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Impact of IoT in improving the proficiency of electric power distribution systems: Linguistic bipolar MABAC technique based on einstein models with priority degrees

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The key theme of this paper is to introduce the linguistic bipolar fuzzy sets, which are a modified version of the linguistic information and bipolar fuzzy set theory. Additionally, we propose the Einstein operational laws based on linguistic bipolar fuzzy information for evaluating the prioritized aggregation operators, such as the linguistic bipolar fuzzy prioritized Einstein averaging operator and linguistic bipolar fuzzy prioritized Einstein geometric operator, with three basic properties for each operator, called idempotency, monotonicity and boundedness. Moreover, we

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aim to construct the multiattributive border approximation area comparison tool and the multiattribute decision-making tool based on the designed operators. Furthermore, we illustrate the design of some numerical analyses for evaluating the decision-making technique and suggest models to describe the authority of the planned theory. Finally, by considering the proposed ranking values, we aim to compare them with the ranking values of the existing models for assessing the validity of the described theory.

Keywords: Decision-making technique; electric power distribution systems; internet of things; linguistic bipolar fuzzy systems; MABAC technique; priority Einstein aggregation operators.

1. Introduction

The Internet of Things (IoTs) is a valuable and efficient system of interconnected physical devices surrounded by a network, sensors and software connectivity that permits or allows them to exchange and arrange information over the internet without human interference. Numerous valuable and common examples are stated; for instance, smart agriculture, industrial IoTs, wearables, connected cars and smart home devices. The function of IoT is working in the following ways: for instance, sensors collect the data and connectivity sends information over the internet for data processing to take action in the form of alerts or signals. The fundamental question is why IoT is important and valuable because of the following reasons: it saves time, energy and cost, enables real-time monitoring, enhances efficiency/accuracy and automates tasks. Table 1 shows the role of IoT in modern systems and its impact.

The electric power distribution system is a power system or electric technique that provides electricity from transmission systems to end users, for instance, industries, homes and businesses. The electric power is the rate at which electrical energy is consumed, generated, or transmitted. The generation, transmission and distribution are the three major and fundamental stages of the electric power systems. The characteristics of the electric power distribution systems are derived in Table 2.

Table 1. Role of IoT in modern systems.

Aspects	Explination	Impact
Communication	The IoT enables devices to share information through networks.	This approach enhanced the coordination systems and enabled coordinated operations.
Automation	These advanced devices enable the system to respond automatically.	The impact of this approach is that it reduces manual effort and enhances system operations.
Data Collection	These advanced devices integrate data from physical objects.	This aspect enhances the system awareness and improves the operational mechanism.
System Integration	It enables system integration and connects physical and digital components.	This approach improves the overall efficiency of the system and enhances its reliability.
Real-time Monitoring	It enables the system to continuously observe the system conditions.	It helps to improve the decision-making approaches and enables the system to operate effectively.

Table 2. Characteristics of electric power distribution systems.

Component	Function	Importnce
Transformers	The transformers help to adjust the voltage level and support a constant flow of electricity.	It enhances the efficiency of the system and also improves safety.
Protection Devices	These devices are used for the detection of faults and isolating failures.	This component helps to prevent system damage and supports long-term system efficiency.
Distribution Lines	These devices deliver electricity from the main source to the end user.	It ensures a reliable power supply and also helps users get proper electricity.
Monitoring Systems	The monitoring devices help to track the load and performance of the system continuously.	It helps to detect faults earlier and reduces the chances of shortfalls.
Substations	It controls the flow of the power and guides it on the appropriate route.	It supports the stability of the system and also enhances system efficiency. It also helps to maintain the effectiveness of the system.

Our main theme is to discuss the impact of IoT in enhancing the validity and proficiency of electric power distribution systems because IoT enables data-driven decision-making, real-time monitoring and intelligent control, which eliminates the limitations and uncertainty of traditional distribution networks. By using the communication systems, sensors and smart meters, IoT helps in efficient load management, predictive maintenance, reduction of power losses, faster restoration and early fault detection. Conventional electric power distribution systems are widely operated because of delayed fault detection, inefficient maintenance practices, lack of real-time visibility and poor demand forecasting. The main theme of IoT in power distribution systems is to embed communication devices, intelligent controllers and sensors throughout the grid. The overall impact on proficiency of distribution systems includes cost-effectiveness, sustainability, flexibility, reliability and efficiency. Some major aspects of the IoT are described as connectivity, sensors and actuators, data collection and analysis, automation and control and interoperability. IoT has many applications, for instance, devices like smart thermostats or homes, healthcare, industry and agriculture. Further, the electric power distribution systems are a powerful technique or network that can provide electricity from the transmission system to the end users, such as industry, homes and businesses. These models and procedures are the last stage of the electricity grid and play an essential role in ensuring the capability and effective supply of power to consumers. Some major components of the electric power distribution systems are explained and mentioned by substations, distribution lines, transformers, switches, circuit breakers, meters, control centers, protection systems and smart grid technology (optional). Some reliable and dominant functions of electric power distribution systems are as follows: voltage regulation, fault detection, load balancing and energy efficiency. The technique of electric power distribution systems has many applications, but they also have some challenges, such as power losses, reliability and renewable integration. In short, the electric power distribution system is a reasonable and valid technique for promoting electricity from power plants to consumers, ensuring it is done reliably, effectively and safely. The role and impact of the IoT in improving the proficiency of electric power distribution systems. Discover how the IoTs is transforming electric power distribution through automation, predictive maintenance and real-time monitoring. In traditional or old power distribution systems, postponing response to errors, the inability to achieve energy requests dynamically and inadequacies often arise from a deficiency of reflectivity. The impact of IoT in improving the proficiency of electric power distribution systems is discussed by many individuals, who explained their importance and effectiveness in real-life problems. After the valuation of the impact of the IoT in enhancing the proficiency of the electric power distribution system, many scholars have worked on it; for instance, Bedi *et al.*[1] investigated the electric power and energy systems based on IoT with a brief literature review. Amin *et al.*[2] established the electric power distribution systems based on IoT with a case study. Sawas and Farag[3] developed the anomaly prediction techniques with IoT-based cyberattacks on power distribution systems. Peter *et al.*[4] presented the normalizing and monitoring technique for power distribution systems with the help of IoT. Zhu *et al.*[5] diagnosed the distribution automation with smart grids and IoT.

A decision-making strategy is a technique or model used to make a decision or evaluate problems under the consideration of available information, attributes and preferences. These models can help us to evaluate or resolve decisions and choose the best one from the collection of information. For the valuation of the different kinds of problems, numerous decision-making techniques have been developed by different scholars; for instance, rational decision-making, intuitive decision-making, heuristic decision-making, multicriteria decision-making, cost-benefit analysis, group decision-making, Game theory, Pareto analysis (80/20 rule), brainstorming and creative decision-making, linear programming and the Delphi method. The traditional decision-making strategy has been used for the valuation of the best decision, but the main problem is that it does not work accurately because of classical information. The classical information has contained just zero and one, but not between and in numerous decision-making scenarios, the expert has required a partial degree instead of just zero and one. Therefore, Zadeh[6] designed the fuzzy set (FS) theory. FS theory is a very famous and dominant model because of the membership function, where the membership function is defined from the universe of discourse to the unit interval. The range of FS theory provides a wide space of information to experts for making their decisions more precisely and accurately, as compared to crisp set theory. After the development of the FS theory, some well-known applications have been derived. For instance, Mahmood and Ali[7] established the fuzzy superior Mandelbrot sets. Demir[8] presented the decision-making technique with global trends. Saqlain[9] diagnosed the bibliometric analysis of FSs and their extensions.

Human opinions are not limited to positive, membership, yes, truth, and so on, but also extended to negative, nonmembership, no, falsity and so on. Because human opinions are so wide and so different to express their feelings in the form of variables or words. The FS contained just a positive truth function but not a negative, which is a very important part of the human decision theory. The negative information plays an important and dominant role in describing the human strategy very efficiently and perfectly. Therefore, in 1994, the novel strategy of bipolar FSs (BFSs) was designed by Zhang.[10] The mathematical form of the BFSs is explained and mentioned as: $\eta = \{(\mu(\mathbb{X}), \xi(\mathbb{X})) : \mathbb{X} \in \mathbb{X}\}$, where the data in set η contained the term of $\mu(\mathbb{X})$ (positive) and $\xi(\mathbb{X})$ (negative), defined from any fixed set $\mathbb{X}$ to $[0, 1]$ and $[-1, 0]$ such as $\mu : \mathbb{X} \to [0, 1]$ and $\xi : \mathbb{X} \to [-1, 0]$. The final and concluding form of BFN is presented and described by: $\eta_j = (\mu(\mathbb{X}_j), \xi(\mathbb{X}_j)),\ j = 1, 2, \ldots, z$. The model of FS theory is part of BFSs if $\xi(\mathbb{X}_j) = 0$, then the tool of the BFS theory will be reduced to the FS theory. The BFS has been given a huge consideration by different scholars and many of them have worked on it. For instance, Alolaiyan *et al.*[11] derived the aggregation operators (AOs) for BFSs by studying their application in software selection. Ahmad *et al.*[12] designed a sustainable energy solution by integrating the BFSs, soft sets and hesitant sets. Sharma[13] invented the VIKOR technique for trapezoidal BFSs with improved stock portfolio selection.

