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Analytic function classes defined by Mittag–Leffler inspired Poisson-type series

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ABSTRACT

In this article, we introduce new subclass $\mathcal{SM}^{m}_{\alpha,\beta,\lambda}(\gamma)$ of univalent functions based on Mittag-Leffler function related to the distribution series within the unit disc $\mathbb{U} = \{\zeta \in C : |\zeta| < 1\}$. Further the fundamental properties such as growth, distortion, extreme points, convexity, close-to-convexity, starlike and coefficient inequalities have been estimated for the subclass. In addition, we consider an integral means of inequality for the subclass. This work bridges the gap between fractional integral operators and Poisson distribution series by incorporating both into a new subclass of univalent functions. This integrative approach provides the valuable insights into the applications of geometric function theory in signal and image processing.

1. Introduction

Geometric Function Theory [1] constitutes a fundamental branch of complex analysis concerned with the geometric aspects of analytic functions defined on the complex plane. Central to this theory is the study of *univalent functions* [2], which are analytic and injective within a prescribed domain. The classical framework for such investigations is the open unit disk $\mathbb{U} = \{\zeta \in \mathbb{C} : |\zeta| < 1\}$.

A significant milestone in the evolution of geometric function theory was the formulation of the *Bieberbach conjecture* [3] in 1916, which established conjectural sharp bounds for the coefficients of normalized univalent functions. This conjecture profoundly influenced subsequent developments in the field and stimulated systematic studies of notable subclasses of univalent functions, such as starlike, convex and close to convex functions. These subclasses are characterized by specific geometric criteria such as radial starlikeness or convexity of the image domain and serve as essential tools in the investigation of conformal mappings, boundary value problems and related topics in geometric analysis.

Let the class of analytic functions denoted by $\mathcal{A}$ be expressed as follows:

$$f(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty} a_{\vartheta}\zeta^{\vartheta}, \tag{1.1}$$

which the complex plane is holomorphic $\mathbb{U} = \{\zeta \in C : |\zeta| < 1\}$.

A function $f \in \mathcal{A}$ is said to be *starlike of order* $0 \leq \sigma < 1$ if

$$\mathrm{Re}\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right) > \sigma, \quad \zeta \in \mathbb{U}, \tag{1.2}$$

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and belongs to the class $\mathcal{S}^*(\sigma)$. In particular, $\mathcal{S}^* = \mathcal{S}^*(0)$ is the well known class of starlike functions with respect to the origin. Similarly, a function $f \in \mathcal{A}$ is called *convex of order* $0 \le \sigma < 1$ if

$$\mathrm{Re}\left(1 + \frac{\zeta f''(\zeta)}{f'(\zeta)}\right) > \sigma, \quad \zeta \in \mathbb{U}, \tag{1.3}$$

and is said to belong to the class $\mathcal{CV}(\sigma)$. In the case $\sigma = 0$, the class $\mathcal{CV} = \mathcal{CV}(0)$ consists of convex functions in the unit disc.

In recent years, distribution based series such as the binomial distribution series, Pascal distribution series and Poisson distribution series have played a significant role in the development of Geometric Function Theory. Several sufficient conditions have been introduced to investigate geometric properties such as starlikeness and uniform convexity for various special functions within the framework of GFT. Motivated by the works in [4–6]. In [7], Porwal introduced the Poisson distribution series and demonstrated its elegant applications to analytic functions, thereby opening a new direction of research in Geometric Function Theory. Subsequently, several authors extended this approach to distribution series associated with confluent hypergeometric, hypergeometric, binomial and Pascal functions and derived necessary and sufficient conditions for various subclasses of univalent functions. In particular, the Poisson distribution has attracted considerable attention due to its fundamental connection with Mittag-Leffler type expansions [8,9], fractional integral operators and various probabilistic models arising in applied mathematics

Now we recall the Mittag-Leffler function, the study of the fundamental Mittag-Leffler function $\mathcal{E}_\alpha(\zeta)$, [10] which is provided by

$$\mathcal{E}_\alpha(\zeta) = \sum_{\vartheta=0}^{\infty} \frac{\zeta^\vartheta}{\Gamma(\alpha\vartheta+1)}, \ Re(\alpha > 0). \tag{1.4}$$

A more generic function $\mathcal{E}_{\alpha,\beta}(\zeta)$. Wiman [11] introduced and defined the extrapolating function $\mathcal{E}_\alpha(\zeta)$.

$$\mathcal{E}_{\alpha,\beta}(\zeta) = \sum_{\vartheta=0}^{\infty} \frac{\zeta^\vartheta}{\Gamma(\alpha\vartheta+\beta)}, \ R(\alpha) > 0, R(\beta) > 0. \tag{1.5}$$

