_2^2 A_2 + \omega_0^2 A_2^2 = 0.$$

Solving the derived algebraic system with the aid of a Computer Algebraic Software, the following parameters families are acquired:

Family 1:

$$\varrho_1 = \frac{\sqrt{\rho\left(4\lambda^2 v^2 \rho\varrho_0\varrho_2 + v^2 - \omega_0^2\right)}}{\rho v \lambda},\ A_0 = -\frac{12\lambda^2 v^2 \rho\varrho_0\varrho_2}{\omega_0^2},$$
$$A_1 = -\frac{12\sqrt{\rho\left(4\lambda^2 v^2 \rho\varrho_0\varrho_2 + v^2 - \omega_0^2\right)}\, v\lambda\varrho_2}{\omega_0^2},\ A_2 = -\frac{12\rho\,\lambda^2 \varrho 2^2 v^2}{\omega_0^2}. \tag{18}$$

Thus, we attain the following solution functions utilizing the parameters in Family 1:

$$\Theta_{1,1}(x,t) = \frac{3v^2 - 3\omega_0^2}{\omega_0^2 \cosh\left(\frac{\sqrt{\frac{v^2-\omega_0^2}{\lambda^2 v^2 \rho}}\,\lambda\Gamma(1+\gamma)(x^\alpha + v\,t^\alpha)}{2\alpha}\right)^2}, \tag{19}$$

$$\Theta_{1,2}(x,t) = \frac{-3v^2 + 3\omega_0^2}{\omega_0^2 \sinh\left(\frac{\sqrt{\frac{v^2-\omega_0^2}{\rho\, v^2 \lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha + v\,t^\alpha)}{2\alpha}\right)^2}. \tag{20}$$

where $\Delta = \varrho_1^2 - 4\varrho_0\varrho_2$ and $\Delta > 0$.

$$\Theta_{1,3}(x,t) = \frac{3v^2 - 3\omega_0^2}{\omega_0^2 \cos\left(\frac{\sqrt{\frac{-v^2+\omega_0^2}{\rho\, v^2 \lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha + v\,t^\alpha)}{2\alpha}\right)^2}, \tag{21}$$

$$\Theta_{1,4}(x,t) = \frac{3v^2 - 3\omega_0^2}{\sin\left(\frac{\sqrt{\frac{-v^2+\omega_0^2}{\rho\, v^2 \lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha + v\,t^\alpha)}{2\alpha}\right)^2 \omega_0^2} \tag{22}$$

where $\Delta = \varrho_1^2 - 4\varrho_0\varrho_2$ and $\Delta < 0$.

Family 2:

$$\varrho_1 = \frac{\sqrt{\rho\left(4\lambda^2 v^2 \rho\varrho_0\varrho_2 - v^2 + \omega_0^2\right)}}{\rho v \lambda},\quad A_0 = -\frac{2\left(6\lambda^2 v^2 \rho\varrho_0\varrho_2 - v^2 + \omega_0^2\right)}{\omega_0^2},$$
$$A_1 = -\frac{12\sqrt{\rho\left(4\lambda^2 v^2 \rho\varrho_0\varrho_2 - v^2 + \omega_0^2\right)}\, v\lambda\varrho_2}{\omega_0^2},\ A_2 = -\frac{12\lambda^2 v^2 \rho\,\varrho_2^2}{\omega_0^2}. \tag{23}$$

We get the following solution functions using the parameters in Family 2:

$$\Theta_{2,1}(x,t)=\frac{\left(v^2-\omega_0^2\right)}{\omega_0^2}\left(2-\frac{3}{\cosh\left(\frac{\sqrt{\frac{-v^2+\omega_0^2}{\rho v^2\lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha+v\,t^\alpha)}{2\alpha}\right)^2}\right), \tag{24}$$

$$\Theta_{2,2}(x,t)=\frac{\left(2\left(\cosh^2\left(\frac{\sqrt{\frac{-v^2+\omega_0^2}{\rho v^2\lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha+v\,t^\alpha)}{2\alpha}\right)\right)+1\right)\left(v^2-\omega_0^2\right)}{\omega_0^2\sinh\left(\frac{\sqrt{\frac{-v^2+\omega_0^2}{\rho v^2\lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha+v\,t^\alpha)}{2\alpha}\right)^2}, \tag{25}$$

where $\Delta=\varrho_1^2-4\varrho_0\varrho_2$ and $\Delta>0$.