Some information is not expressed very efficiently by human beings in the presence of real-world data. Therefore, for the interpretation of those types of data, many experts have used the linguistic variables, which were invented by Zadeh.[14–16] The linguistic variables are very suitable for the representation of complex and natural information, which is also called linguistic term sets (LTSs). The constructive and well-known form of LTS with odd cardinality is mentioned and explained by: $\mathbb{L} = .\{\mathbb{L}_{\mathbb{I}_j} : j = 1, 2, \ldots, z, \mathbb{I} \in [0, \zeta]\}$, numerous conditions for the LTS are explained by: (i) $\mathbb{L}_{\mathbb{I}_j} > \mathbb{L}_{\mathbb{I}'_j} \Leftrightarrow \mathbb{I}_j > \mathbb{I}'_j$, (ii) $Neg(\mathbb{L}_{\mathbb{I}_j}) = \mathbb{L}_{\mathbb{I}'_j}$, where $\mathbb{I}'_j = \mathbb{I}_j + ou$, called negative operators, and (iii) $\max(\mathbb{L}_{\mathbb{I}_j}, \mathbb{L}_{\mathbb{I}'_j}) = \mathbb{L}_{\mathbb{I}_j}$, if $\mathbb{I}_j \geq \mathbb{I}'_j$, and $\min(\mathbb{L}_{\mathbb{I}_j}, \mathbb{L}_{\mathbb{I}'_j}) = \mathbb{L}_{\mathbb{I}_j}$, if $\mathbb{I}_j \leq \mathbb{I}'_j$. The LTS is discussed in many fields; for instance, Xu[17] derived the aggregation techniques based on extended induced linguistic sets. Xu[18] presented the aggregation model for linguistic decision-making frameworks. Herrera and Herrera–Viedma[19] designed the aggregation systems for linguistically weighted information.

All existing models are resourceful and efficient because of their advantages, characteristics and limitations. Therefore, numerous scholars have integrated the above ideas and developed different models. For instance, Bonissone[20] derived the fuzzy model based on linguistic approaches in 1980. Liu *et al.*[21] determined the bipolar LTSs (BLTSs), which contain the linguistic variable with positive and negative truth functions. But we aim to derive the positive and negative functions in the form of linguistic variables, which have not been developed yet. The investigation of the best decision among the family of alternatives is also very complex; numerous scholars have used the decision-making models. In 2015, Pamucar and Cirovic[22] also designed a new model for the valuation of the best decision, called the multiattribute border approximation area comparison (MABAC) technique. The decision-making model and MABAC model are very efficient and reliable procedures, and many people have developed them for FSs based on some AOs. For instance, the prioritized averaging (PRA) operator and the prioritized geometric (PRG) operator were invented by Yager.[23] In this paper, we developed the model of the MABAC technique based on Einstein t-norm (ETN) and Einstein t-conorm (ETCN), which was invented by Menger.[24] In 1994, the tool of AOs for FS theory was designed by Yager.[25] Further, Mardani *et al.*[26] developed the technique of fuzzy AOs with an application. Merigo and Casanovas[27] invented the fuzzy generalized hybrid AOs with decision-making techniques. Wei *et al.*[28] derived a technique of Hamacher AOs for BFS with a decision-making model. Jana *et al.*[29] presented the AOs based on BFSs with a soft structure of information. Jana *et al.*[30] stated the Dombi AOs for BFSs. Jana *et al.*[31] designed the Dombi prioritized AOs for BFSs. Riaz *et al.*[32] invented the innovative bipolar fuzzy sine trigonometric AOs with SIR techniques. Tchangani[33] exposed the bipolar aggregation methods. Ali and Yang[34] developed the AOs for bipolar complex fuzzy soft inference systems. Jana *et al.*[35] presented the bipolar fuzzy approach with economic condition analysis. Jana[36] described the extended bipolar fuzzy MABAC model. Rasool *et al.*[37] developed the CRITIC and EDAS techniques. Al-Agtash and Al-Fahoum[38] designed the computation approaches to electricity. Ali and Yang[39] derived the MABAC models. Bertling *et al.*[40] introduced the power distribution systems. Al-Fahoum and Ghobon[41] invented the applied approaches for speed estimation techniques. Adel *et al.*[42] developed the high-dimensional chaotic Lorenz system. Ibrahim *et al.*[43] derived the simulation of the fractional COVID-19 model based on the spectral collocation approach. Tabassum *et al.*[44] derived the IoT approach and adaptive neuro-fuzzy systems. Nishtar *et al.*[45] presented the improved version of digital grids stability using intelligent neuro-fuzzy logics. Anyhow, we noted that some major problems are affecting the ranking values, which are described:

(1) Why do we initiate the novel technique of linguistic bipolar fuzzy sets (LBFSs)?
(2) Why do we need to develop the MABAC model and the multiattribute decision-making (MADM) model?
(3) Why do we discuss the impact of IoT in improving the proficiency of electric power distribution systems?
(4) Is the analysis of Einstein operators important?

After a long assessment, we noted that many scholars have used the FSs, BFSs and LTSs for the valuation of their major problems, but these ideas do not efficiently and reliably describe human opinions and nature. Therefore, we concentrated on the construction of the linguistic bipolar fuzzy model, which is an efficient and reliable technique for describing human opinions very precisely and efficiently. The LBFSs contained the positive and negative functions in the form of linguistic variables, which described the nature and opinion of human beings efficiently. Further, the analysis of any idea is very easy, but the main problem is how to use it for our real-life problems. Therefore, we know that the idea of LBFS is novel and up to date; no one can derive it. So, we choose the model of Einstein norms for the valuation of the AOs, because the Einstein norms are novel, and many existing norms are special cases of the considered norms. The Einstein operations and operators have not been developed yet for LBFSs. Therefore, we decided to investigate the theory of Einstein operators based on Einstein operations for LBFSs and describe their basic laws. The analysis of any type of model is very complex because of ambiguity and problems; therefore, we choose the technique of the MABAC model and the MADM model for the valuation of the best decision. These models are developed based on Einstein operators, because the operators can help us aggregate the collection of data into a singleton set. These models and techniques fulfill the criteria of the above-mentioned question, and to date, no one can derive them. The linguistic information and BFSs are two different techniques used for coping with awkward and complex information in real-life problems. Both ideas have many advantages and supremacy, and because of these reasons, we decided to develop the idea of linguistic bipolar fuzzy information, where the LTS and BFSs are the special cases of the proposed theory. Further, we decided to initiate the Einstein operators with a priority degree for the investigation of the MABAC technique. Inspired by the above analysis, the main contribution of this paper is described as follows:

(1) To initiate the LBFSs and their fundamental systems, which involves the integration of the LTSs and BFSs to handle awkward and complex data.
(2) To design the Einstein operations for the valuation of the averaging and geometric operators based on LBFSs, called linguistic bipolar fuzzy prioritized Einstein averaging operator (LBFPREA) and linguistic bipolar fuzzy prioritized Einstein geometric (LBFPREG) operator.
(3) To diagnose the MADM model and MABAC model, which are famous for the analysis of the optimal solution among the family of alternatives.
(4) To illustrate the design of some numerical analysis for evaluating the decision-making technique, and suggest models to describe the authority of the planned theory.
(5) To compare them with the ranking values of the existing models for assessing the validity of the described theory. The summary of the invented paper is also described in the shape of Fig. 1.

This paper is arranged as follows. In Sec. 2, we explained the existing notion of the BFSs, ETN, ETCN, PRA operator, PRG operator and their operational laws. At the end of this section, we also explained the theory of LTSs with their operational laws. In Sec. 3, we introduced the LBFSs and their operations. Further, we evaluated the LBFPREA operator and the LBFPREG operator. Moreover, we constructed the MABAC tool and MADM tool based on the designed operators. In Sec. 4, we

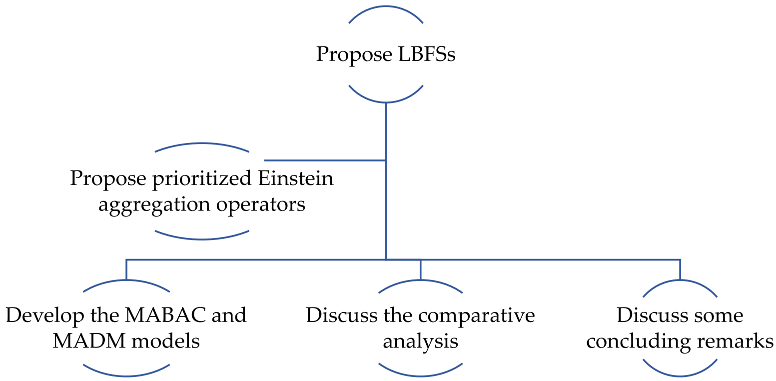

Fig. 1. Representation of the proposed approaches.

illustrated the design of some numerical analyses for evaluating the decision-making technique and suggested models to describe the authority of the planned theory. In Sec. 5, we compared them with the ranking values of the existing models for assessing the validity of the described theory. Some concluding remarks are stated in Sec. 6.

2. Preliminaries

In this section, we briefly explain the existing notion of the BFSs with their flexible and major operational techniques and models. Further, we explained the techniques of ETN and ETCN with the PRA operator and the PRG operator for the construction of our new aggregation models. At the end of this section, we also explained the theory of LTSs with their operational laws.

Definition 1. For any universal set X,[10] we explained and mentioned the existing model of BFSs, such as

$$\eta = \{(\mu(\mathbb{X}), \xi(\mathbb{X})) : \mathbb{X} \in \mathbb{X}\}, \tag{1}$$

where the data in set η contained the term $\mu(\mathbb{X})$ (positive) and $\xi(\mathbb{X})$ (negative), defined from any fixed set $\mathbb{X}$ to [0, 1] and [−1, 0] such as $\mu : \mathbb{X} \to [0,1]$ and $\xi : \mathbb{X} \to [-1,0]$. The final and concluding form of BFN is presented and described by: $\eta_j = (\mu(\mathbb{X}_j), \xi(\mathbb{X}_j))$, $j = 1, 2, \ldots, z$.