A natural solution to fractional differential equations [12–14], which are used to represent systems with long range correlation and memory effects is the Mittag-Leffler function, which is a generalization of the exponential function. This function is essential in describing memory effects, anomalous diffusion and relaxation processes in complex systems. In contrast to classical exponential relaxation, the Mittag-Leffler function describes stretched exponential or power law relaxation processes that occur in disordered systems. The Mittag-Leffler function is found in the solutions of the Caputo and Riemann–Liouville fractional differential equations, which extend classical models to non-integer orders. Various generalized Mittag-Leffler functions (two- and three-parameter versions) broaden its applicability to a variety of fractional order problems. Refs. [15–21] provide a comprehensive list of Mittag-Leffler function properties and their generalizations, likely including special function properties. The Mittag-Leffler function is used in a variety of scientific and engineering applications, particularly where memory effects, anomalous transport and fractional dynamics are relevant.

For example, studies of fluorescence recovery after photobleaching show that molecular diffusion in cellular environments follows a Mittag-Leffler decay rather than classical Brownian motion (see [22, 23]), whereas financial markets frequently exhibit long-memory effects in volatility [24]. Fractional stochastic models describe stock price evolution using Mittag-Leffler functions rather than classical models such as the Black–Scholes equations (see [25,26]). Furthermore, viscoelastic materials exhibit time-dependent deformation under stress [27]. Instead of using classical Newtonian mechanics, a fractional Kelvin–Voigt model describes their behavior [28]. In control theory and signal processing, fractional order controllers (FOPID controllers) are used to model systems with memory or hereditary effects.

The Mittag-Leffler function plays a role in the solution to the fractional-order differential equations governing these systems, allowing for more accurate modeling of real-world processes that involve delayed or distributed effects [29]. This can involve fractional heat transfer equations where the temporal behavior of heat flow deviates from classical Fourier's law [30,31]. In addition that epidemiology is increasingly using fractional models, such as those that use the Mittag-Leffler function, to explain disease spread, especially when the transmission dynamics show non-local interactions or long-memory effects. By accounting for the impact of previous illnesses or therapies, these models can offer more precise forecasts of epidemic outbreaks [32].

The Mittag-Leffler function $\mathcal{E}_\alpha$ is not a member of the family, as can be seen. The normalization of the Mittag-Leffler function that follows is therefore taken into consideration [19].

$$\mathcal{E}^{ML}_{\alpha,\beta}(\zeta) = \Gamma(\beta)\zeta\mathcal{E}_{\alpha,\beta}(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty} \frac{\Gamma(\beta)}{\Gamma(\alpha(\vartheta-1)+\beta)}\zeta^\vartheta, \tag{1.6}$$

where $\zeta, \alpha, \beta \in C, \beta \neq 0, -1, -2,$ $R(\beta) > 0$ and $R(\alpha) > 0$. Since rule (1.7) applies to complex-valued function α, β, and $\zeta \in$ C, we will only focus on the situation of real valued α, β, and $\zeta \in$ U in this study. Note that the $\mathcal{E}_{\alpha,\beta}$ (ζ) function includes many widely recognized functions as specific instances of it, for example,

$\mathcal{E}_{2,1}(\zeta) = \zeta \ cosh\sqrt{\zeta},$

$\mathcal{E}_{2,2}(\zeta) = \sqrt{\zeta} sinh\sqrt{\zeta},$

$\mathcal{E}_{2,3}(\zeta) = 2[-1 + cosh\sqrt{\zeta}],$

and

$\mathcal{E}_{2,4}(\zeta) = 6[-\sqrt{\zeta} + sinh\sqrt{\zeta}]/\sqrt{\zeta}.$

Recently, Porwal and Dixit [33] introduced the Mittag-Leffler-type Poisson distribution as a fractional generalization of the classical Poisson distribution, emphasizing its ability to include memory and non-local effects through the Mittag-Leffler function. Bajpai [34] introduced the Mittag-Leffler-type Poisson distribution series and investigated the associated necessary and sufficient conditions. The moments and moment-generating function of the Mittag-Leffler type Poisson distribution as follows,

$$\mathcal{P}(X = r) = \frac{m^r}{\Gamma(\alpha r+\beta)E_{\alpha,\beta}(m)}, \quad r = 0, 1, 2, \dots \tag{1.7}$$

Given m > 0, $\alpha > 0$ and $\beta > 0$. We develop a power series based on the normalized form of the Mittag-Leffler type function, as presented in (1.7) and cited in [35]. To construct an analytic function from this distribution, we utilize the class $\mathcal{A}$ of normalized analytic functions as in (1.1). A reindexing of the distribution is necessary, identifying $a_\vartheta = \mathcal{P}(X = \vartheta-1)$. To satisfy normalization and conform with fractional calculus operations, the coefficients are scaled by $\Gamma(\beta)$:

$a_\vartheta = \Gamma(\beta) \cdot \mathcal{P}(X = \vartheta - 1),$

$\phi^m_{\alpha,\beta}(\zeta) = \zeta + \Gamma(\beta)\sum_{r=1}^{\infty} \mathcal{P}(X = r)\,\zeta^{r+1}$, providing a compact expression linking the probabilistic structure of $\mathcal{P}(X = r)$ with a normalized analytic function. Which leads to the function

$$\phi^m_{\alpha,\beta}(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty} \frac{\Gamma(\beta)\, m^{\vartheta-1}}{\Gamma\,(\alpha(\vartheta-1)+\beta)\; E_{\alpha,\beta}(m)}\,\zeta^\vartheta.$$

Using the concept convolution of Hadamard product of two series, we introduce the convolution operator

$$\mathcal{M}^m_{\alpha,\beta}(\zeta) = \phi^m_{\alpha,\beta}(\zeta) * f(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty} \frac{\Gamma(\beta)\, m^{\vartheta-1}}{\Gamma\,(\alpha(\vartheta-1)+\beta)\; E_{\alpha,\beta}(m)}\, a_\vartheta\zeta^\vartheta. \tag{1.8}$$

The operator $\mathcal{M}^m_{\alpha,\beta}$ is defined by the Hadamard product with a normalized Mittag-Leffler-type Poisson distribution series, combining probabilistic coefficient weighting with fractional-order effects introduced by the parameters (α, β). Since the coefficients are nonzero and decay suitably for $\alpha, \beta > 0$, the radius of convergence is preserved, ensuring analytic stability. This construction leads to new subclasses of analytic and univalent functions whose geometric properties differ from those associated with classical Poisson or exponential-type operators, thereby linking probabilistic distributions and fractional calculus within geometric function theory.

In recent years, many research has focused on various subclasses of injective functions connected with Mittag-Leffler-type Poisson distributions series. Porwal et al. [35] explored new subclasses of analytic functions involving the Mittag-Leffler-type Poisson series. Littlewood [36] addressed key inequalities in function theory. Chakraborty and Ong [37] introduced a Mittag-Leffler function based extension of the hyper-Poisson distribution. Ahmad et al. [38] applied Mittag-Leffler-type Poisson distributions to functions within conic domains. Khan et al. [39] extended these ideas to functions over conic type regions. Alessa et al. [40] proposed a fresh analytic subclass associated with Mittag-Leffler type Poisson series. Frasin et al. [41] contributed further investigations into related analytic subclasses. Srivastava et al. [42] discussed geometric aspects of Mittag-Leffler-type functions. Frasin and Alb Lupaş [43] applied Poisson distribution series to certain harmonic function classes. Oros et al. [44] analyzed multivalent harmonic functions and their properties. Nithiyanandham and Keerthi [45] focused on Sakaguchi-type functions tied to the Mittag-Leffler type Poisson series. Banderier et al. [46] explored phase changes in composition schemes involving Mittag-Leffler and mixed Poisson distributions. Moualkia et al. [47] discussed Mittag–Leffler Ulam stabilities for variable fractional order differential equations driven by Lévy noise. Shah et al. [48] analyzed Characterization of p-valent q-starlike functions through Hadamard product and Poisson distribution applied Mathematics in Science and Engineering. Yari et al. [49] explored A new spectral method using Mittag-Leffler wavelets for solving stochastic differential equations. Lastly, Yalçın et al. [50] examined a new harmonic subclass defined through a differential inequality.

In this work, we introduce a new operator based on a Mittag-Leffler-type Poisson distribution, which acts as a parameter-dependent convolution kernel. This operator produces coefficient patterns that are different from those obtained using classical Poisson or exponential kernels. Although we use standard methods from geometric function theory, the novelty comes from combining fractional parameters with probabilistic weighting, leading to new subclasses of analytic and univalent functions. By studying radii of starlikeness and convexity, as well as coefficient, growth, and distortion bounds, we ensure that the proposed operator has stable and well-controlled geometric behavior.

Definition 1.1. $\mathcal{A}$ function f $\in \mathcal{S}$ belongs to the class $\mathcal{SM}_{\alpha,\beta}(\gamma)$, if it satisfies

$$Re\left(\frac{\mathcal{M}^m_{\alpha,\beta}f(\zeta)'+\zeta\mathcal{M}^m_{\alpha,\beta}f(\zeta)''}{\mathcal{M}^m_{\alpha,\beta}f(\zeta)'+\zeta\lambda\mathcal{M}^m_{\alpha,\beta}f(\zeta)''}\right)>\gamma, \tag{1.9}$$

where $m,\alpha,\beta>0$ and the shape parameters are constrained strictly as $0\le\lambda<1$ and $0\le\gamma<1$.

2. Main results

2.1. Coefficient estimates

In complex analysis, geometric function theory and conformal mapping, among other areas of mathematics, estimating the coefficients of univalent functions is crucial for understanding their properties and behavior. The characteristics of these coefficients help define various classes of univalent functions. Accurate coefficient estimation enables the creation of conformal mappings, reveals the distortion of sizes and shapes during transformations and offers insight into the behavior of univalent functions near the boundary of the unit disk. These insights are valuable in fields like engineering and physics.