$$\Theta_{2,3}(x,t)=\frac{\left(v^2-\omega_0^2\right)}{\omega_0^2}\left(2-\frac{3}{\cos\left(\frac{\sqrt{\frac{v^2-\omega_0^2}{\rho v^2\lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha+v\,t^\alpha)}{2\alpha}\right)^2}\right), \tag{26}$$

$$\Theta_{2,4}(x,t)=-\frac{\left(2\left(\cos^2\left(\frac{\sqrt{\frac{v^2-\omega_0^2}{\rho v^2\lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha+v\,t^\alpha)}{2\alpha}\right)\right)+1\right)\left(v^2-\omega_0^2\right)}{\omega_0^2\sin\left(\frac{\sqrt{\frac{v^2-\omega_0^2}{\rho v^2\lambda^2}}\,\lambda\Gamma(1+\gamma)(x^\alpha+v\,t^\alpha)}{2\alpha}\right)^2}, \tag{27}$$

where $\Delta=\varrho_1^2-4\varrho_0\varrho_2$ and $\Delta<0$.

Family 3:

$$\begin{aligned}\omega_0=&\sqrt{6\lambda^2\rho\varrho_0\varrho_2+1}\,v,\varrho_1=\sqrt{-2\varrho_0\varrho_2},A_0=-\frac{12\lambda^2\rho\varrho_0\varrho_2}{6\lambda^2\rho\varrho_0\varrho_2+1},\\ A_1=&\frac{24\lambda^2\rho\varrho_0\,\varrho_2^2}{\left(6\lambda^2\rho\varrho_0\varrho_2+1\right)\sqrt{-2\varrho_0\varrho_2}},A_2=-\frac{12\lambda^2\rho\,\varrho_2^2}{6\lambda^2\rho\varrho_0\varrho_2+1}.\end{aligned} \tag{28}$$

We derive the following solution functions utilizing the parameters in Family 3:

$$\Theta_{3,1}(x,t)=-\frac{18\lambda^2\rho\varrho_0\varrho_2}{\left(6\lambda^2\rho\varrho_0\varrho_2+1\right)\cosh\left(\frac{\sqrt{6}\sqrt{-\varrho_0\varrho_2}\,\lambda\Gamma(1+\gamma)(x^\alpha+v\,t^\alpha)}{2\alpha}\right)^2}, \tag{29}$$

$$\Theta_{3,2}(x,t) = \frac{18\lambda^2\rho\varrho_0\varrho_2}{\left(6\lambda^2\rho\varrho_0\varrho_2+1\right)\sinh\left(\frac{\sqrt{6}\sqrt{-\varrho_0\varrho_2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)^2}, \tag{30}$$

where $\varrho_0\varrho_2 < 0$, $\Delta = \varrho_1^2 - 4\varrho_0\varrho_2$ and $\Delta > 0$.

$$\Theta_{3,3}(x,t) = -\frac{18\lambda^2\rho\varrho_0\varrho_2}{\left(6\lambda^2\rho\varrho_0\varrho_2+1\right)\cos\left(\frac{\sqrt{6}\sqrt{\varrho_0\varrho_2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)^2}, \tag{31}$$

$$\Theta_{3,4}(x,t) = -\frac{18\lambda^2\rho\varrho_0\varrho_2}{\left(6\lambda^2\rho\varrho_0\varrho_2+1\right)\sin\left(\frac{\sqrt{6}\sqrt{\varrho_0\varrho_2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)^2}, \tag{32}$$

where $\varrho_0\varrho_2 > 0$, $\Delta = \varrho_1^2 - 4\varrho_0\varrho_2$ and $\Delta < 0$.