Further, we design numerous operations for the construction of the operator and models. Therefore, we consider any two BFNs $\eta_j = (\mu(\mathbb{X}_j), \xi(\mathbb{X}_j))$, $j = 1, 2$, such as

$$\eta_1 \oplus \eta_2 = (\mu(\mathbb{X}_1) + \mu(\mathbb{X}_2) - \mu(\mathbb{X}_1) * \mu(\mathbb{X}_2), -(\xi(\mathbb{X}_1) * \xi(\mathbb{X}_2))), \tag{2}$$

$$\eta_1 \otimes \eta_2 = (\mu(\mathbb{X}_1) * \mu(\mathbb{X}_2), \xi(\mathbb{X}_1) + \xi(\mathbb{X}_2) + \xi(\mathbb{X}_1) * \xi(\mathbb{X}_2)), \tag{3}$$

$$\mathbb{L}\eta_1 = (1 - (1 - \mu(\mathbb{X}_1))^{\mathbb{L}}, -|\xi(\mathbb{X}_1)|^{\mathbb{L}}), \tag{4}$$

$$(\eta_1)^{\mathbb{L}} = ((\mu(\mathbb{X}_1))^{\mathbb{L}}, -1 + (1 + \xi(\mathbb{X}_1))^{\mathbb{L}}), \tag{5}$$

$$S(\eta_1) = \frac{1}{2}(\mu(\mathbb{X}_1) - \xi(\mathbb{X}_1)) \in [0,1], \tag{6}$$

$$A(\eta_1) = \frac{1}{2}(\mu(\mathbb{X}_1) + \xi(\mathbb{X}_1)) \in [-1,1], \tag{7}$$

where $S(\eta_1)$ and $A(\eta_1)$ expressed the Score and Accuracy function for the interpretation of any two BFNs.

Definition 2. Let $\eta_1 \in [0,1]$ and $\eta_2 \in [0,1]$ be any two positive values.[24] Thus

$$\mathrm{ETN}(\eta_1,\eta_2) = \frac{\eta_1\eta_2}{1+(1-\eta_1)(1-\eta_2)}, \tag{8}$$

and

$$\mathrm{ETCN}(\eta_1,\eta_2) = \frac{\eta_1+\eta_2}{1+\eta_1\eta_2}, \tag{9}$$

where $\mathrm{ETN}(\eta_1,\eta_2)$ and $\mathrm{ETCN}(\eta_1,\eta_2)$ expressed the ETN and ETCN.

Definition 3. Let $\eta_j \in [0,1]$, $j = 1,2,\ldots,z$, be a family of positive valued.[23] Thus

$$\mathrm{PRA}(\eta_1,\eta_2,\ldots,\eta_z) = \omega_1\eta_1 \oplus \omega_2\eta_2 \oplus \ldots \oplus \omega_z\eta_z = \oplus_{j=1}^{z}\omega_j\eta_j, \tag{10}$$

and

$$\mathrm{PRG}(\eta_1,\eta_2,\ldots,\eta_z) = \eta_1^{\omega_1} \otimes \eta_2^{\omega_2} \otimes \ldots \otimes \eta_z^{\omega_z} = \otimes_{j=1}^{z}\eta_j^{\omega_j}. \tag{11}$$

The above two expressions represent the PRA operator and PRG operator with priority degree, such as $\omega_j = \frac{\mathtt{A}_j}{\sum_{j=1}^{z}\mathtt{A}_j}$ and $\mathtt{A}_j = \prod_{j=2}^{z}|S(\eta_j)|$ with $\mathtt{A}_1 = 1$.

Definition 4. The constructive and well-known form of LTS with odd cardinality is mentioned and explained by[14–16]

$$\mathbb{L} = \{\mathbb{L}_{\mathbb{I}_j} : j = 1,2,\ldots,\mathrm{z}, \mathbb{I} \in [0,\zeta]\}. \tag{12}$$

Numerous conditions for the LTS are explained by (i) $\mathbb{L}_{\mathbb{I}_j} > \mathbb{L}_{\mathbb{I}'_j} \Leftrightarrow \mathbb{I}_j > \mathbb{I}'_j$, (ii) $Neg(\mathbb{L}_{\mathbb{I}_j}) = \mathbb{L}_{\mathbb{I}'_j}$, where $\mathbb{I}'_j = \mathbb{I}_j + ou$, called negative operators, and (iii) $\max(\mathbb{L}_{\mathbb{I}_j}, \mathbb{L}_{\mathbb{I}'_j}) = \mathbb{L}_{\mathbb{I}_j}$, if $\mathbb{I}_j \geq \mathbb{I}'_j$, and $\min(\mathbb{L}_{\mathbb{I}_j}, \mathbb{L}_{\mathbb{I}'_j}) = \mathbb{L}_{\mathbb{I}_j}$, if $\mathbb{I}_j \leq \mathbb{I}'_j$.

3. Linguistic Bipolar MABAC/MADM Techniques Based on Einstein Models with Priority Degrees

In this section, we derive the LBFSs and their fundamental laws for the valuation of the Einstein AOs based on priority degree. Further, we also design the MADM model and MABAC model for the analysis of the impact of IoT in enhancing the proficiency of electric power distribution systems.

3.1. *LBFSs: Linguistic bipolar fuzzy systems*

We aim to develop the notion of LBFSs and their flexible and major operational techniques. Additionally, we also introduced some operational laws for LBFSs based on ETN and ETCN.

Definition 5. For any universal set X, we explained and mentioned the existing model of LBFSs, such as

$$\eta = \left\{ (\mathbb{L}_{\mu(\mathbb{X})}, \mathbb{L}_{\xi(\mathbb{X})}) : \mathbb{X} \in \mathbb{X}, \mu(\mathbb{X}) \in [0,\zeta], \xi(\mathbb{X}) \in [-\zeta,0], \frac{\mu(\mathbb{X})}{\zeta} \in [0,1], \frac{\xi(\mathbb{X})}{\zeta} \in [-1,0] \right\}, \tag{13}$$

where the data in set η contained the term $L_{\mu(\mathbb{X})}$ (positive) and $L_{\xi(\mathbb{X})}$ (negative), where $\frac{\mu(\mathbb{X})}{\zeta}$ and $\frac{\xi(\mathbb{X})}{\zeta}$ is defined from any fixed set $\mathbb{X}$ to [0, 1] and [-1, 0] such as $\mu : \mathbb{X} \to [0,\zeta]$ and $\xi : \mathbb{X} \to [-\zeta, 0]$. The final and concluding form of LBFN is presented and described by: $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)})$, $j = 1, 2, \ldots, z$.

Further, we design numerous operations for the construction of the operator and models. Therefore, we consider any two LBFNs $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)})$, $j = 1, 2$, such as

$$\eta_1 \oplus \eta_2 = (\mathbb{L}_{\zeta(\frac{\mu(\mathbb{X}_1)}{\zeta} + \frac{\mu(\mathbb{X}_2)}{\zeta} - \frac{\mu(\mathbb{X}_1)}{\zeta} * \frac{\mu(\mathbb{X}_2)}{\zeta})}, \mathbb{L}_{-\zeta(\frac{\xi(\mathbb{X}_1)}{\zeta} * \frac{\xi(\mathbb{X}_2)}{\zeta})}), \tag{14}$$

$$\eta_1 \otimes \eta_2 = (\mathbb{L}_{\zeta(\frac{\mu(\mathbb{X}_1)}{\zeta} * \frac{\mu(\mathbb{X}_2)}{\zeta})}, \mathbb{L}_{\zeta(\frac{\xi(\mathbb{X}_1)}{\zeta} + \frac{\xi(\mathbb{X}_2)}{\zeta} + \frac{\xi(\mathbb{X}_1)}{\zeta} * \frac{\xi(\mathbb{X}_2)}{\zeta})}), \tag{15}$$

$$\mathbb{L}\eta_1 = (\mathbb{L}_{\zeta(1-(1-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}})}, \mathbb{L}_{-\zeta|\frac{\xi(\mathbb{X}_1)}{\zeta}|^{\mathbb{L}}}), \tag{16}$$

$$(\eta_1)^{\mathbb{L}} = (\mathbb{L}_{\zeta(\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}}, \mathbb{L}_{\zeta(-1+(1+\frac{\xi(\mathbb{X}_1)}{\zeta})^{\mathbb{L}})}), \tag{17}$$

$$S(\eta_1) = \frac{1}{2}\left(\frac{\mu(\mathbb{X}_1)}{\zeta} - \frac{\xi(\mathbb{X}_1)}{\zeta}\right) \in [0,1], \tag{18}$$

$$A(\eta_1) = \frac{1}{2}\left(\frac{\mu(\mathbb{X}_1)}{\zeta} + \frac{\xi(\mathbb{X}_1)}{\zeta}\right) \in [-1,1], \tag{19}$$

where $S(\eta_1)$ and $A(\eta_1)$ expressed the Score and Accuracy function for the interpretation of any two BFNs.

Further, we design numerous operations for the construction of the Einstein operator and models. Therefore, we consider any two LBFNs $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)})$, $j = 1, 2$, such as

$$\eta_1 \oplus \eta_2 = \left(\mathbb{L}_{\zeta\left(\frac{\frac{\mu(\mathbb{X}_1)}{\zeta} + \frac{\mu(\mathbb{X}_2)}{\zeta}}{1+\frac{\mu(\mathbb{X}_1)}{\zeta} * \frac{\mu(\mathbb{X}_2)}{\zeta}}\right)}, \mathbb{L}_{-\zeta\left(\frac{\frac{\xi(\mathbb{X}_1)}{\zeta} * \frac{\xi(\mathbb{X}_2)}{\zeta}}{1+(1+\frac{\xi(\mathbb{X}_1)}{\zeta})(1+\frac{\xi(\mathbb{X}_2)}{\zeta})}\right)} \right), \tag{20}$$

$$\eta_1 \otimes \eta_2 = \left(\mathbb{L}_{\zeta\left(\frac{\frac{\mu(\mathbb{X}_1)}{\zeta} * \frac{\mu(\mathbb{X}_2)}{\zeta}}{1+(1-\frac{\mu(\mathbb{X}_1)}{\zeta})(1-\frac{\mu(\mathbb{X}_2)}{\zeta})}\right)}, \mathbb{L}_{\zeta\left(\frac{\frac{\xi(\mathbb{X}_1)}{\zeta} + \frac{\xi(\mathbb{X}_2)}{\zeta}}{1+\frac{\xi(\mathbb{X}_1)}{\zeta} * \frac{\xi(\mathbb{X}_2)}{\zeta}}\right)} \right), \tag{21}$$

$$\mathbb{L}\eta_1 = \left(\mathbb{L}_{\zeta\left(\frac{(1+\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}} - (1-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}}{(1+\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}} + (1-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2|\frac{\xi(\mathbb{X}_1)}{\zeta}|^{\mathbb{L}}}{(2-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\mathbb{L}} + |\frac{\xi(\mathbb{X}_1)}{\zeta}|^{\mathbb{L}}}\right)} \right), \tag{22}$$

$$(\eta_1)^{\mathbb{L}} = \left(\mathbb{L}_{\zeta\left(\frac{2(\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}}{(2-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}+(\frac{\mu(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}}\right)}, \mathbb{L}_{\zeta\left(\frac{(1+\frac{\xi(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}-(1-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}}{(1+\frac{\xi(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}+(1-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\mathbb{L}}}\right)}\right). \tag{23}$$

3.2. *Einstein aggregation operator based on LBFSs with priority degrees*

In this section, we developed the model of the Einstein model based on linguistic bipolar fuzzy systems with priority degree, called the information of the LBFPREA operator and LBFPREG operator. Throughout this section, the LBFNs are mentioned by $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)})$, $j = 1, 2, \ldots, z$ with priority degree such as $\omega_j = \frac{\mathbb{A}_j}{\sum_{j=1}^{z} \mathbb{A}_j}$ and $\mathbb{A}_j = \prod_{j=2}^{z} |S(\eta_j)|$ with $\mathbb{A}_1 = 1$.