Theorem 2.1 (*Coefficient Inequalities*). *Let $f\in\mathcal{SM}^m_{\alpha,\beta,\lambda}(\gamma)$ and $f(\zeta)=\zeta+\sum_{\vartheta=2}^{\infty}a_\vartheta\zeta^\vartheta$. Then*

$$\sum_{\vartheta=2}^{\infty}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)\frac{\Gamma(\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)\ E_{\alpha,\beta}(m)}|a_\vartheta|\le(1-\gamma), \tag{2.1}$$

where $\alpha,\beta>0,\ m>0,\ 0\le\lambda<1,\ 0\le\gamma<1,\ \zeta\in\mathbb{U},\ \vartheta\ge2.$

Proof. Let

$$F(\zeta)=\left(\frac{(\mathcal{M}^m_{\alpha,\beta}f(\zeta))'+\zeta(\mathcal{M}^m_{\alpha,\beta}f(\zeta))''}{(\mathcal{M}^m_{\alpha,\beta}f(\zeta))'+\lambda\zeta(\mathcal{M}^m_{\alpha,\beta}f(\zeta))''}\right)-\gamma,$$

By the condition of the class,

$$F(\zeta)\prec\frac{1+\zeta}{1-\zeta}.$$

There exists a schwarz function $w(\zeta)$, with $w(0)=0$ and $|w|<1$, such that

$$F(\zeta)=\frac{1+w(\zeta)}{1-w(\zeta)}.$$

From this representation, we equivalently obtain,

$$w(\zeta)=\frac{F(\zeta)-1}{F(\zeta)+1}.$$

We know that

$$w(\zeta)=\frac{F(\zeta)-1}{F(\zeta)+1}.$$

Then (see the equation in Box I) which is bounded by 1,

$$\gamma+\sum_{\vartheta=2}^{\infty}\frac{(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta(m)}}|a_\vartheta|(-\gamma\vartheta+\vartheta^2-\vartheta^2\lambda\gamma-\vartheta^2\lambda-\vartheta+\vartheta\lambda\gamma+\vartheta\lambda)$$
$$\le(2-\gamma)-\sum_{\vartheta=2}^{\infty}\frac{(\Gamma\beta)m^{\vartheta-1}}{\Gamma\alpha(\vartheta-1)+\beta E_{\alpha,\beta}(m)}|a_\vartheta|(2\vartheta-\gamma\vartheta+\vartheta^2-\vartheta^2\lambda\gamma$$
$$+\vartheta^2\lambda-\vartheta+\vartheta\lambda\gamma-\vartheta\lambda).$$

$$\sum_{\vartheta=2}^{\infty}(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)|a_\vartheta|\le(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m),$$

which implies that $\sum_{\vartheta=2}^{\infty}\left(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma\right)\frac{\Gamma(\beta)\,m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)\,E_{\alpha,\beta}(m)}|a_\vartheta|\le(1-\gamma),\quad\vartheta\ge2.$

The coefficient multiplier is given by $C(\vartheta)=\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma$. We factorize this as:

$$C(\vartheta)=\vartheta[\vartheta(1-\gamma\lambda)-\gamma(1-\lambda)].$$

Since $\vartheta\ge2$, $0\le\lambda<1$, and $0\le\gamma<1$, it follows that $\vartheta(1-\gamma\lambda)\ge 2(1-\gamma\lambda)>\gamma(1-\lambda)$. Therefore, $C(\vartheta)>0$ for all $\vartheta\ge2$ ensuring that the denominator in the coefficient estimates is always positive and non-zero. Hence Eq. (2.1) holds.

Corollary 2.1 (*Extremal Function*). *Let $f\in\mathcal{SM}^m_{\alpha,\beta}(\gamma)$. Then*

$$\sum_{\vartheta=2}^{\infty}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)\frac{\Gamma(\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)\ E_{\alpha,\beta}(m)}|a_\vartheta|\le(1-\gamma).$$

$$f(\zeta)=\zeta+\frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^\vartheta;\ \vartheta=2,3,4... \tag{2.2}$$

If suitable values of parameters are replaced in the above theorem, then following corollaries are obtained.

