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Unlocking the high dimensional' potential: Comparative analysis of qubits and qutrits in variational quantum neural networks

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ABSTRACT

Quantum machine learning is a promising research area with great potential. In particular, Variational quantum neural networks (VQNN) have shown high performance in many applications. However, while qubits, which are 2-level quantum systems, are the standard building blocks of quantum computing, the development of qudits, i.e. d-level quantum systems, has opened up new opportunities in VQNNs thanks to many properties such as robustness to noise and more quantum information processing with fewer quantum resources. In this study, we present a comparative analysis of qubits and qutrits (3-level quantum systems) systems performance in VQNNs while also exploring the effect of encoding strategies and entanglement on classifier performance. Our findings contribute to a better understanding the benefits and limitations of using qutrits in VQNN and pave the way for future developments in this field.

1. Introduction

In the ever-evolving landscape of technology, a fascinating synergy between two groundbreaking fields – quantum computing and machine learning – has emerged to redefine the limits of what we can achieve. Quantum machine learning (QML), the combination of quantum computing and machine learning, holds the promise of transcending the boundaries of classical computing and solving complex problems previously considered impossible [1]. A key component of QML is quantum classifiers, which are quantum algorithms designed to classify data into different categories [2,3]. One of these classifiers, variational quantum neural networks (VQNN), has attracted significant attention due to their versatility and capacity to adapt to diverse datasets [4–6].

Quantum bits i.e., qubits have been extensively studied and widely used in various quantum computing architectures as the fundamental information units of quantum computing [7–9]. Qubit-based VQNNs have shown promising results in tackling complex classification tasks, demonstrating their potential in image recognition [10–12], drug discovery [13], and optimization problems [14]. However, the inherent fragility of qubits poses significant challenges in maintaining coherence and reducing errors through quantum error correction codes [15].

On the other hand, extending traditional two-level qubits to d-level quantum systems, i.e., qudits, offers additional degrees of freedom and more computational power. They also provide a more extensive state space for storing and processing information, leading to reduced circuit complexity and increased algorithm efficiency [16–18]. By taking advantage of these extra states, qudit-based VQNNs can potentially increase the expressive power of quantum classifiers. Furthermore, qudit systems can potentially mitigate some of the challenges faced by qubit-based systems by exhibiting improved robustness to noise and errors [19].

In this context, there are some studies developed using qudits. Adhikary et al. [20] developed a quantum classifier that encodes input features in a single qudit instead of an entangled multi-qubit system. They introduced a new "single-shot training" method where the classifier is trained simultaneously with all input samples of the same class. Konar et al. [21] proposed QFS-Net, a self-checking model for automatic brain MRI segmentation inspired by quantum computing and utilizing a 3-level quantum system, the qutrit system. Useche et al. [22] proposed a hybrid classical–quantum method for supervised classification and estimation of probability density functions that can run on a high-level quantum computer. In their 2023 study, Wach et al. [23] investigated the potential of employing a data reloading method to evaluate the efficacy of a single qudit for solving classification problems. Their findings revealed that qudit systems with multiple levels are highly adept at addressing multi-class classification problems, as each qudit can encode a specific data class with its corresponding

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base case. Mandilara et al. [24] introduced a novel approach for data classification by utilizing qudits. The proposed model involves a mapping process whereby classical data is encoded onto qudits through rotation, and subsequently, the qudits are projected onto the surface of the Bloch hypersphere. The model further employs rotation encoding and measurement techniques to classify data. Remarkably, the authors demonstrated that this geometrically-inspired qudit model can effectively solve complex classification problems with minimal parameter usage and no reliance on entanglement operations.

Unlike previous studies that often focus in either qubits or high dimensional such as qutrits in isolation, our research provides a comprehensive comparative analysis of both systems within the same framework. This holistic approach allows for a direct comparison of their performance, providing valuable insights into their relative strengths and weaknesses. The main objective of this study is to investigate the impact of system dimension, encoding strategies, and entanglement in the VQNNs' performance and to elucidate the advantages and limitations of these systems. Additionally, we aim to shed light on the potential benefits of using qutrits in quantum machine learning. Our main contributions are as follows:

- It was shown that qutrit systems can outperform qubit systems with fewer parameters using angle encoding.
- Using amplitude encoding, qutrit systems were shown to achieve better performance than qubit systems.
- The importance of entanglement in both qubit and qutrit systems was emphasized to achieve high performance.

Our findings contribute to a better understanding of the benefits and limitations of using qutrits in VQNN, will facilitate researchers and practitioners in selecting the most suitable system for specific applications, and will contribute to the development of more robust and efficient quantum machine learning algorithms.

The paper is structured as follows: First, we briefly overview qubits and qutrits, summarizing their fundamental properties and computational capabilities. Then, we discuss the architecture, training methods, and optimization techniques of VQNNs. Then, we evaluate the effectiveness of VQNNs based on the qubits and qutrits by examining classification accuracy, computational resources, and scalability on well-known datasets. Finally, we analyze the experimental results and discuss the significance of our findings.

2. Quantum computing

Quantum computing relies on the principles of quantum mechanics to enable physical quantum systems to perform computation and information processing. In quantum computing, d-level quantum systems are referred to as qudits, and their quantum state is mathematically represented by Dirac's Bra-Ket notation. A qudit quantum state comprises $|0\rangle, |1\rangle, \dots, |d-1\rangle$ orthonormal bases of the d-dimensional Hilbert space H_d. The general state of a qudit $|\varphi\rangle$ is as follows [25–27]:

$$|\varphi\rangle = \lambda_0 |0\rangle + \lambda_1 |1\rangle + \lambda_2 |2\rangle + \cdots + \lambda_{d-1} |d-1\rangle \tag{1}$$

where $\lambda_i \in \mathbb{C}, i = 0, \dots, d-1$ and $|\lambda_0|^2 + |\lambda_1|^2 + \cdots + |\lambda_{d-1}|^2 = 1$.

Quantum measurements are a crucial aspect of quantum computing, but they come with a challenge. When we measure qudits, they collapse into one of their ground states, which is irreversible. This means that the quantum state is irreversibly changed, and it is no longer possible to manipulate it with quantum gates. Before the measurement, a quantum state can be in one of an infinite number of states and can be manipulated using quantum gates.

Quantum gates are the building blocks of quantum computing, allowing the manipulation and transformation of quantum states. They make it possible to perform operations such as superposition, entanglement, and quantum interference. These gates have a unitary structure that guarantees that closed quantum systems will undergo a unitary evolution that not only preserves the norm of quantum states but also

Table 1
The 2-level quantum gates used in this study and their matrix representations. The subscript (2) under the gate name indicates that it operates in a 2-level quantum system [27].

Quantum gate	Matrix Repr.	Quantum gate	Matrix Repr.
$Rx_{(2)}(\theta)$	$\begin{bmatrix} cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & cos\frac{\theta}{2} \end{bmatrix}$	$Ry_{(2)}(\theta)$	$\begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$
$Rz_{(2)}(\theta)$	$\begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$	$CX_{(2)}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

ensures that the process is reversible. In this context, computational matrix representations of quantum gates operating in qubit that 2-level system, and qutrit that 3-level system, used in this study are presented in Tables 1 and 2, respectively.

Quantum information processing tasks and calculations often require multiple computational resources working together. The state of a quantum system of n qudits can be expressed as the tensor product of the quantum states of individual qudits. The superposition of the d^n basis state for n qudits can be expressed as follows:

$$|\psi\rangle = \sum_{i=0}^{d^n-1} \lambda_i |i\rangle \tag{2}$$

where i is the decimal representation of the basis, $\lambda_i \in \mathbb{C}$ and $\sum_{i=0}^{d^n-1} |\lambda_i|^2 = 1$.

3. Quantum data embedding

Quantum data embedding allows classical data to be represented in its quantum state in Hilbert space. This is accomplished by means of a $\phi : \mathcal{D}- > \mathcal{H}$ property mapping from the data space $\mathcal{D}$ to the Hilbert space $\mathcal{H}$. This mapping takes a classical data point x and translates it into a set of gate parameters in a quantum circuit, creating a quantum state $|\varphi_x\rangle$. This allows quantum algorithms to work on encoded data. This process plays a crucial role in the development of quantum algorithms and has a significant impact on their computational power. The process of embedding quantum data can be accomplished through various means, with the most suitable technique being dependent on the particular requirements and application of the quantum algorithm [29]. In the realm of quantum machine learning, amplitude encoding has emerged as a more favorable approach due to its significant advantages [30]. Our study utilized both angle encoding and amplitude encoding techniques to investigate their respective efficacy.

3.1. Angle encoding

In the angle encoding technique, classical data is encoded in qudits using quantum rotation gates, with the angles of the gates determined by the classical data. This approach enables efficient encoding of classical data into quantum states [31].

$$|\varphi_x\rangle = \bigotimes_i^n R_d(x_i) |0^i\rangle \tag{3}$$

where $R_{(d)}$ can be one or more of $Rx_{(d)}$, $Ry_{(d)}$, and $Rz_{(d)}$. The number of computational resources used for encoding equals the size of the vector x.

3.2. Amplitude encoding

Amplitude encoding is a coding method used in quantum computing in which the amplitudes of the quantum system are used to represent data values. It is also known as wave function coding [30]. Amplitude encoding encodes a vector x of length N into the amplitudes of an

Table 2
The 3-level quantum gates used in this study and their matrix representations. The subscript (3) under the gate name indicates that it operates in a 3-level quantum system [28].