Definition 6. We explained and mentioned the proposed model of the LBFPREA operator, such as

$$\begin{aligned}
&\mathrm{LBFPREA}(\eta_1, \eta_2, \ldots, \eta_z)\\
&\quad = \omega_1\eta_1 \oplus \omega_2\eta_2 \oplus \ldots \oplus \omega_z\eta_z = \oplus_{j=1}^{z}\omega_j\eta_j\\
&\quad = \left(\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z}(2-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)}\right).
\end{aligned} \tag{24}$$

The proof of Definition 6 is described in Appendix A.

Property 1. When $\eta_j = \eta = (\mathbb{L}_{\mu(\mathbb{X})}, \mathbb{L}_{\xi(\mathbb{X})})$, $j = 1, 2, \ldots, z$, then LBFPREA $(\eta_1, \eta_2, \ldots, \eta_z) = \eta$. (Idempotency)

Property 2. When $\eta_j \le \eta'_j$, such as $\mu(\mathbb{X}) \le \mu(\mathbb{X})'$ and $\xi(\mathbb{X}) \le \xi(\mathbb{X})'$, thus $\mathrm{LBFPREA}(\eta_1, \eta_2, \ldots, \eta_z) \le \mathrm{LBFPREA}(\eta'_1, \eta'_2, \ldots, \eta'_z)$. (Monotonicity)

Property 3. When $\eta_- = \min(\eta_1, \eta_2, \ldots, \eta_z)$ and $\eta_+ = \max(\eta_1, \eta_2, \ldots, \eta_z)$, thus $\eta_- \le \mathrm{LBFPREA}(\eta_1, \eta_2, \ldots, \eta_z) \le \eta_+$. (Boundedness)

Definition 7. We explained and mentioned the proposed model of the LBFPREG operator, such as

$$\begin{aligned}
&\mathrm{LBFPREG}(\eta_1, \eta_2, \ldots, \eta_z)\\
&\quad = \eta_1^{\omega_1} \otimes \eta_2^{\omega_2} \otimes \ldots \otimes \eta_z^{\omega_z} = \otimes_{j=1}^{z}\eta_j^{\omega_j}\\
&\quad = \left(\mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z}(\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z}(2-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z}(\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}, \mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}\right).
\end{aligned} \tag{25}$$

The proof of Definition 7 is described in Appendix B.

Property 4. When $\eta_j = \eta = (\mathbb{L}_{\mu(\mathbb{X})}, \mathbb{L}_{\xi(\mathbb{X})})$, $j = 1, 2, \ldots, z$, then LBFPREG $(\eta_1, \eta_2, \ldots, \eta_z) = \eta$. (Idempotency)

Property 5. When $\eta_j \leq \eta'_j$, such as $\mu(\mathbb{X}) \leq \mu(\mathbb{X})'$ and $\xi(\mathbb{X}) \leq \xi(\mathbb{X})'$, thus $\text{LBFPREG}(\eta_1, \eta_2, \ldots, \eta_z) \leq \text{LBFPREG}(\eta'_1, \eta'_2, \ldots, \eta'_z)$. (Monotonicity)

Property 6. When $\eta_- = \min(\eta_1, \eta_2, \ldots, \eta_z)$ and $\eta_+ = \max(\eta_1, \eta_2, \ldots, \eta_z)$, thus $\eta_- \leq \text{LBFPREG}(\eta_1, \eta_2, \ldots, \eta_z) \leq \eta_+$. (Boundedness)

3.3. *LBF-MABAC technique based on Einstein operators with priority degrees*

In this section, we interpreted and developed the novel model of the MABAC technique based on LBFSs, which is called the linguistic bipolar fuzzy MABAC (LBF-MABAC) technique. The LBF-MABAC technique is a very competent and proficient model for depicting vague and uncertain data. The model of the MABAC tool contains many valuable steps. After the construction of the decision matrix, we will have normalized the data, if possible or required. The LBF-MABAC technique contained so many valuable steps in arranging the data sets for the normalizations. Then, we simplify the weighted decision matrix and the aggregated decision matrix by using the data in the normalized decision matrix. Lastly, to justify the major value or the best value among the considered values, we analyze the distance between the weighted aggregated matrix and the aggregated matrix datasets. The steps of the LBF-MABAC procedure are described by:

Step 1: Arrange the datasets of the LBFNs in the shape of a new decision matrix. The decision matrix will be the combination of the rows and columns, and each value in a matrix will be in the form of a linguistic bipolar fuzzy number, such as $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)})$, $j = 1, 2, \ldots, z$.
Step 2: Second, we separate the data in a matrix into two sections, cost types and benefit types. In the case of cost types, we do normalization, such as

$$\eta = \begin{cases} (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)}) & \text{benefit}, \\ (\mathbb{L}_{\xi(\mathbb{X}_j)}, \mathbb{L}_{\mu(\mathbb{X}_j)}) & \text{cost}. \end{cases} \tag{26}$$

From the data in Eq. (26), the benefit type of information is not required to be normalized, but in the case of cost type of data, we will normalize the decision matrix datasets.
Step 3: Third, using the Einstein operational laws, we compute the weighted decision matrix. For this, we use the information in Eqs. (22) and (23), which are called Einstein operational laws.
Step 4: Fourth, using the LBFPREA operator and LBFPREF operator, which are described in Eqs. (24) and (25), we aim to aggregate the data in the weighted decision matrix with the help of the invented operators.

Step 5: Fifth, we derive or design the distance values by using the tool of the distance function and evaluate the aggregated values and weighted aggregated values, such as

$$\eta_{ij} = \begin{cases} \mathrm{DD}(\eta_i, \eta_j) & \text{if } \eta_i > \eta_j, \\ 0 & \text{if } \eta_i = \eta_j, \\ -\mathrm{DD}(\eta_i, \eta_j) & \text{if } \eta_i < \eta_j, \end{cases} \tag{27}$$

where

$$\mathrm{DD}(s\eta_i, \eta_j) = \frac{1}{2}\left(\left|\frac{\mu(\mathbb{X}_i)}{\zeta} - \frac{\mu(\mathbb{X}_j)}{\zeta}\right| + \left|\frac{\xi(\mathbb{X}_i)}{\zeta} - \frac{\xi(\mathbb{X}_j)}{\zeta}\right|\right). \tag{28}$$

Step 6: In the second-to-last step, we considered the distance values and evaluated the appraisal values of the alternatives according to their distance values based on the appraisal function, such as

$$S_j = \frac{1}{n}\sum_{i=1}^{n} \mathrm{DD}(\eta_i, \eta_j). \tag{29}$$

Step 7: At the last, we design the ranking table for evaluating the best and worst optimal between the considered data.

3.4. *LBF-MADM technique based on Einstein operators with priority degrees*

In this section, we interpreted and developed the novel model of the MADM technique based on LBFSs, which is called the linguistic bipolar fuzzy MADM (LBF-MADM) technique. The LBF-MADM technique is a very competent and proficient model for depicting vague and uncertain data. The model of the MADM tool contains many valuable steps. After the construction of the decision matrix, we will have normalized the data, if possible or required. The LBF-MADM technique contained so many valuable steps in arranging the data sets for the normalizations. Then, we simplify the aggregated decision matrix by using the data in a normalized decision matrix. Lastly, to justify the major value or the best value among the considered values, we analyze the score value of the aggregated matrix datasets. The steps of the LBF-MADM procedure are deliberated by:

Step 1: Arrange the datasets of the LBFNs in the shape of a new decision matrix. The decision matrix will be the combination of the rows and columns, and each value in a matrix will be in the form of a linguistic bipolar fuzzy number, such as $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)})$, $j = 1, 2, \ldots, z$.
Step 2: Second, we separate the data in a matrix into two sections, cost types and benefit types. In the case of cost types, we do normalization, such as

$$\eta = \left[\begin{array}{ll} (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)}) & \text{benefit}, \\ (\mathbb{L}_{\xi(\mathbb{X}_j)}, \mathbb{L}_{\mu(\mathbb{X}_j)}) & \text{cost}. \end{array}\right. \tag{30}$$

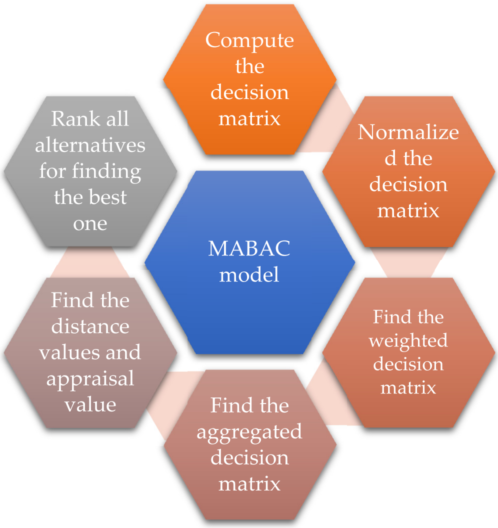

Fig. 2. Geometrical shape of the proposed MABAC model.