Family 4:

$$\begin{aligned} \omega &= \sqrt{-4\rho\,\lambda^2\varrho_0\varrho_2+\rho\,\lambda^2\varrho_1^2+1}\,\nu, A_0 = \frac{2\lambda^2\rho\left(2\varrho_0\varrho_2+\varrho_1^2\right)}{4\rho\,\lambda^2\varrho_0\varrho_2-\rho\,\lambda^2\varrho_1^2-1},\\ A_1 &= \frac{12\lambda^2\rho\varrho_1\varrho_2}{4\rho\,\lambda^2\varrho_0\varrho_2-\rho\,\lambda^2\varrho_1^2-1}, A_2 = \frac{12\lambda^2\rho\,\varrho_2^2}{4\rho\,\lambda^2\varrho_0\varrho_2-\rho\,\lambda^2\varrho_1^2-1}. \end{aligned} \tag{33}$$

We generate the following solution functions utilizing the parameters in Family 4:

$$\Theta_{4,1}(x,t) = \frac{\left(4\varrho_0\varrho_2-\varrho_1^2\right)\lambda^2\rho\left(-2+\frac{3}{\cosh\left(\frac{\sqrt{-4\varrho_0\varrho_2+\varrho_1^2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)^2}\right)}{4\rho\,\lambda^2\varrho_0\varrho_2-\rho\,\lambda^2\varrho_1^2-1}, \tag{34}$$

$$\Theta_{4,2}(x,t) = -\frac{\lambda^2\rho\left(2\left(\cosh^2\left(\frac{\sqrt{-4\varrho_0\varrho_2+\varrho_1^2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)\right)+1\right)\left(4\varrho_0\varrho_2-\varrho_1^2\right)}{\left(-1+\lambda^2\rho\left(4\varrho_0\varrho_2-\varrho_1^2\right)\right)\sinh\left(\frac{\sqrt{-4\varrho_0\varrho_2+\varrho_1^2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)^2}, \tag{35}$$

where $\Delta = \varrho_1^2 - 4\varrho_0\varrho_2$ and $\Delta > 0$.

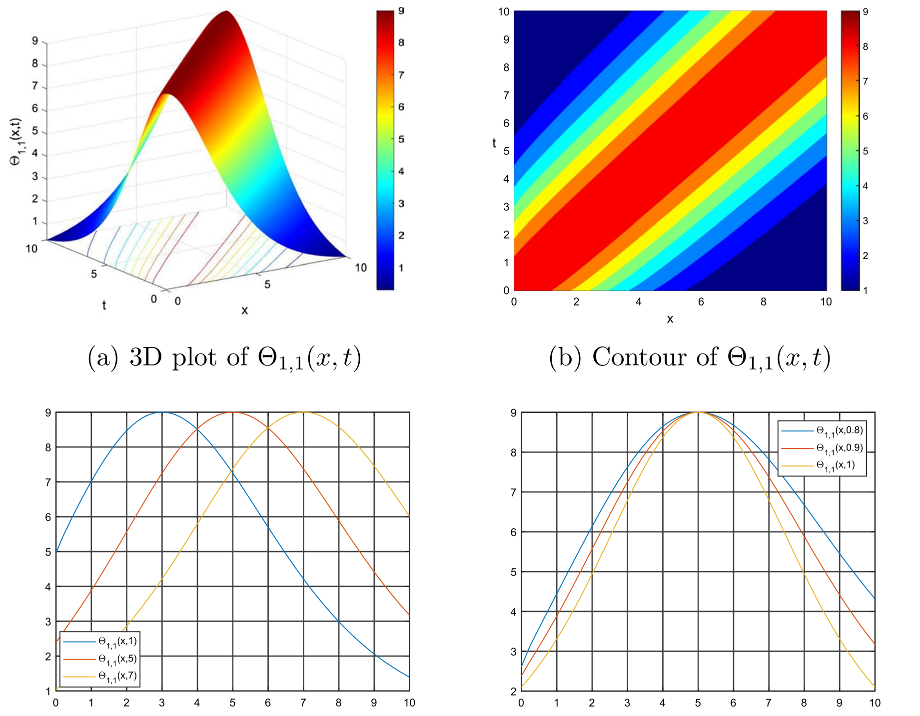
$$\Theta_{4,3}(x,t) = -\frac{12\lambda^2\rho\,\varrho_2^2\alpha^2}{\left(\varrho_2\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)+k\alpha\right)^2}, \tag{36}$$