Quantum gate	Matrix Repr.	Quantum gate	Matrix Repr.	Quantum gate	Matrix Repr.
$Rx_{(3)}^{(01)}(\theta)$	$\begin{bmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} & 0 \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$Ry_{(3)}^{(01)}(\theta)$	$\begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$Rz_{(3)}^{(01)}(\theta)$	$\begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 & 0 \\ 0 & e^{i\frac{\theta}{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$Rx_{(3)}^{(02)}(\theta)$	$\begin{bmatrix} \cos\frac{\theta}{2} & 0 & i\sin\frac{\theta}{2} \\ 0 & 1 & 0 \\ -i\sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{bmatrix}$	$Ry_{(3)}^{(02)}(\theta)$	$\begin{bmatrix} \cos\frac{\theta}{2} & 0 & -\sin\frac{\theta}{2} \\ 0 & 1 & 0 \\ \sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{bmatrix}$	$Rz_{(3)}^{(02)}(\theta)$	$\begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$
$Rx_{(3)}^{(12)}(\theta)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ 0 & -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$	$Ry_{(3)}^{(12)}(\theta)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$	$Rz_{(3)}^{(12)}(\theta)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\frac{\theta}{2}} & 0 \\ 0 & 0 & e^{i\frac{\theta}{2}} \end{bmatrix}$
$CX_{(3)}$	$\begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0&0 \end{bmatrix}$				

$\lceil \log_d N \rceil$ computational source:

$$|\varphi_x\rangle = \sum_{i=0}^{N-1} x_i\,|i\rangle \tag{4}$$

where $\{|i\rangle\}$ is the computational basis for the Hilbert space $\mathcal{H}_d$. Since classical data constitute the amplitudes of a quantum state, the input must satisfy the

$$\sum_i |x_i|^2 = 1 \tag{5}$$

normalization condition.

4. Variational quantum neural networks

Variational quantum neural networks (VQNNs) are a class of quantum machine learning models that combine the tools of both quantum computation and classical neural networks. The basic idea behind VQNNs is to use quantum circuits as a trainable part of a neural network, allowing both classical and quantum computations to work together, potentially offering advantages over classical neural networks for specific tasks [32].

The working principle of a VQNN is presented in Fig. 1. Basically, it can be explained as follows: First, the input data is encoded into the quantum state represented by the computational resources. A parameterized quantum circuit passes through the encoded quantum state. This circuit consists of a series of quantum gates manipulating the quantum state based on specific parameters. The parameters of the quantum circuit are tunable and act as learnable weights of the VQNN. After evaluating the data in parameterized quantum layers, measurements are made on the quantum state to obtain classical outputs. The measurement results are transmitted to the next classical output layer. The results from the output layer use a loss function to measure the discrepancy between the predicted and target labels. Based on the gradient of the loss function or other relevant information, the parameters in the model are updated in a manner compatible with classical optimization algorithms using the automatic differentiation technique offered by Pennylane [33]. This cycle continues until the cost function converges or a satisfactory solution is obtained. Once the VQNN has been trained and the parameters optimized, it can be used to make predictions on new data. The trained VQNN can be used for tasks such as classification, regression, and optimization problems.

VQNNs have a critical advantage in using quantum entanglement and interference to detect complex patterns and correlations in data. This variable procedure makes these algorithms relatively fault-tolerant and ideal for executing on noisy medium-scale quantum computers, which are under development on various platforms and are slowly being made available. As quantum technology continues to evolve, the potential of VQNNs in various quantum machine-learning tasks will become increasingly evident [4].

5. Experimental methodology

Our experimental methodology is designed to systematically compare the performance of qubit- and qutrit-based variational quantum neural networks (VQNNs). We employed this methodology to train and assess both qubit- and qutrit-based VQNNs using three distinct datasets. Each experiment was conducted under identical conditions to ensure fair and consistent comparisons between qubit and qutrit systems. For each dataset, the experiments progressed through various phases, including data preprocessing, model initialization, training, and evaluation. The essential elements of our methodology are illustrated in Fig. 2.

5.1. Datasets

The popular Iris [34], Wine [35], and Breast Cancer [36] datasets – which are frequently used to train and evaluate machine learning algorithms – were utilized for this study. The Iris, Wine, and Breast Cancer datasets were obtained from publicly available repositories, such as the UCI Machine Learning Repository. These datasets are widely recognized and used in the machine learning community, ensuring their reliability and validity.

The 150 iris flower specimens in the Iris dataset are divided into three species: Setosa, Versicolor, and Virginia. For every floral sample, there are four available features. The Wine dataset contains 12 features and one target variable. The features are usually related to the chemical components of wines and represent wine samples belonging to three different classes. These classes are usually red wine, white wine, and a third type (sometimes called rosé). There are 178 samples in this dataset. The Breast Cancer dataset is a machine learning and data analytics dataset used for breast cancer diagnosis. This dataset contains 569 data samples and each sample contains 30 different features. These features include measurements of factors such as cell nucleus characteristics and tumor tissue characteristics. Each sample also contains a label indicating whether the tumor is benign or malignant.

The chosen datasets represent a progression in complexity and dimensionality, allowing us to systematically evaluate the performance

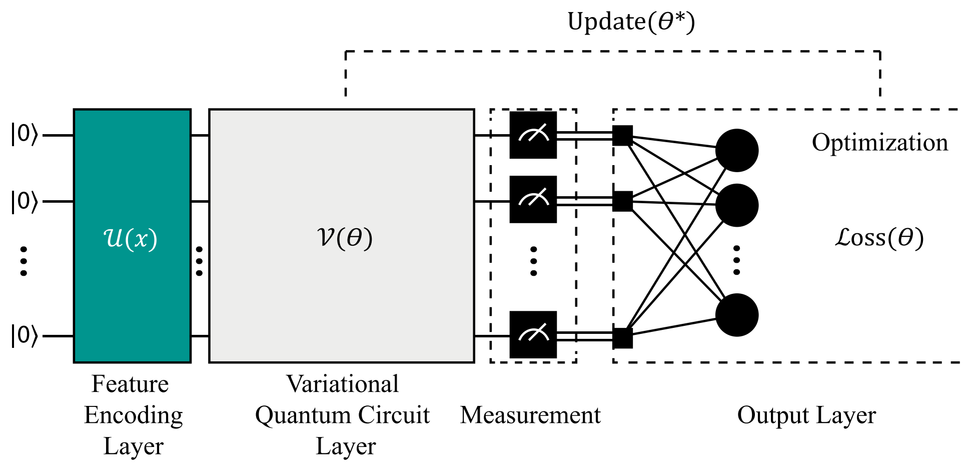

Fig. 1. VQNN architecture.

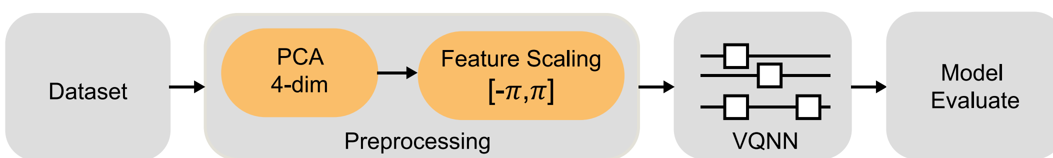

Fig. 2. The essential elements of experimental methodology.

of VQNNs across a range of classification tasks. By starting with the simpler Iris dataset and progressing to the more complex Breast Cancer dataset, we can gain insights into the strengths and limitations of VQNNs at different levels of data complexity. Additionally, these datasets are well-established benchmarks in the machine learning community. Their widespread use ensures that our results are relevant and can be easily compared with existing literature, facilitating the evaluation of the potential advantages offered by quantum neural networks.

5.2. Preprocessing

The datasets used in this study went through the following preprocessing stages.

1. **Dimensionality Reduction:** To make a precise comparison between qubit and qutrit systems, it is essential to ensure that the same number of computational resources and parameters are utilized. Thus, we used principal component analysis (PCA) for dimensionality reduction. The Iris dataset naturally has four features that are consistent with the input dimensionality required for quantum data encoding without further reduction. PCA was applied only to datasets with higher-dimensional features to reduce their dimensionality to four, facilitating a consistent input dimensionality in our experiments. PCA is a statistical method used to diminish the dimensionality of features in a dataset. It achieves this by creating a smaller set of new features, known as principal components while retaining the most significant information in the data.
2. **Feature Scaling:** Feature scaling is used to reduce these differences when different features in the data set have very different scales compared to each other. For this reason, all features were scaled to the interval $[-\pi, \pi]$.

5.3. VQNN models

This section presents quantum circuits for VQNN without and with entanglement models using both angle and amplitude encoding methods in Figs. 3 and 4, respectively.

With N = 4 features, the number of computational resources required for the amplitude encoding method is $\lceil log_d N \rceil = 2$ for both qubit and qutrit. On the other hand, the number of computational resource features required for angle encoding is N. In this context, the quantum circuit for VQNN models consists of four stages. First, all computational resources are initialized in the initial state $|0\rangle$. Then, the encoding layer was applied to represent the quantum state of classical data. Specifically, in the angle encoding method, each component of the features is embedded in the angles of the rotation gates $Ry_{(d)}(x_i)$, $i = 0, 1, 2, 3$ after passing them through the $\arctan$ function. In the amplitude encoding technique, feature normalization was performed before encoding them as amplitudes. Subsequently, a variational layer sequence was implemented with trainable parameters. The main component of the variational layer here is the gate

$$R_{(d)}(\theta, \omega, \lambda) = Rz_{(d)}(\lambda) Ry_{(d)}(\omega) Rz_{(d)}(\theta) \tag{6}$$

where $Ry_{(d)}$, $Rz_{(d)}$ gates for the 3-level system are as follows:

$$Ry_{(3)} = Ry^{(01)}_{(3)} Ry^{(02)}_{(3)} Ry^{(12)}_{(3)} \tag{7}$$

$$Rz_{(3)} = Rz^{(01)}_{(3)} Rz^{(02)}_{(3)} Rz^{(12)}_{(3)} \tag{8}$$

In addition, $CX_{(d)}$ gates are used for the entanglement. Utilizing the amplitude encoding method for both qubit and qutrit systems produces a total of $6l$ trainable parameters, whereas the angle encoding method results in $12l$ parameters. These parameters are dictated solely by the chosen encoding method and the number of layers l, rather than the specific dimensionality of the system (qubit or qutrit). This approach allows for consistent parameterization across different quantum systems. Finally, to obtain the classical outputs, the expected value of

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{9}$$

in the 2-level system and the expected value of Gell-Mann observable

$$\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

in the 3-level system are calculated, and the obtained values are transmitted to the output layer. The number of neurons in the output layer is the number of classes in the dataset.