From the data in Eq. (30), the benefit type of information does not require normalization, but in the case of cost type of data, we will normalize the decision matrix datasets.
Step 3: Third, using the LBFPREA operator and LBFPREF operator, which are described in Eqs. (24) and (25), we aim to aggregate the data in the normalized decision matrix with the help of the invented operators.
Step 4: In the second-to-last step, using the score function, which is described in Eqs. (18) and (19), we design the score values of the aggregated data.
Step 5: At the last, we design the ranking table for evaluating the best and worst optimal between the considered data. The geometrical shape of the MABAC model is derived in Fig. 2.

4. Impact of IoT in Improving the Proficiency of Electric Power Distribution Systems

A decision-making strategy is a technique or model used to make a decision or evaluate problems under the consideration of available information, attributes and preferences. These models can help us, or individuals, evaluate or resolve decisions and choose the best one from the collection of information. The role and impact of the IoTs in improving the proficiency of electric power distribution systems explores how the IoTs are transforming electric power distribution through automation, predictive maintenance and real-time monitoring. In the traditional or old power distribution systems, postponing response to errors, the inability to achieve energy requests dynamically, and inadequacies often arise from a deficiency of reflectivity. IoT has many applications, for instance, devices like smart thermostats or homes, healthcare, industry and agriculture. Further, the electric power distribution systems are a powerful technique or network that can provide electricity from the transmission system to the end users, such as the industry, homes and businesses. These models and procedures are the last stage of the electricity grid and play an essential role in ensuring the capability and effective supply of power to consumers. Some major components of the electric power distribution systems are explained and mentioned by substations, distribution lines, transformers, switches, circuit breakers, meters, control centers, protection systems and smart grid technology (optional). Some reliable and dominant functions of electric power distribution systems are as follows: voltage regulation, fault detection, load balancing and energy efficiency. The key theme of this application is to introduce some numerical analysis for evaluating the decision-making technique and suggest models to describe the authority of the planned theory. The impact of IoT in improving the proficiency of electric power distribution systems is discussed by many individuals and explained their importance and effectiveness in real-life problems. To evaluate the above problems, we consider five alternatives in the form of IoT to improve efficiency, such as

(1) Real-Time Monitoring and Data Collection

The IoTs devices installed throughout the power distribution network arrange information on temperature, current level, equipment status and voltage levels. This information is transmitted to central control techniques, allowing the techniques to monitor the health of the network and distinguish potential problems before they cause disruptions.

(2) Predictive Maintenance

By scrutinizing the information from IOT sensors, modified or extended analytics can predict equipment failures before they occur, permitting practical preservation. This decreases the downtime, increases the lifespan of equipment and reduces repair expenses.

(3) Fault Detection and Isolation

The IoTs-enabled or empowered systems can quickly detect faults in the network or systems, such as equipment failures, isolating affected areas, short circuits and reducing the influence on the rest of the network. This can enhance the proficiency and reliability of the power distribution systems.

(4) Load Management and Demand Response

The IoTs can communicate with smart meters and be utilized in industry, business and homes to adjust energy consumption in real-life problems. This permits better integration and load balancing of renewable energy sources, called wind or solar, which have variable output and also helps in increasing peak demand, thereby enhancing grid capability and efficiency.

(5) Grid Automation and Optimization

The IoTs enables or permits the automation of grid operations, called circuit reconfiguration, voltage regulation and power flow control. This procedure reduces the requirement for improved operational capability and manual intervention by quickly adjusting to changes in demand or supply.

Further, to consider the best preference and the worst optimal among the collection of all the above is a very challenging task for experts or decision-makers. For the evaluation of the best and worst ones, we consider some attributes of the above information, such as

(1) Operational Efficiency

The simple meaning of operational efficiency is doing any work with the best accuracy and in the fastest way. This method is always very helpful for big organizations to reduce costs, time and resources. Good efficiency is a powerful tool for the reduction of waste in daily operations. These techniques are always very helpful to enhance the quality as well as the service of the product. An efficient system is always helpful for the employees to work in a better and easier way. Operational efficiency always saves energy and plays the most crucial role in increasing the overall productivity. This efficiency system always meets the customer, and companies need it on time. With the help of good decision-making and better planning, always achieve accurate and smooth operations. An efficient system always improves decision-making, and it is valuable for long-term business growth. Operational efficiency plays a fundamental role in a stable and successful management system.

(2) Reliability and Resilience

First, we explain the simple meaning of reliability. In this process, the system works correctly with the best accuracy every time. It gives a consistent and trusted performance in the whole working process. A reliable system always reduces failures and breakdowns in the main system. A reliable system always improves confidence and accuracy in the operating system. The simple and best meaning of resilience is to recover from problems in a timely and accurate manner. This ability to do the work is called resilience. It always continues the work despite any faults and difficulties. This is very supportive of the system and helpful for the system to adapt to new situations. The combination of both reliable and resilient systems reduces the cost and time. Reliability and resilience always play a key and fundamental role in the success of the modern system.

(3) Energy Efficiency and Sustainability

Energy efficiency means doing the same work with very little energy. This one is helpful for safe natural resources and fuels. This one always gives a new way to reduce energy waste, and this one is very beneficial for home and industrial uses. Efficient energy always plays the most important role in reducing harmful emissions. Sustainability means protecting our natural resources for the future. This one is helpful for future generations. Sustainable practice always protects the environment and supports the green rule and clean energy resources. Energy efficiency supports sustainable development and reduces dependency on fossil fuels. It always promotes green prosecution, which always supports long-term economic growth.

(4) Scalability and Flexibility

The basic definition of scalability is that a system can grow and develop in a very easy way. This one is always helpful to do more work without any problems. A scalable system performs very well and in a good way when the demand increases. This one also plays a critical role in reducing the cost and money in very large organizations. Flexibility means how we can change the system in a very quick way. Using this way, we can give the best response to the market when the technology changes. Flexibility always reduces the risk in the operational system. It always allows the consistent, stable and smooth process adjustment in both big and small organizations. The combination of scalability and flexibility gives guarantees for long-term success.

(5) Customer Experience and Engagement

Customer experience means the feedback of the customer about the service and product. A good experience means customers feel happy and very satisfied. This one is the main identification of the successful businessman. Customer engagement means how we can give the best and most welcoming interaction with different customers. Engaged customers always improve the trust and loyalty in the business and organization. Positive and valuable service always improves the customers' trust as well as their loyalty. Engagement always understands the customers' needs in a better way, and this methodology encourages the customers to repeat the purchase. A quick and ethical response always improves customer satisfaction. Good and valuable experience increases the demand for the brand values. Engaged customers always give the best and most positive feedback about the service and product quality. String and positive engagement always support the long-term growth as well as lead to business success.

Thus, considering the procedure of the MABAC model and selecting the model of the MADM technique, we aim to resolve our considered problems by using the linguistic bipolar fuzzy information.

4.1. *MABAC technique based on LBF Einstein models with priority degrees*

The key theme of this section is to resolve the above problems based on the initiated models. Therefore, the steps of the LBF-MABAC procedure are deliberated by:

Step 1: Arrange the datasets of the LBFNs in the shape of a new decision matrix. The decision matrix will be the combination of the rows and columns, and each value in a matrix will be in the form of a linguistic bipolar fuzzy number, such as $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)})$, $j = 1, 2, \ldots, z$, see the data in Table 3.
Step 2: Second, we separate the data in a matrix into two sections, cost types and benefit types. In the case of cost types, we do normalization. From the data in Eq. (26), the benefit type of information is not required to be normalized, but in the case of cost type of data, we will normalize the decision matrix datasets. However, the data in Table 3 are ok, so we do not need normalization.
Step 3: Third, using the Einstein operational laws, we compute the weighted decision matrix. For this, we use the information in Eqs. (22) and (23), which are called Einstein operational laws, see Table 4.
Step 4: Fourth, using the LBFPREA operator and LBFPREF operator, which are described in Eq. (24) and Eq. (25), we aim to aggregate the data in the weighted decision matrix with the help of invented operators, see the data in Table 5.
Step 5: Fifth, we derive or design the distance values by using the tool of the distance function and evaluate the aggregated values and the data in weighted aggregated values, see the data in Table 6.

Table 3. Linguistic bipolar fuzzy matrix of data.

	η_1^{at}	η_2^{at}	η_3^{at}	η_4^{at}	η_5^{at}
η_1	$(\mathbb{L}_2, \mathbb{L}_{-1})$	$(\mathbb{L}_3, \mathbb{L}_{-3})$	$(\mathbb{L}_4, \mathbb{L}_{-2})$	$(\mathbb{L}_5, \mathbb{L}_{-4})$	$(\mathbb{L}_6, \mathbb{L}_{-2})$
η_2	$(\mathbb{L}_3, \mathbb{L}_{-3})$	$(\mathbb{L}_4, \mathbb{L}_{-4})$	$(\mathbb{L}_5, \mathbb{L}_{-5})$	$(\mathbb{L}_2, \mathbb{L}_{-6})$	$(\mathbb{L}_6, \mathbb{L}_{-1})$
η_3	$(\mathbb{L}_7, \mathbb{L}_{-7})$	$(\mathbb{L}_6, \mathbb{L}_{-1})$	$(\mathbb{L}_5, \mathbb{L}_{-2})$	$(\mathbb{L}_4, \mathbb{L}_{-7})$	$(\mathbb{L}_7, \mathbb{L}_{-8})$
η_4	$(\mathbb{L}_4, \mathbb{L}_{-5})$	$(\mathbb{L}_3, \mathbb{L}_{-6})$	$(\mathbb{L}_5, \mathbb{L}_{-4})$	$(\mathbb{L}_5, \mathbb{L}_{-5})$	$(\mathbb{L}_3, \mathbb{L}_{-3})$
η_5	$(\mathbb{L}_7, \mathbb{L}_{-7})$	$(\mathbb{L}_6, \mathbb{L}_{-4})$	$(\mathbb{L}_2, \mathbb{L}_{-3})$	$(\mathbb{L}_1, \mathbb{L}_{-2})$	$(\mathbb{L}_3, \mathbb{L}_{-1})$

Table 4. Linguistic bipolar fuzzy weighted matrix.