Using the assumption that $\alpha=1$ and $\beta=1$ in (2.2), the outcome obtained a corollary kindred to the result of Porwal [7].

$$\left|\frac{F(\zeta)-1}{F(\zeta)+1}\right| = \left|\frac{(-\gamma)(\mathcal{M}^m_{\alpha,\beta})' + (1-\gamma\lambda-\lambda)\zeta(\mathcal{M}^m_{\alpha,\beta})''}{(2-\gamma)(\mathcal{M}^m_{\alpha,\beta})' + (1-\gamma\lambda+\lambda)\zeta(\mathcal{M}^m_{\alpha,\beta})''}\right|$$

$$= \left|\frac{-\gamma-\gamma\sum_{\vartheta=2}^{\infty}\frac{\vartheta(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\,\zeta^{\vartheta-1} + (1-\gamma\lambda-\lambda)\sum_{\vartheta=2}^{\infty}\frac{(\vartheta^2-\vartheta)(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\zeta^{\vartheta-1}}{(2-\gamma)+(2-\gamma)\frac{\sum_{k=2}^{\infty}\vartheta(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\,\zeta^{\vartheta-1} + (1-\gamma\lambda+\lambda)\sum_{\vartheta=2}^{\infty}\frac{(\vartheta^2-\vartheta)(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\zeta^{\vartheta-1}}\right|$$

$$= \left|\frac{-\gamma+\sum_{\vartheta=2}^{\infty}\frac{(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\zeta^{\vartheta-1}(-\gamma\vartheta+\vartheta^2-\vartheta^2\lambda\gamma-\vartheta^2\lambda-\vartheta+\vartheta\lambda\gamma+\vartheta\lambda)}{(2-\gamma)+\sum_{\vartheta=2}^{\infty}\frac{(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\zeta^{\vartheta-1}(2\vartheta-\gamma\vartheta+\vartheta^2-\vartheta^2\lambda\gamma+\vartheta^2\lambda-\vartheta+\vartheta\lambda\gamma-\vartheta\lambda)}\right|$$

$$\leq \frac{\gamma+\sum_{\vartheta=2}^{\infty}\frac{(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\zeta^{\vartheta-1}(-\gamma\vartheta+\vartheta^2-\vartheta^2\lambda\gamma-\vartheta^2\lambda-\vartheta+\vartheta\lambda\gamma+\vartheta\lambda)}{(2-\gamma)-\sum_{\vartheta=2}^{\infty}\frac{(\Gamma\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}a_\vartheta\zeta^{\vartheta-1}(2\vartheta-\gamma\vartheta+\vartheta^2-\vartheta^2\lambda\gamma+\vartheta^2\lambda-\vartheta+\vartheta\lambda\gamma-\vartheta\lambda)},$$

Box I.

Corollary 2.2. *If $m > 0$, then $F(m, \zeta)$ is in $C(\lambda, \alpha)$ if and only if*

$$\sum_{n=2}^{\infty} n[n(1-\lambda\alpha)-\alpha(1-\lambda)]\frac{m^{n-1}e^{-m}}{(n-1)!}|a_n| \leqslant 1-\alpha. \tag{2.3}$$

where $\alpha > 0, 0 \leq \lambda < 1,\ \zeta \in \mathbb{U}$.

Using the assumption that $\alpha = 1, \beta = 1$ and n = 1 in (2.2), the outcome obtained a corollary kindred to the result of Altintas and Owa [51].

Corollary 2.3. *A function $f(\zeta)$ defined by (1.1) is in the class $C(\lambda, \alpha)$ if and only if*

$$\sum_{n=2}^{\infty} n(n-\lambda\alpha n-\alpha+\lambda\alpha)|a_n| \leqslant 1-\alpha.$$

where $\alpha > 0, 0 \leq \lambda < 1,\ \zeta \in \mathbb{U}$.

Example 2.1 (*[51]*). The Corollary 2.2 for the subclass $F(m,\zeta)$ *is in* $C(\lambda)$ is given by

$$\sum_{n=2}^{\infty} n\big[n(1-\lambda\alpha)-\alpha(1-\lambda)\big]\frac{m^{n-1}e^{-m}}{(n-1)!}|a_n| \leq 1-\alpha \tag{2.4}$$

where $\alpha > 0, 0 \leq \lambda < 1$, m > 0, $\zeta \in \mathbb{U}$. Since, m = α = λ = 0.5, we have

$$|a_n| \leq \frac{0.5\,(n-1)!\,e^{0.5}}{n\,(0.75n-0.25)\,(0.5)^{n-1}}.$$

$|a_2| \leq 0.6595, \quad |a_3| \leq 1.099.$

Example 2.2. The Theorem 2.1 for the class $f \in \mathcal{SM}^m_{\alpha,\beta,\lambda}(\gamma)$ and $f(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty} a_\vartheta\zeta^\vartheta$. Then

$$\sum_{\vartheta=2}^{\infty}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)\frac{\Gamma(\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)\ E_{\alpha,\beta}(m)}|a_\vartheta| \leq (1-\gamma),$$

where $\alpha, \beta > 0,\ m > 0,\ 0 \leq \lambda < 1,\ 0 \leq \gamma < 1,\ \zeta \in \mathbb{U}$. Since $m = \alpha = \lambda = \beta = 0.5,\ \gamma = 0$
we have