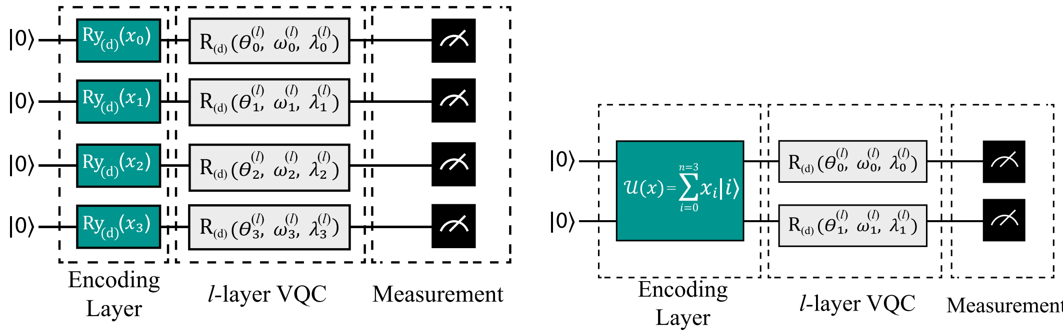

Fig. 3. Circuit diagrams of VQNN without entanglement utilizing angle encoding (left) and amplitude encoding (right) methods.

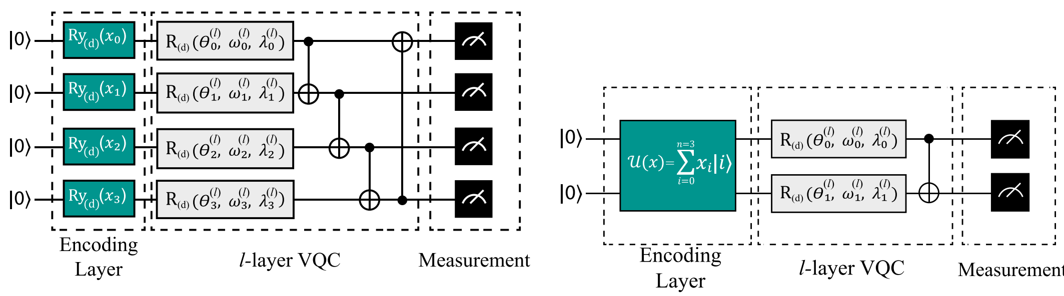

Fig. 4. Circuit diagrams of VQNN with entanglement utilizing angle encoding (left) and amplitude encoding (right) methods.

Table 3
Other relevant details such as the number of qubits and qutrits used in the experiments, the number of trainable parameters, circuit depth, and the number of gates.

Dataset	Experiment	Encoding method	System type	#Qubit/ Qutrit	#Trainable parameters (per layer)	#Rotation gate (per layer)	#CNOT gate	#Circuit depth (per layer)
Iris Wine Breast Cancer	Without entanglement	Angle	Qubit	4	12	4	4	8
		Angle	Qutrit	4	12	4	4	8
		Amplitude	Qubit	2	6	2	1	3
		Amplitude	Qutrit	2	6	2	1	3
	With entanglement	Angle	Qubit	4	12	4	4	8
		Angle	Qutrit	4	12	4	4	8
		Amplitude	Qubit	2	6	2	1	3
		Amplitude	Qutrit	2	6	2	1	3

5.4. Model training and evaluate procedure

For this study, we ensured reproducibility by using the same starting parameters for each experiment and initializing them with a fixed random seed. We used the Adam optimization algorithm [37] with a learning rate of 0.01 to train the VQNN models. The learning rate remained constant throughout the training process. We used the Sparse Categorical Cross Entropy function as the loss function and trained each model for 25 epochs without applying early stopping. We used a batch size of 8 for training. We varied the number of layers in each model from one to five for training. We employed a 5-fold cross-validation approach for each dataset, splitting the data into five subsets for testing, while the remaining subsets were used for training. This process was repeated five times, and the results were averaged to provide a robust estimate of the model's performance. We have provided a comprehensive breakdown of the number of qubits and qutrits used in the experiments, as well as other relevant details such as the number of trainable parameters, circuit depth, and number of gates in Table 3.

6. Results and discussion

In this section, we perform a comparative analysis of the performance of two VQNNs on the Iris, Wine and Breast Cancer datasets. We use two VQNNs, one without and one with entanglement, and investigate the performance of qubit and qutrit systems by varying the number of variational layers. We will also analyze the impact of angle and amplitude encoding methods on performance and examine the effect of entanglement.

6.1. Comparison of qubit and qutrit systems

This section presents a comparison of qubit and qutrit systems in VQNNs using angle and amplitude encoding methods. The performance of both systems is evaluated on the Iris, Wine and Breast Cancer datasets. Loss plots for both systems for training and testing on each dataset are presented in Figs. 8–13.

Analyzing the training and testing loss graphs presented in Figs. 8–13 shows that the VQNN without entanglement using angle encoding exhibits superior performance in the qutrit system compared to the qubit system for one-layer models in all three datasets. In contrast, its performance on the qutrit system shows a drop in accuracy after the third layer compared to the version designed for the qubit system on the Iris and Breast Cancer dataset. Similarly, on the Wine dataset, a decrease in accuracy is observed starting from the second layer. In addition, it is observed that the one-, two-, and three-layer model of the VQNN without entanglement using angle encoding performs better on the qutrit system than on the qubit system. However, after the four-layer model, its performance decreases in the qutrit system compared

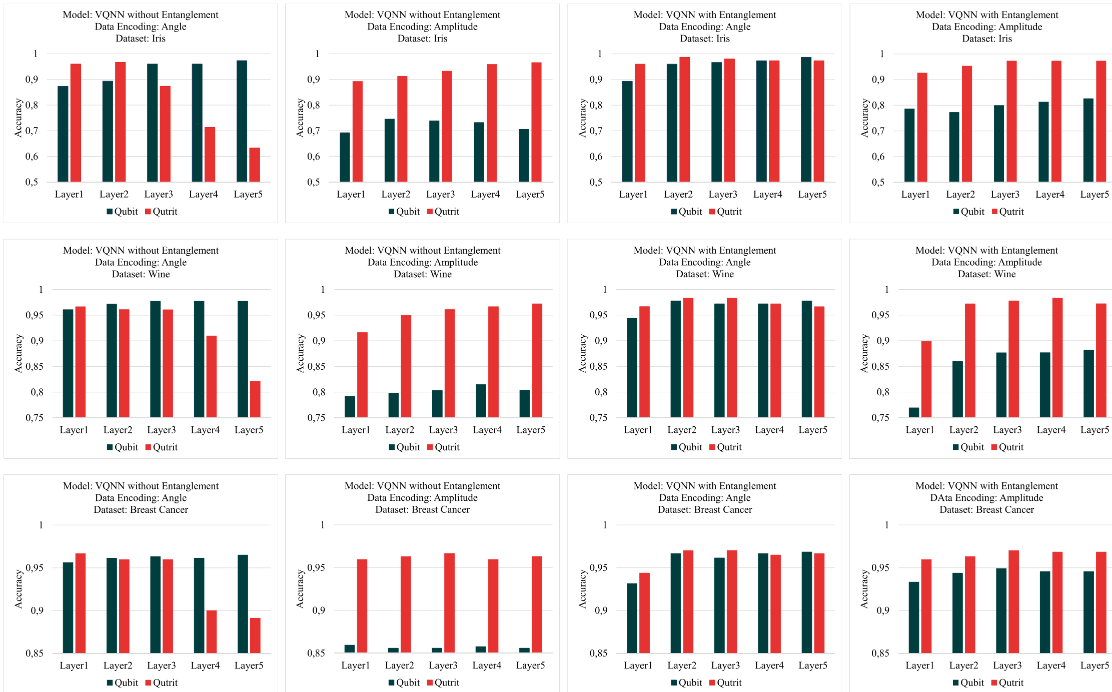

Fig. 5. Average accuracy rates for qubit and qutrit systems on the Iris, Wine and Breast Cancer datasets. These were evaluated on VQNNs utilizing angle and amplitude encoding.

to the qubit system. It was also observed that in all three datasets, the qutrit system outperformed the qubit system in both VQNNs when amplitude encoding was used. In addition, it was observed that the performance of the qutrit system increased as the number of layers increased. However, increasing the number of layers did not have a significant effect on the qubit system. Similar results are seen in the test results in Fig. 5.

As a result, when VQNNs are utilized with the angle encoding method, it is seen that higher performance can be achieved with fewer layers in the qutrit system than in the qubit system. This means that fewer trainable parameters are needed. However, it requires more computational resources. Also, the qubit system needs more layers to outperform the qutrit system. This increases the computational complexity of the quantum circuit. Furthermore, VQNNs perform better on the qutrit system when using amplitude encoding. This advantage is due to the higher dimensional state space offered by qutrits, which allows amplitude encoding to represent more information in a single quantum state. In qutrits systems, the added dimension provides a more compact data representation, reducing the need for additional computational resources. This is particularly advantageous as it minimizes the number of gates used in the circuit and, consequently, the number of parameters. While amplitude encoding is powerful, it involves complex quantum state preparations and manipulations that can be computationally expensive. However, the increased computational effort is justified by higher performance gains in qutrit systems. This is advantageous when there are limited computational resources. However, it requires more layers to achieve higher performance.

6.2. Effect of encoding strategies

This section discusses the effect of angle and amplitude encoding methods on the performance of VQNNs operating on qubit and qutrit systems. The performance of both encoding methods is evaluated on the Iris, Wine and Breast Cancer datasets and the loss plots for training and testing are presented in Figs. 14–19.