	η_1^{at}	η_2^{at}	η_3^{at}	η_4^{at}	η_5^{at}
η_1	$(\mathbb{L}_{0.4065}, \mathbb{L}_{-6.424})$	$(\mathbb{L}_{0.6228}, \mathbb{L}_{-7.27})$	$(\mathbb{L}_{0.8574}, \mathbb{L}_{-6.964})$	$(\mathbb{L}_{1.1216}, \mathbb{L}_{-7.48})$	$(\mathbb{L}_{1.4361}, \mathbb{L}_{-6.964})$
η_2	$(\mathbb{L}_{0.6228}, \mathbb{L}_{-7.27})$	$(\mathbb{L}_{0.8574}, \mathbb{L}_{-7.48})$	$(\mathbb{L}_{1.1216}, \mathbb{L}_{-7.637})$	$(\mathbb{L}_{0.4065}, \mathbb{L}_{-7.76})$	$(\mathbb{L}_{1.4361}, \mathbb{L}_{-6.424})$
η_3	$(\mathbb{L}_{1.845}, \mathbb{L}_{-7.86})$	$(\mathbb{L}_{1.4361}, \mathbb{L}_{-6.424})$	$(\mathbb{L}_{1.1216}, \mathbb{L}_{-6.964})$	$(\mathbb{L}_{0.8574}, \mathbb{L}_{-7.86})$	$(\mathbb{L}_{1.845}, \mathbb{L}_{-7.944})$
η_4	$(\mathbb{L}_{0.8574}, \mathbb{L}_{-7.637})$	$(\mathbb{L}_{0.6228}, \mathbb{L}_{-7.76})$	$(\mathbb{L}_{1.1216}, \mathbb{L}_{-7.48})$	$(\mathbb{L}_{1.1216}, \mathbb{L}_{-7.637})$	$(\mathbb{L}_{0.6228}, \mathbb{L}_{-7.27})$
η_5	$(\mathbb{L}_{1.845}, \mathbb{L}_{-7.86})$	$(\mathbb{L}_{1.4361}, \mathbb{L}_{-7.48})$	$(\mathbb{L}_{0.4065}, \mathbb{L}_{-7.27})$	$(\mathbb{L}_{0.2008}, \mathbb{L}_{-6.964})$	$(\mathbb{L}_{0.6228}, \mathbb{L}_{-6.424})$

Table 5. Linguistic bipolar fuzzy aggregated matrix.

MABAC	LBFPREA operator	LBFPREG operator	Priority degree
η_1	$(\mathbb{L}_{0.4921}, \mathbb{L}_{-3.804})$	$(\mathbb{L}_{0.4682}, \mathbb{L}_{-6.622})$	0.7756,0.1291,0.0561,0.0268,0.0125
η_2	$(\mathbb{L}_{0.6987}, \mathbb{L}_{-4.04})$	$(\mathbb{L}_{0.6809}, \mathbb{L}_{-7.34})$	0.7319,0.1487,0.0723,0.0328,0.0143
η_3	$(\mathbb{L}_{1.6892}, \mathbb{L}_{-4.099})$	$(\mathbb{L}_{1.6619}, \mathbb{L}_{-7.633})$	0.7041,0.1658,0.0745,0.0361,0.0196
η_4	$(\mathbb{L}_{0.8464}, \mathbb{L}_{-4.132})$	$(\mathbb{L}_{0.8364}, \mathbb{L}_{-7.641})$	0.7151,0.1572,0.0751,0.0365,0.016
η_5	$(\mathbb{L}_{1.593}, \mathbb{L}_{-4.147})$	$(\mathbb{L}_{1.4515}, \mathbb{L}_{-7.72})$	0.6925,0.185,0.0789,0.0314,0.0123

Table 6. Linguistic bipolar fuzzy distance matrix.

	η_1^{at}		η_2^{at}		η_3^{at}		η_4^{at}		η_5^{at}	
η_1	0.1503	0.0144	0.1837	0.0071	0.2176	0.0228	0.1954	0.0014	0.2203	0.0296
η_2	0.1999	0.0446	0.2	0.0176	0.1432	0.0797	0.214	0.0185	0.1939	0.0142
η_3	0.1958	0.0406	0.2234	0.041	0.1907	0.0672	0.2013	0.0248	0.2394	0.0831
η_4	0.2392	0.084	0.2229	0.0386	0.2552	0.0573	0.21	0.0161	0.2338	0.1115
η_5	0.228	0.0727	0.1734	0.0929	0.2222	0.0275	0.1868	0.0324	0.1804	0.1181

Step 6: In the second-to-last step, we considered the distance values and evaluated the appraisal values of the alternatives according to their distance values based on the appraisal function, see the data in Table 7.
Step 7: At the last, we design the ranking table for evaluating the best and worst optimal between the considered data, see the data in Table 8.

The most preferable decision is η_5 which is obtained with the help of an invented theory, called the LBF-MABAC technique, based on the LBFPREA operator and LBFPREG operator. Moreover, we deliberated the efficiency and validity of the invented theory by evaluating the data in Table 3 with the help of the LBF-MADM

Table 7. Linguistic bipolar fuzzy appraisal matrix.

MABAC	LBFPREA operator	LBFPREG operator
η_1	0.2026	0.0513
η_2	0.2007	0.0394
η_3	0.2058	0.0509
η_4	0.2015	0.0186
η_5	0.2136	0.0713

Table 8. Linguistic bipolar fuzzy ranking matrix.

Proposed model	Ranking values	Best optimal
LBFPREA operator	$\eta_5 > \eta_3 > \eta_1 > \eta_4 > \eta_2$	η_5
LBFPREG operator	$\eta_5 > \eta_1 > \eta_3 > \eta_2 > \eta_4$	η_5

technique and comparing the ranking values of both models to describe the validity of the designed approaches.

4.2. *MADM technique based on LBF Einstein models with priority degrees*

In this section, we considered the above problems and evaluated them with the help of the MABAC models. Therefore, the steps of the LBF-MADM procedure are deliberated by

Step 1: Arrange the datasets of the LBFNs in the shape of a new decision matrix. The decision matrix will be the combination of the rows and columns, and each value in a matrix will be in the form of a linguistic bipolar fuzzy number, such as $\eta_j = (L_{\mu(\mathbb{X}_j)}, L_{\xi(\mathbb{X}_j)}) j = 1, 2, \ldots, z$, see the data in Table 3.
Step 2: Second, we separate the data in a matrix into two sections, cost types and benefit types. In the case of cost types, we do normalization. From the data in Eq. (26), the benefit type of information is not required to be normalized, but in the case of cost type of data, we will normalize the decision matrix datasets. But the data in Table 3 is ok, so we do not need normalization.
Step 3: Third, using the LBFPREA operator and LBFPREF operator, which are described in Eqs. (24) and (25), we aim to aggregate the data in the normalized decision matrix with the help of the invented operators, see the data in Table 9.
Step 4: In the second-to-last step, using the score function, which is described in Eqs. (18) and (19), we design the score values of the aggregated data, see the data in Table 10.
Step 5: At the last, we design the ranking table for evaluating the best and worst optimal between the considered data, see the data in Table 11.

The most preferable decision is η_5 which is obtained with the help of an invented theory, called the LBFPREA operator, but according to the theory of the LBFPREG operator, the most preferable one is η_3. It means that the proposed theory of the MABAC-LBFPREA operator, MABAC-LBFPREG operator and LBFPREA operator gives the same ranking values, but the technique of the LBFPREG operator is different. Additionally, we deliberated the proficiency and validity of the derived

Table 9. Linguistic bipolar fuzzy aggregated matrix.

MADM	LBFPREA operator	LBFPREG operator	Priority degree
η_1	$(\mathbb{L}_{2.1368}, \mathbb{L}_{-0.966})$	$(\mathbb{L}_{2.096}, \mathbb{L}_{-1.155})$	0.9196,0.0511,0.017,0.0085,0.0038
η_2	$(\mathbb{L}_{3.275}, \mathbb{L}_{-2.37})$	$(\mathbb{L}_{3.2054}, \mathbb{L}_{-3.344})$	0.7805,0.1156,0.0642,0.0285,0.0111
η_3	$(\mathbb{L}_{6.6027}, \mathbb{L}_{-2.844})$	$(\mathbb{L}_{6.4846}, \mathbb{L}_{-6.035})$	0.6444,0.1949,0.0758,0.0463,0.0386
η_4	$(\mathbb{L}_{3.9658}, \mathbb{L}_{-3.219})$	$(\mathbb{L}_{3.9166}, \mathbb{L}_{-5.082})$	0.6814,0.1703,0.0852,0.0473,0.0158
η_5	$(\mathbb{L}_{6.4451}, \mathbb{L}_{-3.392})$	$(\mathbb{L}_{6.071}, \mathbb{L}_{-6.076})$	0.6343,0.2741,0.0761,0.0127,0.0028

Table 10. Linguistic bipolar fuzzy score matrix.

MADM	LBFPREA operator	LBFPREG operator
η_1	0.1724	0.1806
η_2	0.3136	0.3638
η_3	0.5248	0.6955
η_4	0.3992	0.4999
η_5	0.5465	0.6748

Table 11. Linguistic bipolar fuzzy ranking matrix.

Proposed model	Ranking values	Best optimal
LBFPREA operator	$\eta_5 > \eta_3 > \eta_4 > \eta_2 > \eta_1$	η_5
LBFPREG operator	$\eta_3 > \eta_5 > \eta_4 > \eta_2 > \eta_1$	η_3

approaches by comparing the ranking values of the invented models with the ranking values of the existing techniques to state the art of the drive theory.