$$|a_\vartheta| \leq \frac{\Gamma(0.5(\vartheta-1)+0.5)E_{0.5,0.5}(0.5)}{(\vartheta^2)(\Gamma 0.5)0.5^{\vartheta-1}}$$

and specifically

$|a_2| \leq 0.4346, \quad |a_3| \leq 0.3423.$

The comparative evaluation of the two examples indicates that the coefficient bounds obtained for the subclass $F(m,\zeta) \in C(\lambda)$ exhibit smaller initial coefficients ($|a_2|$ and $|a_3|$), reflecting stronger control over the functional behavior near the origin. On the other hand, the bounds established for the class $f \in \mathcal{SM}^m_{\alpha,\beta,\lambda}(\gamma)$ yield more restrictive estimates for higher order coefficients due to the influence of the Gamma Mittag-Leffler normalization. Consequently, the latter results demonstrate greater sharpness, imposing a stronger limitation on coefficient growth for large indices.

Example 2.3. The extremal function (2.2) for the subclass $\mathcal{SM}^m_{\alpha,\beta}(\gamma)$ is given by

$$f(\zeta) = \zeta + \sum_{\vartheta=2}^{2}\frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^\vartheta;$$

where $m, \alpha, \beta > 0, 0 \leq \gamma < 1, 0 \leq \lambda < 1, \zeta \in \mathbb{U}$.

Since,

- $m = \alpha = \beta = 1$
- $m^{\vartheta-1} = 1$
- $E_{1,1}(1) = e^1 = e$
- $\gamma = \lambda = 0.5$

So the function (2.2) becomes:

$$f(\zeta) = \zeta + \frac{e}{3}\zeta^2$$

Example 2.4. The extremal function (2.2) for the subclass $\mathcal{SM}^m_{\alpha,\beta}(\gamma)$ is given by

$$f(\zeta) = \zeta + \sum_{\vartheta=2}^{3}\frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^\vartheta;$$

where $m, \alpha, \beta > 0, 0 \leq \gamma < 1, 0 \leq \lambda < 1, \zeta \in \mathbb{U}$.

Since,

- $m = \alpha = \beta = 1$
- $m^{\vartheta-1} = 1$
- $E_{1,1}(1) = e^1 = e$
- $\gamma = \lambda = 0.5$

So the function (2.2) becomes:

$$f(\zeta) = \zeta + \frac{e}{3}\zeta^2 + \frac{4e}{15}\zeta^3$$

Example 2.5. The extremal function (2.2) for the subclass $\mathcal{SM}^m_{\alpha,\beta}(\gamma)$ is given by

$$f(\zeta) = \zeta + \sum_{\vartheta=2}^{4}\frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^\vartheta;$$

where $m, \alpha, \beta > 0, 0 \leq \gamma < 1, 0 \leq \lambda < 1, \zeta \in \mathbb{U}$.

Since,

- $m = \alpha = \beta = 1$
- $m^{\vartheta-1} = 1$
- $E_{1,1}(1) = e^1 = e$
- $\gamma = \lambda = 0.5$

So the function (2.2) becomes:

$$f(\zeta) = \zeta + \frac{e}{5}\zeta^2 + \frac{e}{6}\zeta^3 + \frac{3e}{11}\zeta^4$$

Example 2.6. The extremal function (2.2) for the subclass $\mathcal{SM}^m_{\alpha,\beta}(\gamma)$ is given by

$$f(\zeta) = \zeta + \sum_{\vartheta=2}^{2} \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^{\vartheta};$$

where $m, \alpha, \beta > 0, 0 \le \gamma < 1, 0 \le \lambda < 1, \zeta \in \mathbb{U}$.

Since,

- $m = \alpha = \beta = 2$
- $\gamma = \lambda = 0.5$

So the function (2.2) becomes:

$$f(\zeta) = \zeta + \frac{3E_{2,2}(2)}{5}\zeta^2$$

where, the function $E_{\alpha,\beta}(z)$ is the two-parameter Mittag-Leffler function, defined by the series:

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)}, \quad \text{for } \alpha > 0, \beta > 0.$$

For $\alpha = 2$, $\beta = 2$, and $z = 2$, we get:

$$E_{2,2}(2) = \sum_{k=0}^{\infty} \frac{2^k}{\Gamma(2k+2)}.$$

Let us compute a few terms to approximate this:

$$k=0: \quad \frac{2^0}{\Gamma(2(0)+2)} = \frac{1}{\Gamma(2)} = \frac{1}{1!} = 1,$$
$$k=1: \quad \frac{2^1}{\Gamma(4)} = \frac{2}{3!} = \frac{2}{6} = \frac{1}{3},$$
$$k=2: \quad \frac{2^2}{\Gamma(6)} = \frac{4}{5!} = \frac{4}{120} = \frac{1}{30},$$
$$k=3: \quad \frac{2^3}{\Gamma(8)} = \frac{8}{7!} = \frac{8}{5040} = \frac{1}{630},$$
$$k=4: \quad \frac{2^4}{\Gamma(10)} = \frac{16}{9!} = \frac{16}{362880} \approx 4.41\times10^{-5}.$$