Analyzing the training and test loss graphs presented in Figs. 14–19 reveals that increasing the number of layers has a limited impact on performance when the angle encoding method is used in the VQNN without entanglement implemented in the qubit system. Similarly, no effect was observed when using the amplitude encoding method. However, it is clear that the angle encoding method performs much better than the amplitude encoding method in the qubit system. On the other hand, it is observed that one- and two-layer the VQNN without entanglement models using the angle encoding method outperform the models using the amplitude encoding method in the qutrit system. This trend started to reverse after the three-layer model. Increasing the number of layers positively affects the performance of VQNN without entanglement using the amplitude encoding method. On the other hand, it has a negative effect in VQNN utilizing the angle encoding method. Additionally, if the angle encoding method is used in the VQNN with entanglement implemented on the qubit system, it is observed that the performance improves as the number of layers increases. This is also valid if the amplitude encoding method is used. However, in the qubit system, the superiority of the angle encoding method over the amplitude encoding method is clearly seen in terms of performance.

On the other hand, it was observed that the VQNN with entanglement utilizing the angle encoding method outperformed the VQNN with entanglement utilizing the amplitude encoding method in one-, two- and three-layer models in the qutrit system. This situation is reversed for four- and five-layer models. In addition, the problem of high variance arises for the five-layer the VQNN with entanglement utilizing the angle encoding method. This is not the case for the VQNN with entanglement utilizing the amplitude encoding method. Increasing the number of layers positively affects the performance of VQNN with entanglement utilizing both encoding methods. The test results in Fig. 6 also support these results.

As a result, the choice of encoding method directly impacts the efficiency and performance of VQNNs. The angle encoding method outperforms the amplitude encoding method on the qubit system. This can be attributed to the fact that angle encoding leverages the rotational properties of qubits on the Bloch sphere, providing a more natural and efficient way to embedding classical data into quantum states. While higher performance is achieved with fewer layers using angle encoding on the qutrit system, increasing the number of layers improves the performance of the VQNN utilizing the amplitude encoding method.

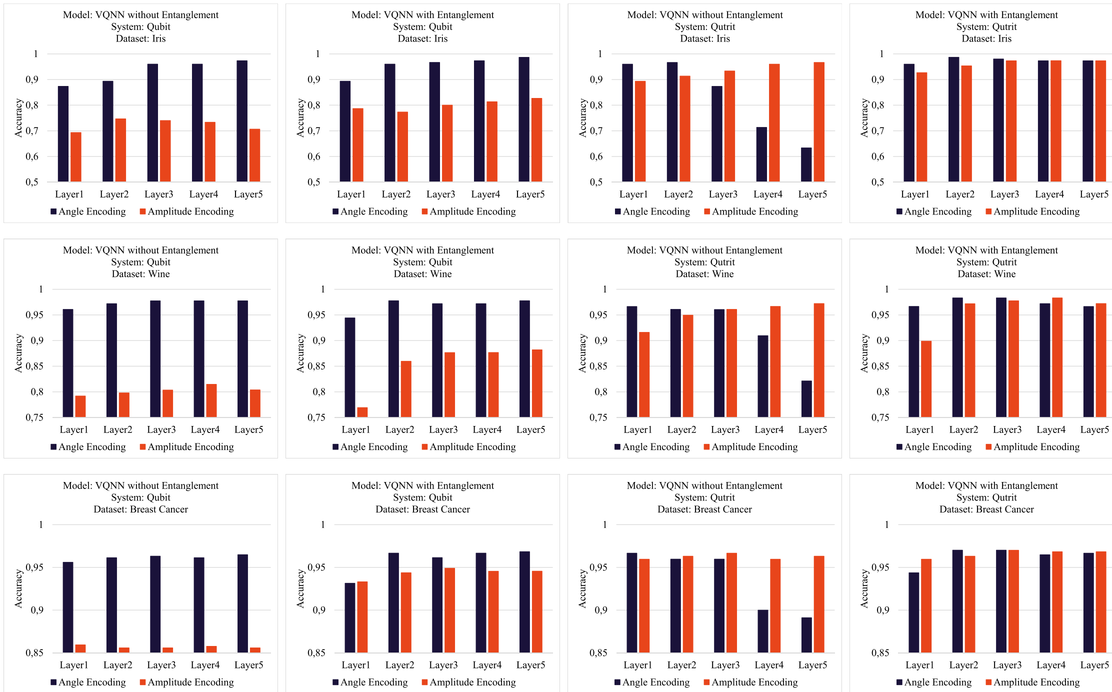

Fig. 6. Average accuracy rates for angle and amplitude encoding on the Iris, Wine and Breast Cancer datasets. These were evaluated on the qubit and qutrit systems using VQNNs.

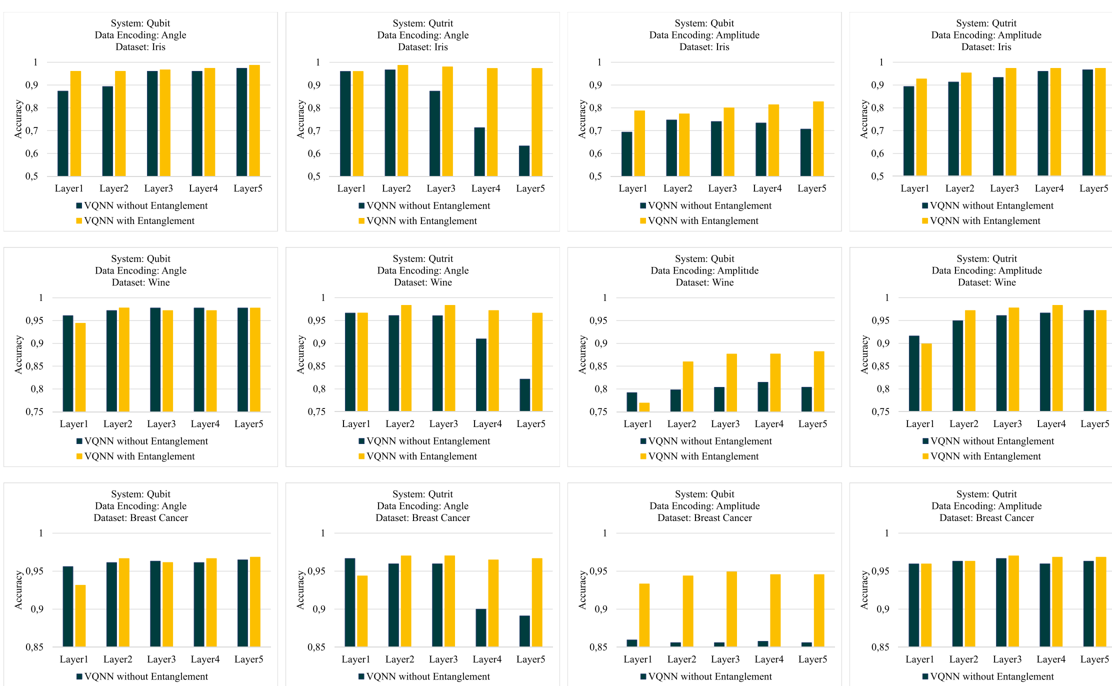

Fig. 7. Average accuracy rates for VQNNs on the Iris, Wine and Breast Cancer datasets. These were evaluated on the qubit and qutrit systems using angle and amplitude encoding.

However, it negatively affects the performance of VQNN utilizing angle encoding.

6.3. Role of entanglement

In this section, we investigate the effect of entanglement on the performance of VQNNs utilizing angle and amplitude encoding on the qubit and qutrit systems. The performance of both models is evaluated on the Iris, Wine, and Breast Cancer datasets and the loss plots for training and testing are presented in Figs. 20–25.

Analyzing the training and test loss graphs presented in Figs. 20–25, it was observed that increasing the number of layers did not lead to any performance improvement for the VQNN without entanglement utilizing angle and amplitude encoding on the qubit system. However, increasing the number of layers led to a significant performance improvement in the VQNN with entanglement. On the other hand, increasing the number of layers in the VQNN without entanglement utilizing the angle encoding method negatively affects the performance of the qutrit system. Furthermore, we observe that using more layers in VQNN with entanglement positively affects the performance of the model. However, in the five-layer model, this effect is replaced by a high variance problem. In addition, increasing the number of layers on the qutrit system has a significant positive effect in VQNNs utilizing amplitude encoding. This effect is more pronounced in VQNN with entanglement. The VQNN with entanglement outperforms the VQNN without entanglement, except for the single-layer model. This observation is also present in the test results shown in Fig. 7.

In our experiment, we observed that entanglement improves VQNN performance as the number of layers increases. With more layers, entanglement provides richer interactions between quantum states, which increases the expressive power of the network and enables it to learn complex data patterns more effectively. Thus improving the generalization ability of the model. However, in networks with fewer layers, entanglement can lead to performance degradation. This effect is due to the fact that entanglement introduces additional complexity without sufficient depth to take full advantage of its potential benefits. In addition, it can reduce the network's ability to generalize by entangling states too early, before the network has sufficient representative capacity to use the information effectively. Therefore, while entanglement can provide a significant performance boost at higher layers, its impact on lower layers needs to be carefully managed.

As a result, entanglement plays a crucial role in enhancing the performance of VQNNs. Our experiments showed that incorporating entanglement between qubits or qutrits improves the expressiveness of the quantum neural network, enabling it to capture more complex correlations in the data.

7. Limitations and future work

In this work, we focus on qutrits, 3-level quantum systems, to explore the benefits of increased dimensionality in VQNNs. Qutrits represent a natural step after qubits. This allows us to explore the benefits of increased data representation capacity without overly complicating the quantum state. Qutrits strike a balance between providing additional dimensionality and maintaining manageability, both in theory and experiment. In addition, they provide sufficient flexibility in quantum neural networks while keeping the model manageable in terms of computational complexity. Higher dimensional qudits (such as 4-level, 5-level, or higher) offer more potential for encoding classical data and can lead to more compact quantum representations, while also potentially unlocking more advantages for specific tasks.