5. Comparative Analysis

In this section, we compute the ranking values of the existing techniques for comparison with the proposed ranking values to designate the rationality and power of the derived approaches. The comparative systems are very efficient and proficient for the assessment and investigation of the supremacy and validity between proposed and existing models. The existing techniques have their own advantages, but to show that the invented model is superior to existing models, the comparative analysis is a very reliable way to do it. Anyhow, to state the validity and efficiency of the derived theory, we arrange some existing techniques for the valuation of the comparative analysis between proposed and existing models, such as Jana *et al.*,[30] who stated the Dombi AOs for BFSs. Jana *et al.*[31] designed the Dombi prioritized AOs for BFSs. Riaz *et al.*[32] invented the innovative bipolar fuzzy sine trigonometric AOs with SIR techniques. Tchangani[33] exposed the bipolar aggregation methods. Ali and Yang[34] developed the AOs for bipolar complex fuzzy soft inference systems. Jana *et al.*[35] presented the bipolar fuzzy approach with economic condition analysis. Jana[36] described the extended bipolar fuzzy MABAC model. Thus, using the data in Table 3, the comparative analysis is stated in Table 12.

The most preferable decision is η_5 which is obtained with the help of an invented theory, called the MABAC-LBFPREA operator and MABAC-LBFPREG operator. Further, the most preferable decision is η_5 which is obtained with the help of an invented theory, called the LBFPREA operator, but according to the theory of the LBFPREG operator, the most preferable one is η_3. It means that the proposed theory of the MABAC-LBFPREA operator, MABAC-LBFPREG operator and LBFPREA operator gives the same ranking values, but the technique of the

Table 12. LBF comparative analysis.

Methods	Score values	Ranking values
Jana *et al.*[30]	Not possible	No
Jana *et al.*[31]	Not possible	No
Riaz *et al.*[32]	Not possible	No
Tchangani[33]	Not possible	No
Ali and Yang[34]	Not possible	No
Jana *et al.*[35]	Not possible	No
Jana[36]	Not possible	No
MABAC-LBFPREA model	0.2026,0.2007,0.2058,0.2015,0.2136	$\eta_5 > \eta_3 > \eta_1 > \eta_4 > \eta_2$
MABAC-LBFPREG model	0.0513,0.0394,0.0509,0.0186,0.0713	$\eta_5 > \eta_1 > \eta_3 > \eta_2 > \eta_4$
LBFPREA operator	0.1724,0.3136,0.5248,0.3992,0.5465	$\eta_5 > \eta_3 > \eta_4 > \eta_2 > \eta_1$
LBFPREG operator	0.1806,0.3638,0.6955,0.4999,0.6748	$\eta_3 > \eta_5 > \eta_4 > \eta_2 > \eta_1$

LBFPREG operator is different. Additionally, the prevailing techniques and models have many limitations and problems, and for this reason, they failed to evaluate the data in Table 3, because some of them are special cases of the proposed theory, and some of them are out of scope. The theoretical comparative analysis between the proposed and existing models is described in Table 13.

From the data in Table 13, we concluded that the existing techniques have so many problems; therefore, we developed the idea of LBFSs, because LBFSs contain the positive and negative functions in the form of linguistic variables. Further, we also invented the mathematical comparison of the derived and existing techniques to clearly state the validity and supremacy of the derived theory in the shape of Table 14.

From our point of view and based on the information in Tables 12–14, we concluded that the proposed model is a very efficient and complete structure for coping with vague and complex data. Anyhow, the proposed theory is novel and up to date; no one can propose any types of operators and methods based on it, and for this reason, they are more reliable and superior compared to existing models.

Table 13. Theoretical interpretation of the invented models.

Authors	Ideas	Positive function	Negative function	Linguistic	Priority degree	Einstein function	MABAC model
Zadeh[6]	FSs	Yes	No	No	No	No	No
Zhang[10]	BFSs	Yes	Yes	No	No	No	No
Zadeh[14]	LTSs	No	No	No	No	No	No
Pamucar and Cirovic[22]	MABAC model	No	No	No	No	No	Yes
Yager[23]	Prioritized operators	No	No	No	Yes	No	No
Menger[24]	ETN and ETCN	No	No	No	No	Yes	
Proposed work	MABAC, Einstein operators for LBFSs	Yes	Yes	Yes	Yes	Yes	Yes

Table 14. Representation of the mathematical comparison.

Authors	Ideas	Mathematical structure	Limitations
Zadeh[6]	FSs	For any universal set $\mathbb{X}$, we explained and mentioned the existing model of FSs, such as $$\eta = \{(\mathbb{X}, \mu(\mathbb{X})) : \mathbb{X} \in \mathbb{X}\}$$ Where the data in set η contained the term $\mu(\mathbb{X})$ (positive), defined from any fixed set $\mathbb{X}$ to $[0, 1]$ such as $\mu : \mathbb{X} \to [0, 1]$. The final and concluding form of FN is presented and described by $$\eta_j = (\mathbb{X}_j, \mu(\mathbb{X}_j))j = 1, 2, \ldots, z.$$	The FS theory is reliable, but due to the absence of a negative function, it is incomplete.
Zhang[10]	BFSs	For any universal set $\mathbb{X}$, we explained and mentioned the existing model of BFSs, such as $$\eta = \{(\mu(\mathbb{X}), \xi(\mathbb{X})) : \mathbb{X} \in \mathbb{X}\}$$ Where the data in set η contained the term $\mu(\mathbb{X})$ (positive) and $\xi(\mathbb{X})$ (negative), defined from any fixed set $\mathbb{X}$ to $[0, 1]$ and $[-1, 0]$ such as $\mu : \mathbb{X} \to [0, 1]$ and $\xi : \mathbb{X} \to [-1, 0]$. The final and concluding form of BFN is presented and described by $\eta_j = (\mu(\mathbb{X}_j), \xi(\mathbb{X}_j))j = 1, 2, \ldots, z$.	BFS is more proficient, but due to the absence of linguistic variables, the technique of BFS cannot describe human opinions precisely.
Zadeh[14]	LTSs	The constructive and well-known form of LTS with odd cardinality is mentioned and explained by $$\mathbb{L} = \{\mathbb{L}_{\mathbb{I}_j} : j = 1, 2, \ldots, \mathrm{z}, \mathbb{I} \in [0, \zeta]\}$$ Numerous conditions for the LTS are explained by: (i) $\mathbb{L}_{\mathbb{I}_j} > \mathbb{L}_{\mathbb{I}'_j} \Leftrightarrow \mathbb{I}_j > \mathbb{I}'_j$, (ii) $Neg(\mathbb{L}_{\mathbb{I}_j}) = \mathbb{L}_{\mathbb{I}'_j}$, where $\mathbb{I}'_j = \mathbb{I}_j + ou$, called negative operators, and (iii) $\max(\mathbb{L}_{\mathbb{I}_j}, \mathbb{L}_{\mathbb{I}'_j}) = \mathbb{L}_{\mathbb{I}_j}$, if $\mathbb{I}_j \geq \mathbb{I}'_j$, and $\min(\mathbb{L}_{\mathbb{I}_j}, \mathbb{L}_{\mathbb{I}'_j}) = \mathbb{L}_{\mathbb{I}_j}$, if $\mathbb{I}_j \leq \mathbb{I}'_j$.	Perfect, but due to the absence of positive or negative information, we cannot consider it for the solution of awkward problems.

Table 14. (*Continued*)

Authors	Ideas	Mathematical structure	Limitations
Proposed models	Proposed models	For any universal set $\mathbb{X}$, we explained and mentioned the existing model of LBFSs, such as $\eta = \{(\mathbb{L}_{\mu(\mathbb{X})}, \mathbb{L}_{\xi(\mathbb{X})}) : \mathbb{X} \in \mathbb{X}, \mu(\mathbb{X}) \in [0,\zeta], \xi(\mathbb{X}) \in [-\zeta, 0], \frac{\mu(\mathbb{X})}{\zeta} \in [0,1], \frac{\xi(\mathbb{X})}{\zeta} \in [-1,0]\}$ Where the data in set η contained the term $L_{\mu(\mathbb{X})}$ (positive) and $L_{\xi(\mathbb{X})}$ (negative), where $\frac{\mu(\mathbb{X})}{\zeta}$ and $\frac{\xi(\mathbb{X})}{\zeta}$ is defined from any fixed set $\mathbb{X}$ to [0, 1] and [−1, 0] such as $\mu : \mathbb{X} \to [0,\zeta]$ and $\xi : \mathbb{X} \to [-\zeta, 0]$. The final and concluding form of LBFN is presented and described by $\eta_j = (\mathbb{L}_{\mu(\mathbb{X}_j)}, \mathbb{L}_{\xi(\mathbb{X}_j)}) j = 1, 2, \ldots, z$.	The proposed model of LBFS is valuable because it is a complete structure to describe human opinion very efficiently.

6. Conclusion

The positive and negative functions play an important role in coping with vague and complex opinions. We aim to develop the technique of LBFSs for providing a better way for decision-makers to make their decision more precisely and efficiently. The main contribution of this paper is described as follows:

(1) We initiated the LBFSs and their fundamental systems, which are the integration of the LTSs and BFSs to handle awkward and complex data.
(2) We designed the Einstein operations for the valuation of the averaging and geometric operators based on LBFSs, called the LBFPREA operator and the LBFPREG operator.
(3) We diagnosed the MADM model and MABAC model, which are famous for the analysis of the optimal solution among the family of alternatives.
(4) We illustrated the design of some numerical analysis for evaluating the decision-making technique, and suggested models to describe the authority of the planned theory.
(5) We compared them with the ranking values of the existing models for assessing the validity of the described theory.

The invented approach of LBFSs is a very efficient and realistic model because of their characteristics, but in numerous situations, it does not work properly because when an expert gives information in the shape of positive and negative truth and falsity functions with linguistic variables, then the LBFSs are not able to evaluate the data, because of ambiguity and problems. For handling such types of problems, we need to develop a model of linguistic bipolar intuitionistic FSs, which can easily describe and evaluate the awkward and complex problems.

In the upcoming times, we concentrate on the integration of the LTSs, BFSs and soft sets to develop the linguistic bipolar fuzzy soft sets, which are more general than the existing LBFSs, to describe the supremacy and validity of the invented model more precisely and accurately. Further, we also work on the implementation of the proposed model in the environment of decision-making models, big data, hydrogen energy, supply chain strategy and generative AI models.