where $\Delta = 0$.

$$\Theta_{4,4}(x,t)=\frac{\left(4\varrho_0\varrho_2-\varrho_1^2\right)\lambda^2\rho\left(-2+\frac{3}{\cos\left(\frac{\sqrt{4\varrho_0\varrho_2-\varrho_1^2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)^2}\right)}{4\rho\,\lambda^2\varrho_0\varrho_2-\rho\,\lambda^2\varrho_1^2-1}, \tag{37}$$

$$\Theta_{4,5}(x,t)=\frac{\lambda^2\rho\left(2\left(\cos^2\left(\frac{\sqrt{4\varrho_0\varrho_2-\varrho_1^2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)\right)+1\right)\left(4\varrho_0\varrho_2-\varrho_1^2\right)}{\left(-1+\lambda^2\rho\left(4\varrho_0\varrho_2-\varrho_1^2\right)\right)\sin\left(\frac{\sqrt{4\varrho_0\varrho_2-\varrho_1^2}\,\lambda\Gamma(1+\gamma)(x^\alpha+\nu\,t^\alpha)}{2\alpha}\right)^2}, \tag{38}$$

where $\Delta=\varrho_1^2-4\varrho_0\varrho_2$ and $\Delta<0$.



(a) 3D plot of $\Theta_{1,1}(x,t)$

(b) Contour of $\Theta_{1,1}(x,t)$

(c) 2D of $\Theta_{1,1}(x,t)$ for various t

(d) 2D of $\Theta_{1,1}(x,t)$ for various α values at $t=5$

Fig. 1 The plots of bright soliton of $\Theta_{2,1}(x,t)$ in Eq. (19) for $\varrho_0=0.5, \varrho_2=0.5, \gamma=0.5, \alpha=0.9, \lambda=1, \rho=2, \nu=-1, \omega=0.5$