While our findings demonstrate the potential advantages of qutrit-based VQNNs, implementing these models in real quantum hardware poses significant challenges. While qudit systems can alleviate some of the challenges faced by qubit-based systems by demonstrating improved robustness to noise and errors, they still face practical challenges [16]. Qudit-based architectures are challenging to maintain coherence due to additional state levels that require precise control and calibration of quantum gates and higher-dimensional entanglement. To mitigate these issues and maintain the baseline performance observed in simulations, error correction methods tailored specifically for multi-level quantum systems need further research [38,39]. In addition, current quantum hardware primarily supports qubit-based systems and the development of qudit-based hardware is still in its infancy [40]. Therefore, the entire study was conducted on the noisy Pennylane quantum computing simulator [33]. Although the realization of qudit is difficult with current technologies, it is obvious that using high-dimensional systems will produce more efficient solutions, considering the efficiency of high-dimensional systems in reducing noise and complexity, allowing fewer quantum resources. Especially in recent years, experimentally developed high-dimensional photonic studies have been promising [41–43].

Fully exploiting the advantages of qutrit-based systems requires the development of optimized algorithms. Future work should focus on designing efficient quantum algorithms and optimizing existing ones for qutrit-based architectures. Beyond qutrits, exploring the potential of even higher-dimensional qudits for VQNNs could unlock further improvements in performance and efficiency. Future studies should investigate the trade-offs and benefits of utilizing higher-dimensional quantum systems in quantum neural network architectures. Finally, applying qutrit-based VQNNs to real-world problems and datasets can provide valuable insights into their practical utility and performance. Conducting experiments in diverse application domains, such as image recognition, drug discovery, and optimization problems, can help validate the theoretical advantages of qutrit-based systems. By addressing these future research directions, we can continue to advance the field of quantum machine learning and unlock the full potential of qudit-based VQNNs in practical applications.

In our study, Principal Component Analysis (PCA) was chosen for its simplicity and effectiveness in preprocessing, as it provides a simple way to capture key features of the data and reduce its dimensionality before encoding. However, some research has suggested that PCA may not be ideal for this purpose. Qi et al. [44], and Chen et al. [45] have shown that tensor network-based architectures are better able to preserve complex, high-order data correlations, which can be advantageous when encoding data for quantum circuits. While tensor network-based methods can offer potential advantages, our current work is focused on dimensionality reduction with a widely used classical technique. Future work could explore tensor network-based dimensionality reduction as an alternative. Investigating these methods may reveal improvements in the overall performance of variational quantum neural networks.

8. Conclusion

Our primary objective in this study is to examine how the performance of VQNNs is affected by system dimension, encoding strategies, and entanglement, and to clarify the strengths and weaknesses of these systems. In this context, we conducted a comparative analysis of variational quantum neural networks (VQNNs) utilizing qubits and qutrits. Our findings highlight the potential advantages of qutrit-based VQNNs in terms of parameter efficiency and performance, particularly in tasks involving angle and amplitude encoding. In this framework, when the angle encoding method is used in VQNNs, it is seen that the qutrit system provides superior performance with fewer variational layers compared to the qubit system. However, it should be noted that this method requires more computational resources. On the contrary, the qubit system requires more variational layers to achieve optimal performance, thus increasing the computational complexity of the quantum circuit. It should also be noted that the performance of the VQNN utilizing angle encoding on the qutrit system may be negatively affected by the increase in the number of layers. Considering the qubit system, the angle encoding method outperformed the amplitude encoding method in performance. On the other hand, the qutrit system showed superior performance compared to the qubit system when amplitude encoding was used. This is especially advantageous when computational resources are limited. However, it is essential to note that achieving high-performance levels through amplitude encoding requires more variational layers. In addition, VQNNs utilizing angle and amplitude encoding require the inclusion of entanglement on both qubit and qutrit systems to maximize performance. These results are essential to understand the trade-offs between the two systems and to make informed decisions when choosing the most appropriate method for a given application. Furthermore, the choice of encoding method should be made carefully considering the specific system and its requirements.

CRediT authorship contribution statement

Erdi Acar: Writing – review & editing, Writing – original draft, Visualization, Methodology, Conceptualization. **İhsan Yılmaz:** Writing – review & editing, Writing – original draft, Resources, Project administration, Conceptualization.

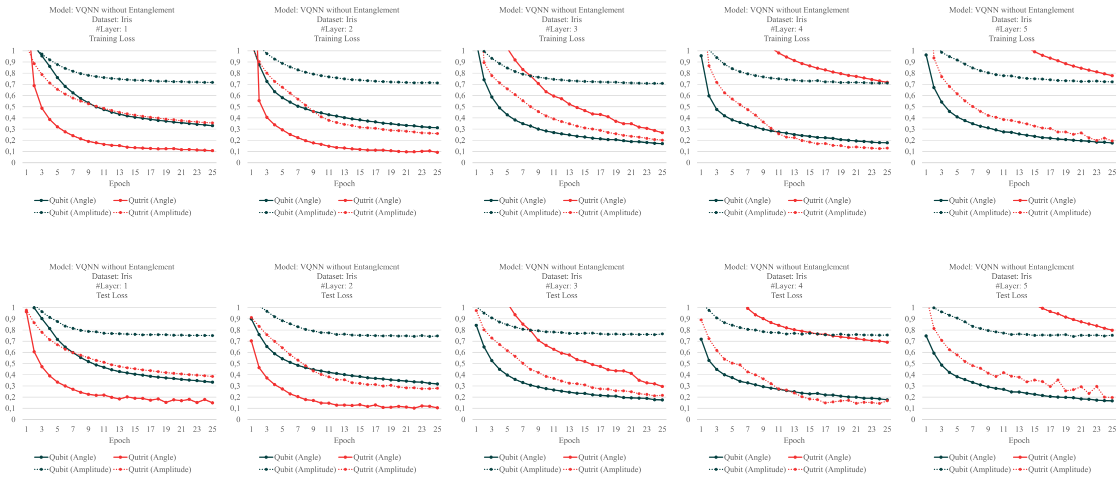

Fig. 8. Performance variation in training and test loss rates of the qubit and qutrit systems when VQNN without entanglement is used for one to five layers on the Iris dataset.

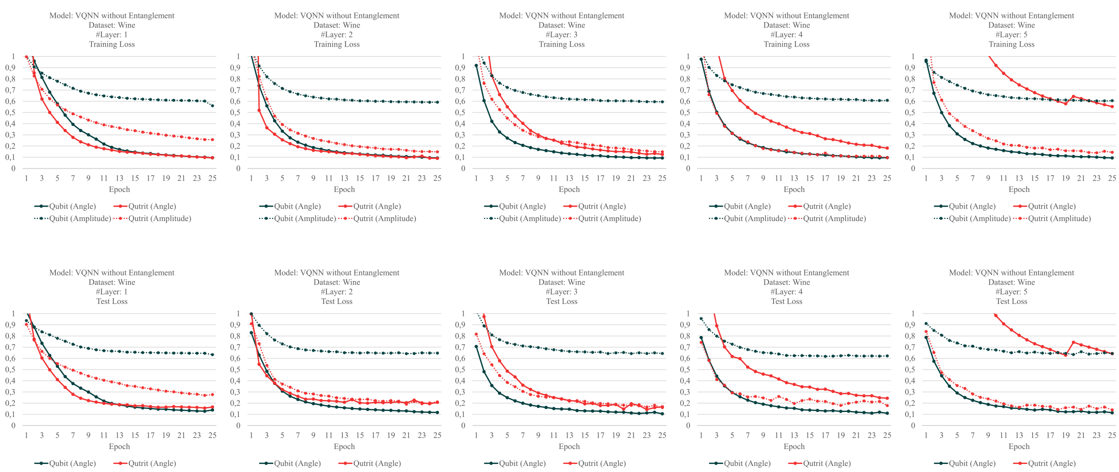

Fig. 9. Performance variation in training and test loss rates of the qubit and qutrit systems when VQNN without entanglement is used for one to five layers on the Wine dataset.

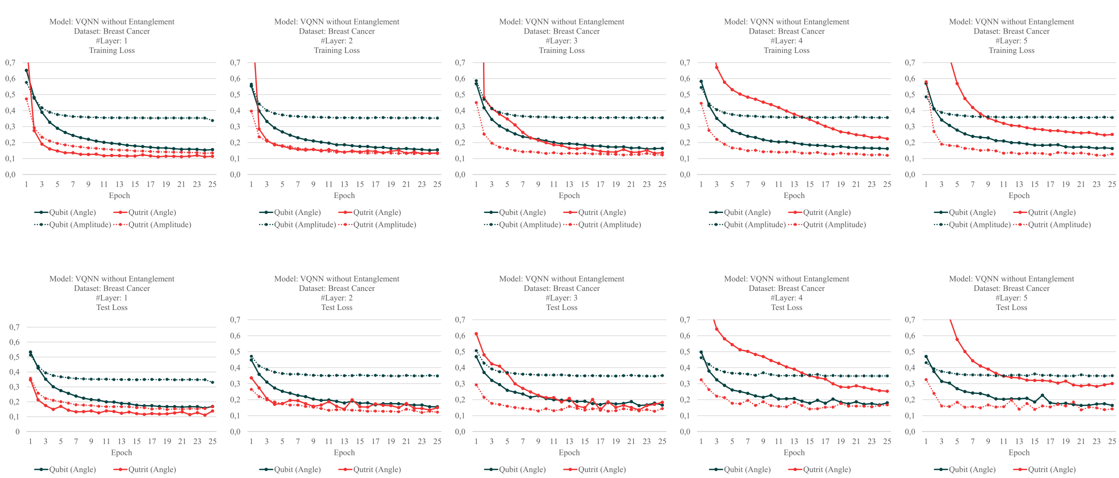

Fig. 10. Performance variation in training and test loss rates of the qubit and qutrit systems when VQNN without entanglement is used for one to five layers on the Breast Cancer dataset.