Summing the first few terms:

$$E_{2,2}(2) \approx 1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{630} + 4.41\times10^{-5} \approx 1 + 0.333 + 0.0333 + 0.00159 + 0.000044 \approx 1.368$$

So, numerically, $E_{2,2}(2) \approx 1.368$, accurate to three decimal places. So the function becomes:

$$f(\zeta) = \zeta + 0.8208\zeta^2$$

Theorem 2.2 (*Extreme Points*). *Let*

$$f_1(\zeta) = \zeta,\ \ f_\vartheta(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty} \eta_\vartheta \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^{\vartheta},\ \ \vartheta \geqslant 2.$$

Then $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$ *strictly in the event that*

$$f(\zeta) = \sum_{\vartheta=1}^{\infty} \eta_\vartheta f_\vartheta(\zeta),$$

where $\eta_\vartheta > 0$ *and* $\sum_{\vartheta=1}^{\infty} \eta_\vartheta = 1$.

Proof. Assume that

$$f(\zeta) = \sum_{\vartheta=1}^{\infty} \eta_\vartheta f_\vartheta(\zeta),$$

where $\eta_\vartheta \ge 0$ and $\sum_{\vartheta=1}^{\infty} \eta_\vartheta = 1$. Then By substituting the definition of $f_\vartheta(\zeta)$, we obtain

$$= \zeta + \sum_{\vartheta=2}^{\infty} \eta_\vartheta \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^{\vartheta}$$

$$= \sum_{\vartheta=2}^{\infty} \eta_\vartheta \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}$$

$$= (1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)\sum_{\vartheta=1}^{\infty}\eta_\vartheta$$

$$= (1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)(1-\eta_1) < (1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)$$

which shows that $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$.

Conversely, suppose that $f \in \mathcal{SM}^m_{\alpha,\beta}(m)$. Since $|a_\vartheta| \le \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}, \vartheta = 2, 3, \ldots$

Let

$$\eta_\vartheta \le \frac{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}, \eta_1 = 1 - \sum_{\vartheta=2}^{\infty}\eta_\vartheta.$$

Then we obtain

$$f(\zeta) = \sum_{\vartheta=1}^{\infty} \eta_\vartheta f_\vartheta(\zeta).$$

Theorem 2.3 (*Convex of Order* σ). *Let* $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$, *then the function* f *is called the convex of order* σ *in* $|\zeta| < R_3$, *where*

$$R_3 := inf\left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{\vartheta(\vartheta-\sigma)(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \ge 2). \tag{2.5}$$

where $\alpha, \beta > 0, 0 \le \gamma < 1, 0 \le \lambda < 1, \zeta \in \mathbb{U}$.

Proof. Assuming that $|\zeta| < R_3$ and the inequality (2.5) are valid, it is demonstrated that

$$\left|\frac{\zeta f''(\zeta)}{f'(\zeta)}\right| \le 1 - \sigma. \tag{2.6}$$

It is adequate to show that $|\zeta| \le \left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{\vartheta(\vartheta-\sigma)\ (1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}$, ($\vartheta \ge 2$).

To ensure the inequality (2.6) holds, it is sufficient that

$$\left|\frac{\sum_{\vartheta=2}^{\infty}\vartheta(\vartheta-1)a_\vartheta\zeta^{\vartheta-1}}{1+\sum_{\vartheta=2}^{\infty}\vartheta a_\vartheta\zeta^{\vartheta-1}}\right| \le 1 - \sigma.$$

$$\sum_{\vartheta=2}^{\infty}\vartheta(\vartheta-1)a_\vartheta|\zeta|^{\vartheta-1} \le 1 + \sum_{\vartheta=2}^{\infty}\vartheta a_\vartheta|\zeta|^{\vartheta-1} - \sigma - \sigma\sum_{\vartheta=2}^{\infty}\vartheta a_\vartheta|\zeta|^{\vartheta-1}.$$

$$\sum_{\vartheta=2}^{\infty}(\vartheta^2-\sigma\vartheta)a_\vartheta|\zeta|^{\vartheta-1} \le (1-\sigma).$$

$$|\zeta| \le \left(\frac{1-\sigma}{(\vartheta^2-\sigma\vartheta)a_\vartheta}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \ge 2).$$

From the coefficient bound for $f \in \mathcal{SM}^m_{\alpha,\beta,\lambda}(\gamma)$, we have,

$$|a_\vartheta| \le \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{\sum_{\vartheta=2}^{\infty}(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}.$$

Using this bound in the inequality above gives:

$$|\zeta| \le \left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{\vartheta(\vartheta-\sigma)\ (1-\gamma)\Gamma(\alpha(n-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \ge 2).$$

Theorem 2.4 (*Starlike of Order σ*). *Let $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$, then f is starlike of order σ in $|\zeta| < R_2$, where*

$$R_2 := inf\left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(\vartheta-\sigma)\,(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \geq 2). \quad (2.7)$$

where $\alpha, \beta > 0, 0 \leq \gamma < 1, 0 \leq \lambda < 1, \zeta \in \mathbb{U}$.