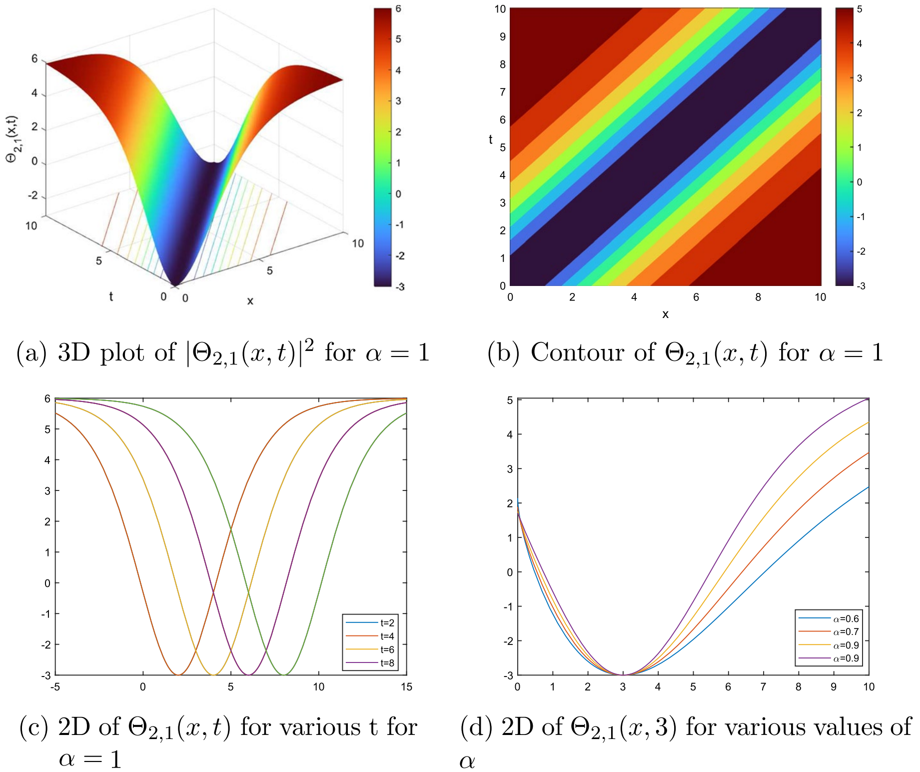
6 Results and discussion

In this part, we introduce the graphs of the solutions in Eqs. (19), (24), and (32) to commentate some physical properties of the M-LWE. The portraits of the solutions are depicted in Matlab. Utilizing proper values of the parameters, we give four shapes to explain the manner of the solutions.

For the parameters $\varrho_0 = 0.5, \varrho_2 = 0.5, \gamma = 0.5, \alpha = 0.9, \lambda = 1, \rho = 2, \nu = -1, \omega = 0.5$, the chart of $\Theta_{1,1}(x,t)$ in Eq. (19) for is given in Fig. 1. In Fig. 1a, b, we illustrate the 3D and contour graphs for $\Theta_{1,1}(x,t)$ in Eq. (19), respectively. 2D visualisation for $t = 1, 5, 7$ is also shown in Fig. 1c. From Fig. 1c, we say that the soliton moves to the left if t decreases.

for the parameters $\varrho_0 = 2, \varrho_2 = 1, \gamma = 1, \lambda = 1.75, \rho = -2, \nu = -1, \omega = 0.5$, the chart of $\Theta_{2,1}(x,t)$ in Eq. (24) is presented in Fig. 2. In Fig. 2a, b, we illustrate the 3D and contour charts for $\Theta_{2,1}(x,t)$ in Eq. (24), respectively. Besides, 2D chart for $t = 2, 4, 6$ and $t = 8$ is represented in Fig. 2c. Figure 3d shows 2D portrait for $\alpha = 0.6, 0.7, 0.8$, and $\alpha = 0.9$. From Fig. 2c, we interpret that the soliton acts to the right if t increases. It can be noticed from Fig. 2d that the soliton horizontally narrows as α increases.

For the parameters $\varrho_0 = 2.5, \varrho_2 = -1.5, \gamma = 2, \lambda = -1.5, \rho = 3, \nu = -1$, the chart of $\Theta_{3,4}(x,t)$ in Eq. (32) is proposed in Fig. 3. In Fig. 3a, b, we illustrate the 3D and contour graphs of $\Theta_{3,4}(x,t)$ in Eq. (32), respectively. Furthermore, 2D visualization for $t = 2, 4, 6$ and $t = 8$ is represented in Fig. 3c. Figure 3d shows 2D portrait for $\alpha = 0.6, 0.7, 0.8$, and



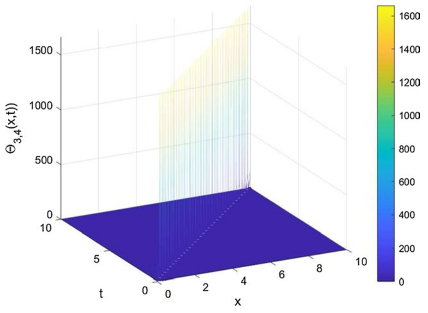
(a) 3D plot of $|\Theta_{2,1}(x,t)|^2$ for $\alpha = 1$