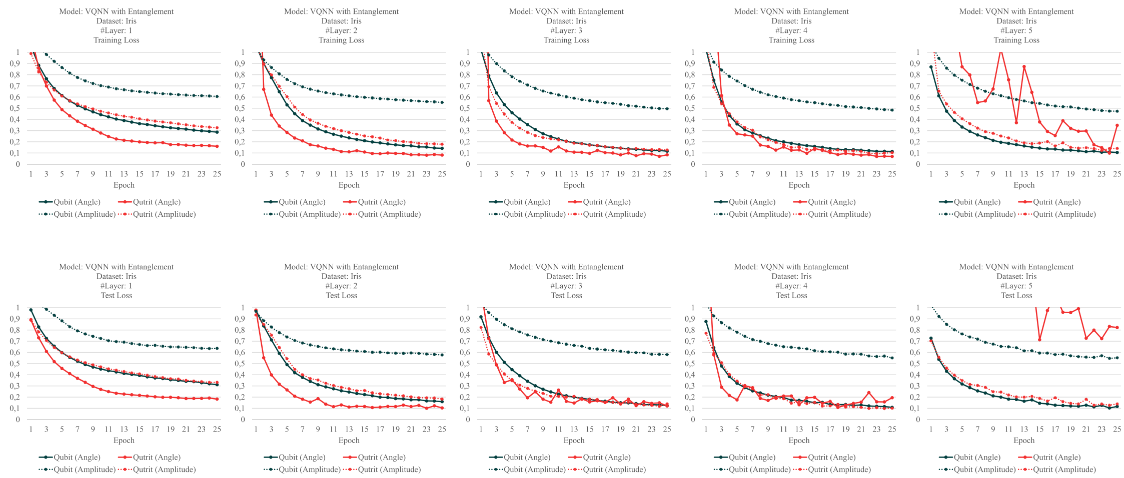

Fig. 11. Performance variation in training and test loss rates of the qubit and qutrit systems when VQNN with entanglement is used for one to five layers on the Iris dataset.

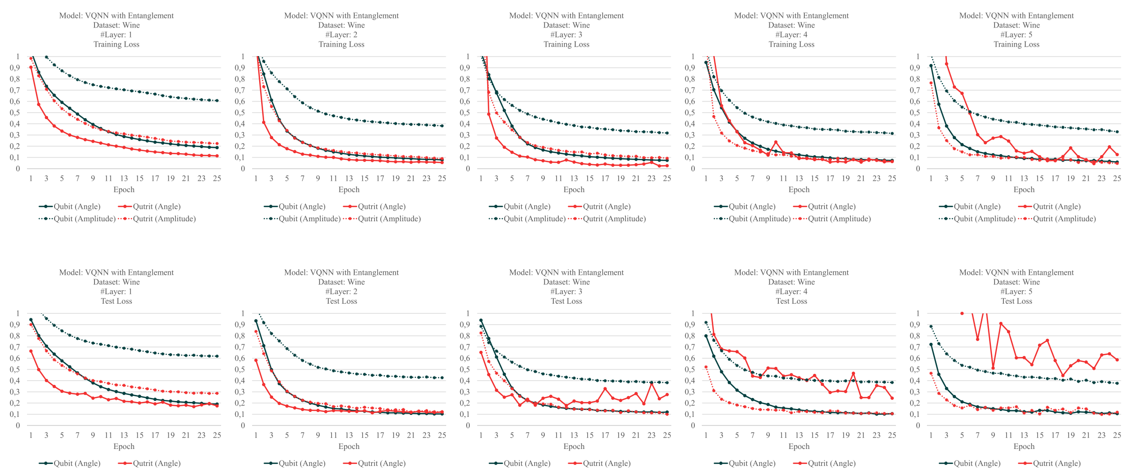

Fig. 12. Performance variation in training and test loss rates of the qubit and qutrit systems when VQNN with entanglement is used for one to five layers on the Wine dataset.

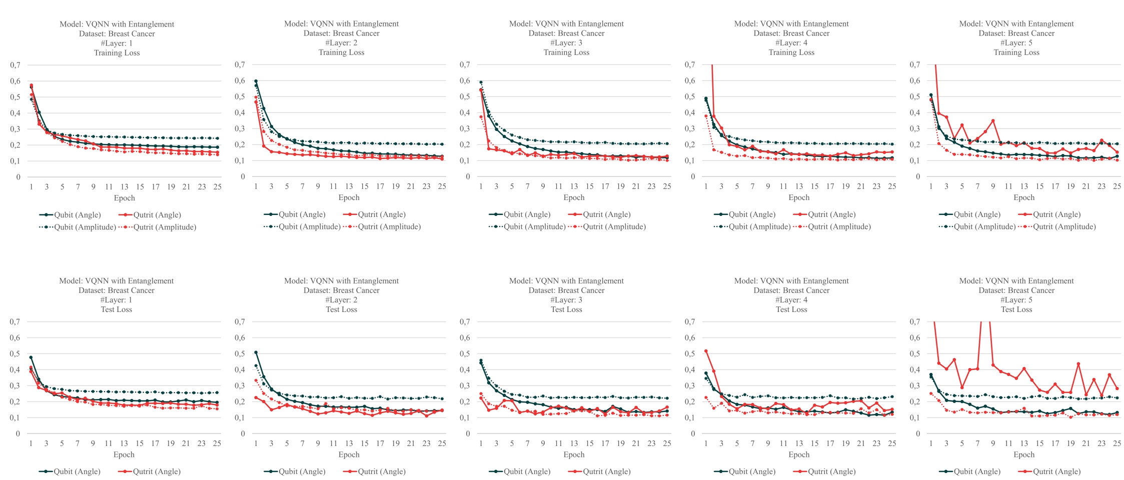

Fig. 13. Performance variation in training and test loss rates of the qubit and qutrit systems when VQNN with entanglement is used for one to five layers on the Breast Cancer dataset.

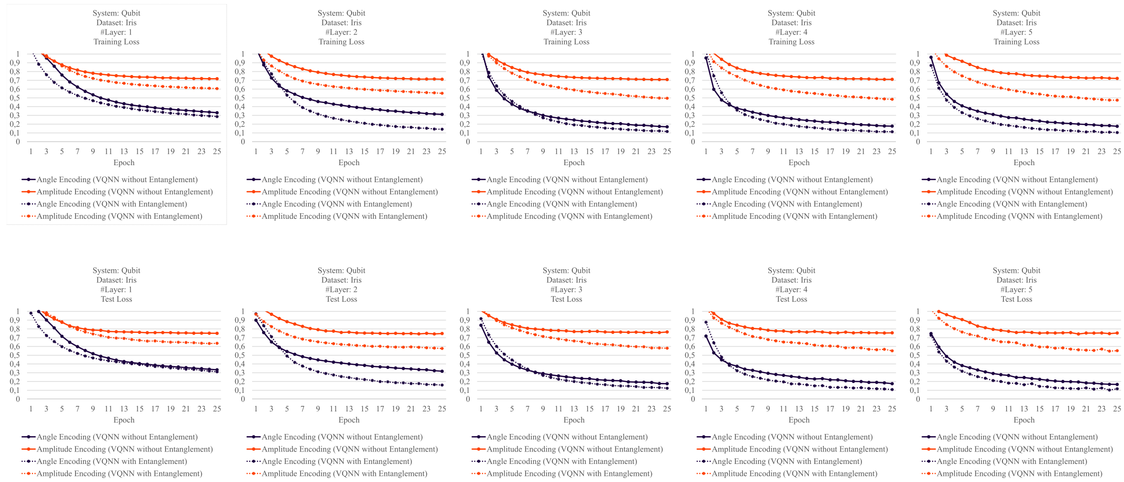

Fig. 14. Variation in the performance rates of angle and amplitude encoding methods during training and testing when using a qubit system for one to five layers on the Iris dataset.

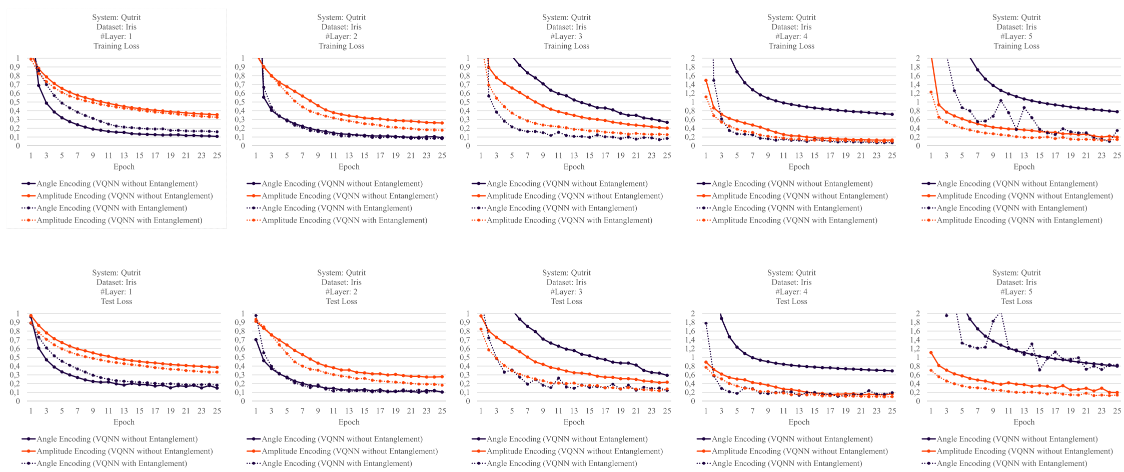

Fig. 15. Variation in the performance rates of angle and amplitude encoding methods during training and testing when using a qutrit system for one to five layers on the Iris dataset.