Data availability

Not applicable.

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Appendix A.

Assuming the induction technique of mathematics, we aim to evaluate the above information. Let $z = 2$, then

$$\omega_1\eta_1 = \left(\mathbb{L}_{\zeta\left(\frac{(1+\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}-(1-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}}{(1+\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}+(1-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2|\frac{\xi(\mathbb{X}_1)}{\zeta}|^{\omega_1}}{(2-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}+|\frac{\xi(\mathbb{X}_1)}{\zeta}|^{\omega_1}}\right)}\right)$$

and

$$\omega_2\eta_2 = \left(\mathbb{L}_{\zeta\left(\frac{(1+\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}-(1-\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}}{(1+\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}+(1-\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2|\frac{\xi(\mathbb{X}_2)}{\zeta}|^{\omega_2}}{(2-\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}+|\frac{\xi(\mathbb{X}_2)}{\zeta}|^{\omega_2}}\right)}\right).$$

Thus,

$$\begin{aligned}
&\mathrm{LBFPREA}(\eta_1, \eta_2)\\
&\quad = \omega_1\eta_1 \oplus \omega_2\eta_2\\
&\quad = \left(\mathbb{L}_{\zeta\left(\frac{(1+\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}-(1-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}}{(1+\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}+(1-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2|\frac{\xi(\mathbb{X}_1)}{\zeta}|^{\omega_1}}{(2-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}+|\frac{\xi(\mathbb{X}_1)}{\zeta}|^{\omega_1}}\right)}\right)\\
&\qquad \oplus \left(\mathbb{L}_{\zeta\left(\frac{(1+\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}-(1-\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}}{(1+\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}+(1-\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2|\frac{\xi(\mathbb{X}_2)}{\zeta}|^{\omega_2}}{(2-\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}+|\frac{\xi(\mathbb{X}_2)}{\zeta}|^{\omega_2}}\right)}\right)\\
&\quad = \left(\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{2}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{2}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{2}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{2}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{2}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{2}(2-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{2}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)}\right).
\end{aligned}$$

It means that the considered theory is ok for $z = 2$. Assume that the considered theory is also ok for $z = z'$, such as

$$\begin{aligned}
&\mathrm{LBFPREA}(\eta_1, \eta_2, \ldots, \eta_{z'})\\
&\quad = \left(\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z'}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z'}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z'}(2-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)}\right).
\end{aligned}$$

Thus, we evaluate that the considered theory is also ok for $z = z' + 1$, such as

$$\begin{aligned}
&\mathrm{LBFPREA}(\eta_1, \eta_2, \ldots, \eta_{z'+1})\\
&\quad = \omega_1\eta_1 \oplus \omega_2\eta_2 \oplus \ldots \oplus \omega_{z'}\eta_{z'} \oplus \omega_{z'+1}\eta_{z'+1}\\
&\quad = \oplus_{j=1}^{z'}\omega_j\eta_j \oplus \omega_{z'+1}\eta_{z'+1}\\
&\quad = \left(\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z'}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z'}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z'}(2-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)}\right)\\
&\qquad \oplus \omega_{z'+1}\eta_{z'+1}\\
&\quad = \left(\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z'}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z'}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z'}(2-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)}\right)\\
&\qquad \oplus \left(\mathbb{L}_{\zeta\left(\frac{(1+\frac{\mu(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}-(1-\frac{\mu(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}}{(1+\frac{\mu(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}+(1-\frac{\mu(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2|\frac{\xi(\mathbb{X}_{z'+1})}{\zeta}|^{\omega_{z'+1}}}{(2-\frac{\xi(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}+|\frac{\xi(\mathbb{X}_{z'+1})}{\zeta}|^{\omega_{z'+1}}}\right)}\right)\\
&\quad = \left(\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'+1}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z'+1}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z'+1}(1+\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'+1}(1-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}, \mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'+1}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z'+1}(2-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'+1}|\frac{\xi(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)}\right).
\end{aligned}$$

Hence, the considered theory is proved correctly for the possible values of $z' + 1$. Hence, the proposed theory is ok for all possible values of z.

Assuming the induction technique of mathematics, we aim to evaluate the above information. Let $z = 2$, then

$$\eta_1^{\omega_1} = \left(\mathbb{L}_{-\zeta\left(\frac{2|\frac{\mu(\mathbb{X}_1)}{\zeta}|^{\omega_1}}{(2-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}+|\frac{\mu(\mathbb{X}_1)}{\zeta}|^{\omega_1}}\right)}, \mathbb{L}_{\zeta\left(\frac{(1+\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}-(1-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}}{(1+\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}+(1-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}}\right)}\right)$$

and

$$\eta_2^{\omega_2} = \left(\mathbb{L}_{-\zeta\left(\frac{2|\frac{\mu(\mathbb{X}_2)}{\zeta}|^{\omega_2}}{(2-\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}+|\frac{\mu(\mathbb{X}_2)}{\zeta}|^{\omega_2}}\right)}, \mathbb{L}_{\zeta\left(\frac{(1+\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}-(1-\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}}{(1+\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}+(1-\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}}\right)}\right).$$

Thus,

$$\begin{aligned}&\mathrm{LBFPREG}(\eta_1,\eta_2)\\&=\eta_1^{\omega_1}\otimes\eta_2^{\omega_2}\\&=\left(\mathbb{L}_{-\zeta\left(\frac{2|\frac{\mu(\mathbb{X}_1)}{\zeta}|^{\omega_1}}{(2-\frac{\mu(\mathbb{X}_1)}{\zeta})^{\omega_1}+|\frac{\mu(\mathbb{X}_1)}{\zeta}|^{\omega_1}}\right)},\mathbb{L}_{\zeta\left(\frac{(1+\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}-(1-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}}{(1+\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}+(1-\frac{\xi(\mathbb{X}_1)}{\zeta})^{\omega_1}}\right)}\right)\\&\quad\otimes\left(\mathbb{L}_{-\zeta\left(\frac{2|\frac{\mu(\mathbb{X}_2)}{\zeta}|^{\omega_2}}{(2-\frac{\mu(\mathbb{X}_2)}{\zeta})^{\omega_2}+|\frac{\mu(\mathbb{X}_2)}{\zeta}|^{\omega_2}}\right)},\mathbb{L}_{\zeta\left(\frac{(1+\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}-(1-\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}}{(1+\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}+(1-\frac{\xi(\mathbb{X}_2)}{\zeta})^{\omega_2}}\right)}\right)\\&=\left(\mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{2}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{2}(2-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{2}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)},\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{2}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{2}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{2}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{2}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}\right).\end{aligned}$$

It means that the considered theory is ok for $z=2$. Assume that the considered theory is also ok for $z=z'$, such as

$$\begin{aligned}&\mathrm{LBFPREG}(\eta_1,\eta_2,\ldots,\eta_{z'})\\&=\left(\mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z'}(2-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)},\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z'}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z'}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}\right).\end{aligned}$$

Thus, we evaluate that the considered theory is also ok for $z=z'+1$, such as

$$\begin{aligned}&\mathrm{LBFPREG}(\eta_1,\eta_2,\ldots,\eta_{z'+1})\\&=\eta_1^{\omega_1}\otimes\eta_2^{\omega_2}\otimes\ldots\otimes\eta_{z'}^{\omega_{z'}}\otimes\eta_{z'+1}^{\omega_{z'+1}}\\&=\otimes_{j=1}^{z'}\eta_j^{\omega_j}\otimes\eta_{z'+1}^{\omega_{z'+1}}\\&=\left(\mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z'}(2-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)},\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z'}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z'}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}\right)\\&\quad\otimes\ \eta_{z'+1}^{\omega_{z'+1}}\\&=\left(\mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}{\prod_{j=1}^{z'}(2-\frac{\mu(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}|\frac{\mu(\mathbb{X}_j)}{\zeta}|^{\omega_j}}\right)},\mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}-\prod_{j=1}^{z'}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}{\prod_{j=1}^{z'}(1+\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}+\prod_{j=1}^{z'}(1-\frac{\xi(\mathbb{X}_j)}{\zeta})^{\omega_j}}\right)}\right)\\&\quad\otimes\left(\mathbb{L}_{-\zeta\left(\frac{2|\frac{\mu(\mathbb{X}_{z'+1})}{\zeta}|^{\omega_{z'+1}}}{(2-\frac{\mu(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}+|\frac{\mu(\mathbb{X}_{z'+1})}{\zeta}|^{\omega_{z'+1}}}\right)},\mathbb{L}_{\zeta\left(\frac{(1+\frac{\xi(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}-(1-\frac{\xi(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}}{(1+\frac{\xi(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}+(1-\frac{\xi(\mathbb{X}_{z'+1})}{\zeta})^{\omega_{z'+1}}}\right)}\right)\end{aligned}$$

$$= \left(\mathbb{L}_{-\zeta\left(\frac{2\prod_{j=1}^{z'+1}\left|\frac{\mu(\mathbb{X}_j)}{\zeta}\right|^{\omega_j}}{\prod_{j=1}^{z'+1}\left(2-\frac{\mu(\mathbb{X}_j)}{\zeta}\right)^{\omega_j}+\prod_{j=1}^{z'+1}\left|\frac{\mu(\mathbb{X}_j)}{\zeta}\right|^{\omega_j}}\right)}, \mathbb{L}_{\zeta\left(\frac{\prod_{j=1}^{z'+1}\left(1+\frac{\xi(\mathbb{X}_j)}{\zeta}\right)^{\omega_j}-\prod_{j=1}^{z'+1}\left(1-\frac{\xi(\mathbb{X}_j)}{\zeta}\right)^{\omega_j}}{\prod_{j=1}^{z'+1}\left(1+\frac{\xi(\mathbb{X}_j)}{\zeta}\right)^{\omega_j}+\prod_{j=1}^{z'+1}\left(1-\frac{\xi(\mathbb{X}_j)}{\zeta}\right)^{\omega_j}}\right)} \right).$$

Hence, the considered theory is proved correctly for the possible values of $z' + 1$. Hence, the proposed theory is ok for all possible values of z.

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