(b) Contour of $\Theta_{2,1}(x,t)$ for $\alpha = 1$

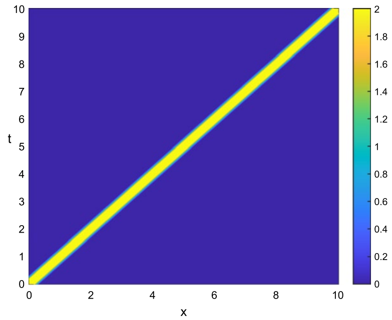
(c) 2D of $\Theta_{2,1}(x,t)$ for various t for $\alpha = 1$

(d) 2D of $\Theta_{2,1}(x,3)$ for various values of α

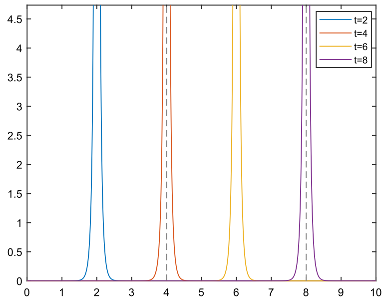
Fig. 2 The plots of anti-bell-shape soliton of $\Theta_{2,1}(x,t)$ in Eq. (24) for $\varrho_0 = 2, \varrho_2 = 1, \gamma = 1, \lambda = 1.75, \rho = -2, \nu = -1, \omega = 0.5$



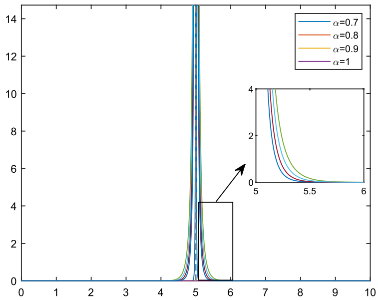
(a) 3D chart of $\Theta_{3,4}(x, t)$ for $\alpha = 1$



(b) Contour of $\Theta_{3,4}(x, t)$ for $\alpha = 1$



(c) 2D of $\Theta_{3,4}(x, t)$ for various t for $\alpha = 1$



(d) 2D of $\Theta_{3,4}(x, t)$ for various values of α at $t = 5$

Fig. 3 The portraits of singular soliton of $\Theta_{3,4}(x, t)$ in Eq. (32) for the parameters $\varrho_0 = 2.5, \varrho_2 = -1.5, \gamma = 2, \lambda = -1.5, \rho = 3, \nu = -1$

$\alpha = 0.9$. From Fig. 3c, we see that the soliton acts to the right as t increases. It can be noticed from Fig. 3d that the soliton horizontally narrows as α increases.

7 Conclusion

In this study, some solutions of the M-LWE in a magneto-electro-elastic circular rod utilizing the unified Riccati equation expansion method and the new Kudryashov method have been productively established for the first time. The choice of the presented methods is that it allows more than one soliton solution, these methods can be applied easily and effectively, and they can also be applied for the fractional form. A number of soliton solutions have been acquired, and 3D, contour and 2D dimensional portraits of the resulting soliton solutions have been exhibited. Singular, bright, and dark solitons have been acquired utilizing the proposed methods, and the impact of some parameters in the solutions has been examined. Moreover, it has been exhibited that the α parameter has an effect. The presented graphics will aid in the understanding of the physical properties of M-LWE. It is seen that soliton solutions of numerous fractional nonlinear models in mathematical physics can be effectively derived via the presented methods in this study.

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Declarations

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Ethical approval The Corresponding Author proclaims that this article is original, has not been previously published, and is not presently being considered for publication elsewhere. The Corresponding Author approves that the manuscript has been read and confirmed by all the named authors and there are no other persons who satisfied the criteria for authorship but are not listed. I further approve that the order of authors listed in the manuscript has been confirmed by all of us. We understand that the Corresponding Author is the sole contact for the Editorial process and is responsible for communicating with the other authors about progress, submissions of revisions, and final approval of proofs.

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