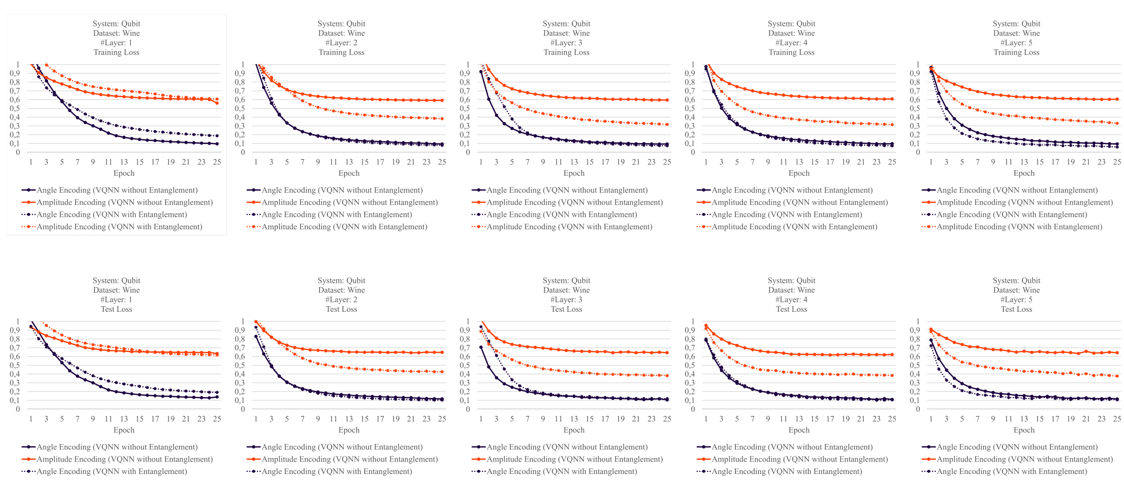

Fig. 16. Variation in the performance rates of angle and amplitude encoding methods during training and testing when using a qubit system for one to five layers on the Wine dataset.

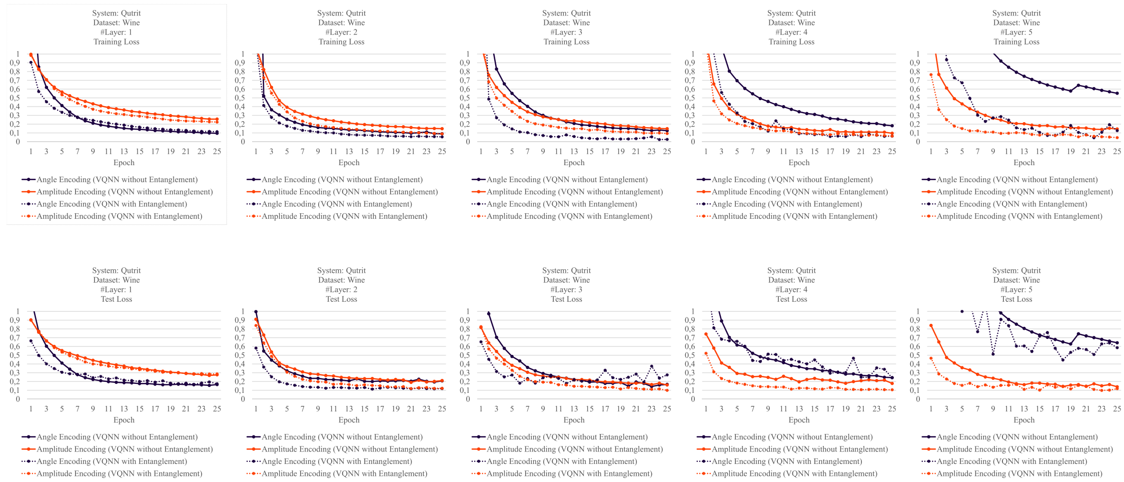

Fig. 17. Variation in the performance rates of angle and amplitude encoding methods during training and testing when using a qutrit system for one to five layers on the Wine dataset.

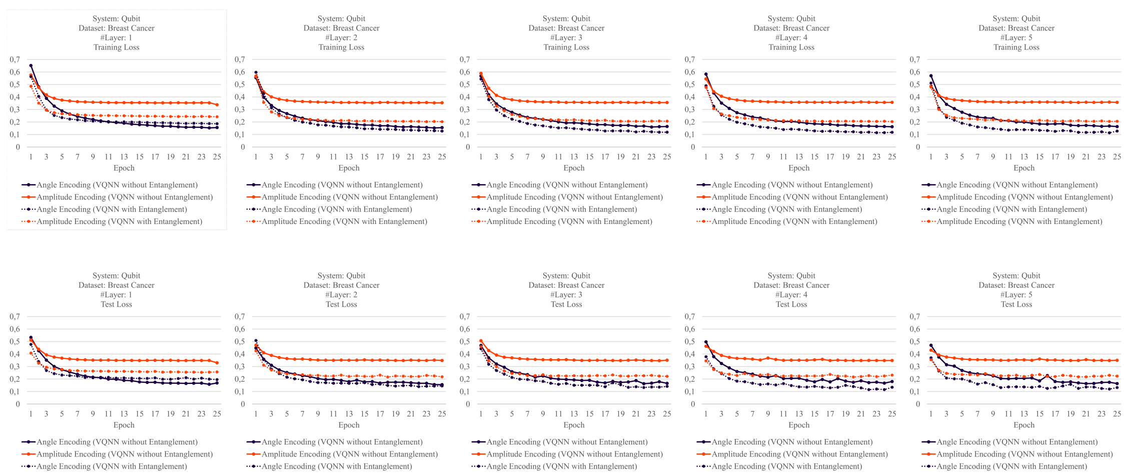

Fig. 18. Variation in the performance rates of angle and amplitude encoding methods during training and testing when using a qubit system for one to five layers on the Breast Cancer dataset.

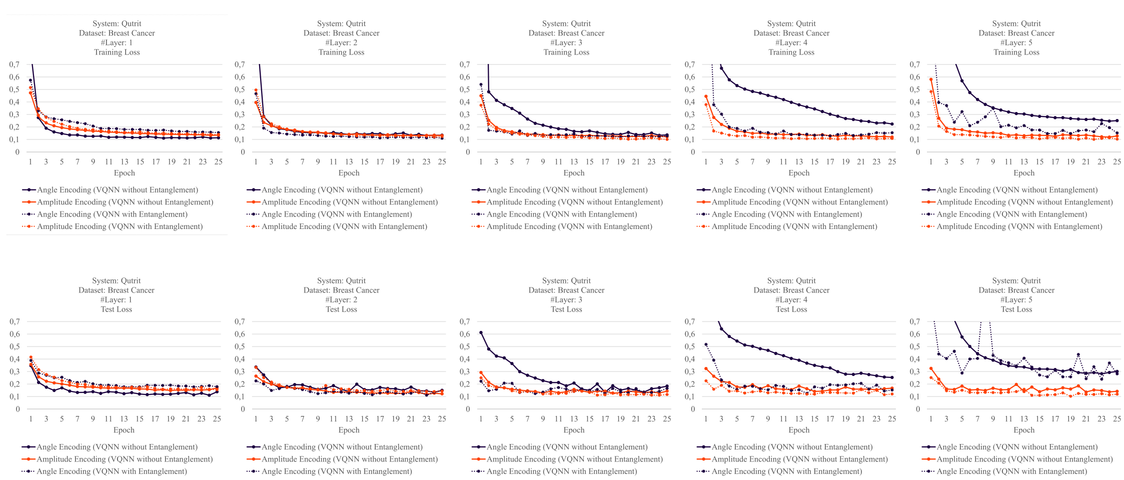

Fig. 19. Variation in the performance rates of angle and amplitude encoding methods during training and testing when using a qutrit system for one to five layers on the Breast Cancer dataset.

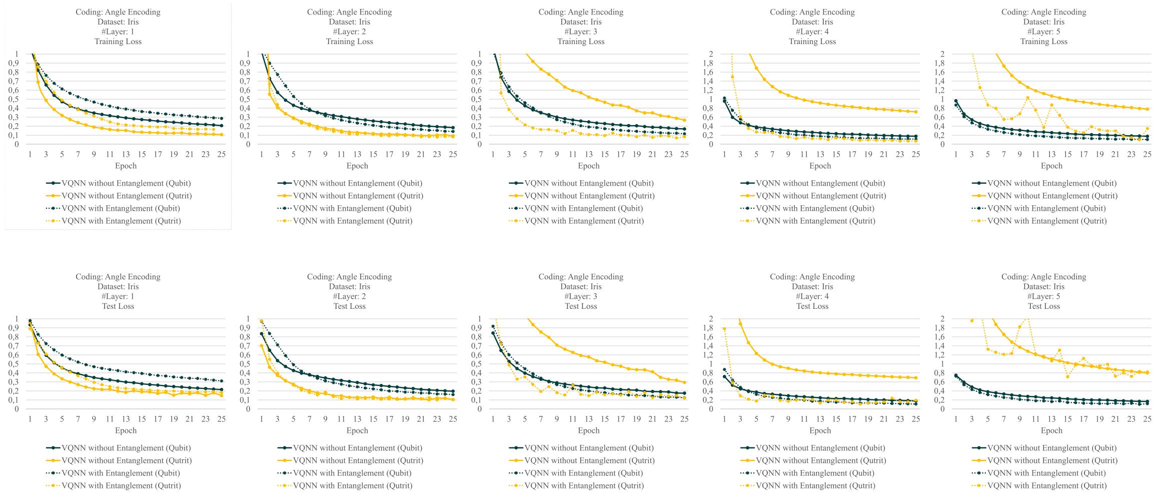

Fig. 20. Performance variation in training and test loss rates of VQNN without and with entanglement on qubit and qutrit systems using angle encoding method on the Iris dataset.

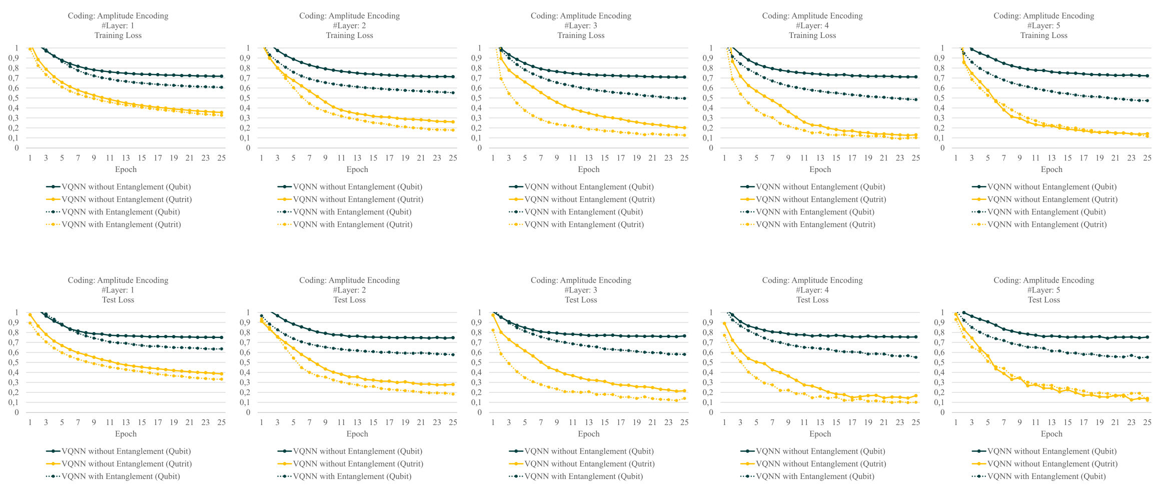

Fig. 21. Performance variation in training and test loss rates of VQNN without and with entanglement on qubit and qutrit systems using amplitude encoding method on the Iris dataset.

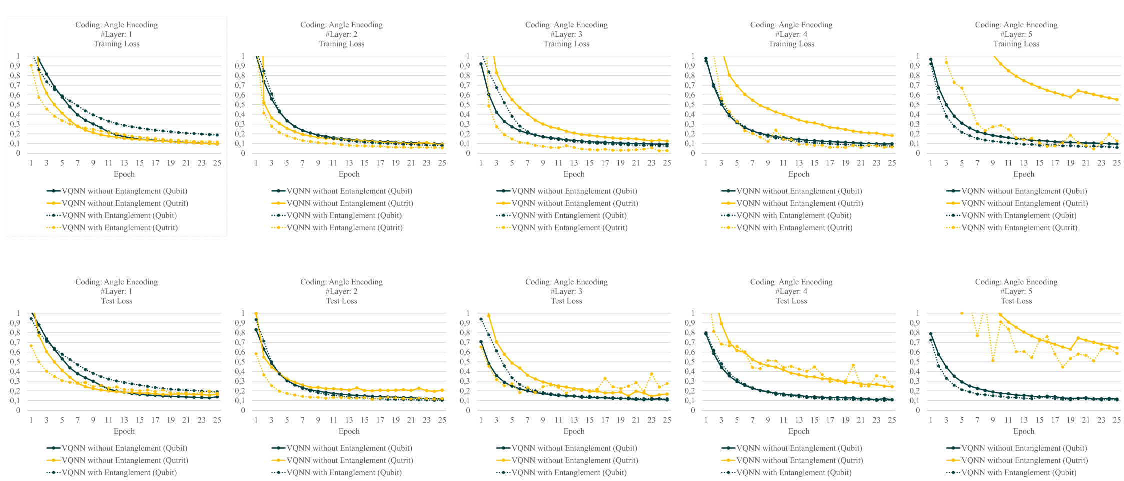

Fig. 22. Performance variation in training and test loss rates of VQNN without and with entanglement on qubit and qutrit systems using angle encoding method on the Wine dataset.

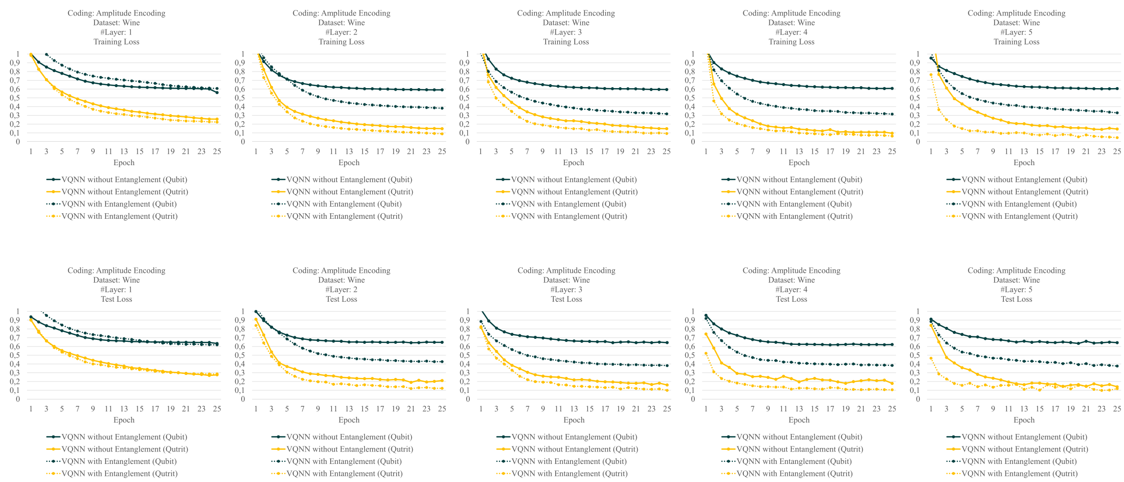

Fig. 23. Performance variation in training and test loss rates of VQNN without and with entanglement on qubit and qutrit systems using amplitude encoding method on the Wine dataset.

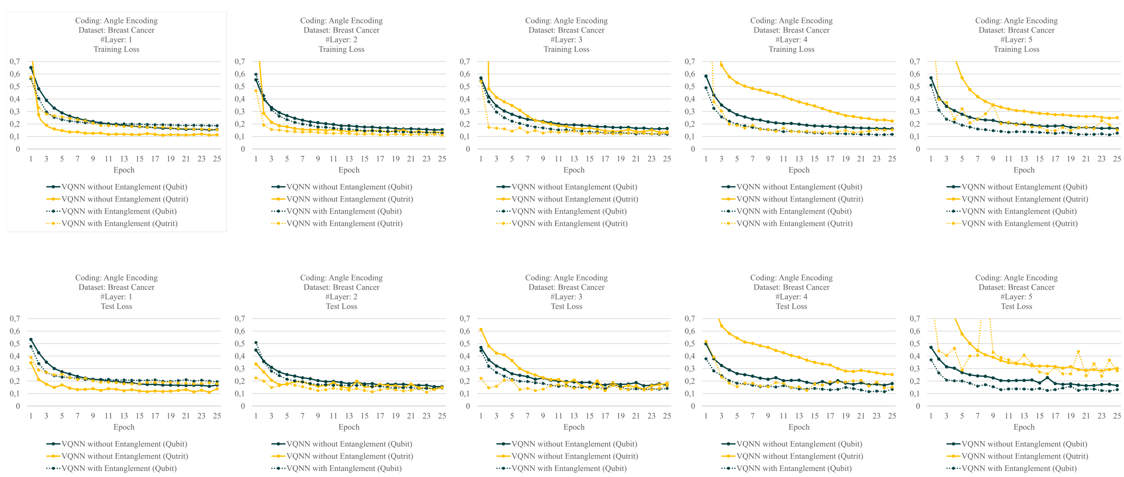

Fig. 24. Performance variation in training and test loss rates of VQNN without and with entanglement on qubit and qutrit systems using angle encoding method on the Breast Cancer dataset.

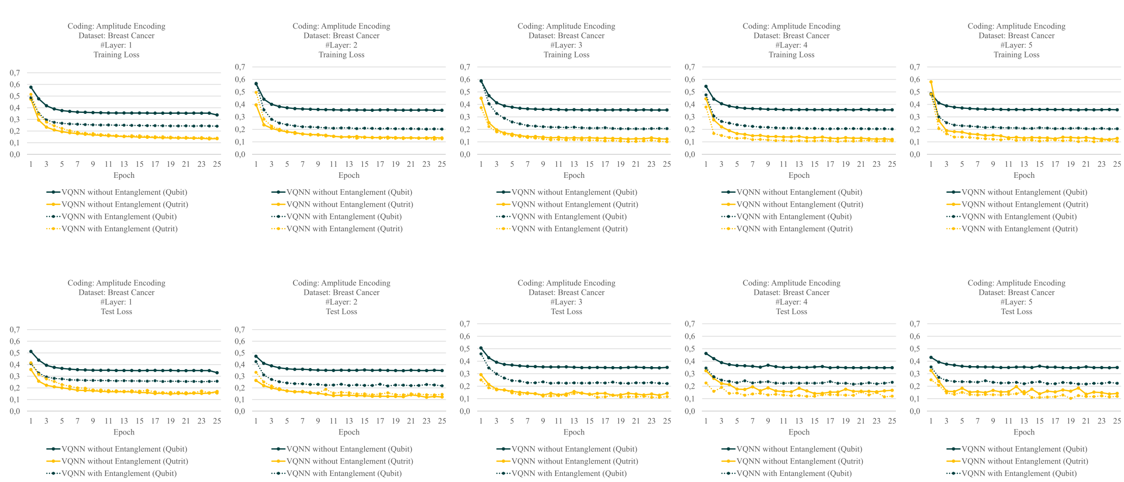

Fig. 25. Performance variation in training and test loss rates of VQNN without and with entanglement on qubit and qutrit systems using amplitude encoding method on the Breast Cancer dataset.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the author(s) used Grammarly in order to grammar check. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Training and test loss plots for comparison of performance of qubit and qutrit systems

See Figs. 8–13.

Appendix B. Training and test loss plots for comparison of performance of angle and amplitude encoding methods

See Figs. 14–19.

Appendix C. Training and test loss plots for comparison of performance of VQNN without entanglement and VQNN with entanglement

See Figs. 20–25.

Data availability

All datasets used in this study were obtained from the UCI Machine Learning Repository.

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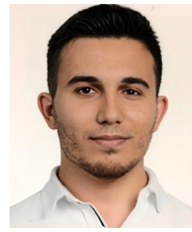

Erdi Acar was born in Istanbul in 1994. In 2017, he graduated from the Department of Mathematics, Faculty of Arts and Sciences at Çanakkale Onsekiz Mart University. In 2021, he completed his Master's degree in Computer Engineering at School of Graduate Studies, Çanakkale Onsekiz Mart University with a thesis titled "Covid-19 Detection with Hybrid Quantum-Classic Machine Learning". He is currently pursuing a Ph.D. in Computer Engineering at School of Graduate Studies, Çanakkale Onsekiz Mart University and has been working as a Research Assistant in the Department of Computer Engineering at Istanbul Gelişim University since September 2023. His academic work focuses on quantum computing and quantum machine learning.

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