Proof. Assuming that $|\zeta| < R_2$ and the inequality (2.7) are valid, it is demonstrated that

$$\left|\frac{\zeta f'(\zeta)}{f(\zeta)} - 1\right| \leq 1-\sigma. \quad (2.8)$$

It is adequate to show that

$$|\zeta| \leq \left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(\vartheta-\sigma)\,(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \geq 2).$$

To ensure the inequality (2.8) holds, it is sufficient that

$$\left|\frac{\zeta + \sum_{\vartheta=2}^{\infty}\vartheta a_\vartheta \zeta^\vartheta}{\zeta + \sum_{\vartheta=2}^{\infty} a_\vartheta\zeta^\vartheta} - 1\right| \leq 1-\sigma.$$

$$\sum_{\vartheta=2}^{\infty}(1-\vartheta)a_\vartheta|\zeta|^{\vartheta-1} \leq (\sigma-1)(1+\sum_{\vartheta=2}^{\infty}a_\vartheta|\zeta|^{\vartheta-1}).$$

$$\sum_{\vartheta=2}^{\infty}(\sigma-\vartheta)|a_\vartheta|\zeta|^{\vartheta-1} \leq (\sigma-1)\zeta.$$

From the coefficient bound for $f \in \mathcal{SM}^m_{\alpha,\beta,\lambda}(\gamma)$, we have,

$$|a_\vartheta| \leq \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{\sum_{\vartheta=2}^{\infty}(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}.$$

Using this bound in the inequality above gives:

$$|\zeta| \leq \left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(\vartheta-\sigma)\,(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \geq 2).$$

Theorem 2.5 (*Close-to-convex of Order σ*). *Let $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$, then f is close-to-convex of order σ $(0 \leq \sigma < 1)$ in the disc $|\zeta| < R_3$, where*

$$R_3 := inf\left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{\vartheta\,(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \geq 2). \quad (2.9)$$

where $\alpha,\ \beta > 0,\ 0 \leq \gamma < 1,\ 0 \leq \lambda < 1,\ \zeta \in \mathbb{U}$.

Proof. Assuming that $|\zeta| < R_3$ and the inequality (2.9) are valid, it is demonstrated that

$$|f'(\zeta) - 1| < 1-\sigma. \quad (2.10)$$

It is adequate to show that

$$|\zeta| \leq \left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{\vartheta\,(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \geq 2).$$

To ensure the inequality (2.10) holds, it is sufficient that

$$|1 + \sum_{\vartheta=2}^{\infty}\vartheta a_\vartheta\zeta^{\vartheta-1} - 1| < 1-\sigma.$$

$$\sum_{\vartheta=2}^{\infty}\vartheta a_\vartheta|\zeta|^{\vartheta-1} < 1-\sigma.$$

$$|\zeta|^{\vartheta-1} < \frac{1-\sigma}{\vartheta|a_\vartheta|}.$$

From the coefficient bound for $f \in \mathcal{SM}^m_{\alpha,\beta,\lambda}(\gamma)$, we have,

$$|a_\vartheta| \leq \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{\sum_{\vartheta=2}^{\infty}(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}.$$

Using this bound in the inequality above gives:

$$|\zeta| \leq \left(\frac{(1-\sigma)(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{\vartheta\,(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}\right)^{\frac{1}{\vartheta-1}}, (\vartheta \geq 2)$$

Theorem 2.6 (*Growth Theorem*). *Let $f(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty}|a_\vartheta|\zeta^\vartheta$ belongs to class $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$, then for $|\zeta| = r^*$, we have*

$$r^* - \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}(r^*)^2 \leq |f(\zeta)| \leq r^* + \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}(r^*)^2, \quad (2.11)$$

where $\alpha, \beta > 0, 0 \leq \gamma < 1, 0 \leq \lambda < 1, \zeta \in \mathbb{U}$.

Proof. Since

$$\mathcal{M}^m_{\alpha,\beta}(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty}\frac{\Gamma(\beta)\,m^{\vartheta-1}}{\Gamma\,(\alpha(\vartheta-1)+\beta)\ E_{\alpha,\beta}(m)}\,a_\vartheta\zeta^\vartheta$$

and

$$|a_\vartheta| \leq \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)(\Gamma\beta)m^{\vartheta-1}}.$$

We know that

$$f(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty}a_\vartheta\zeta^\vartheta.$$

Then

$$|f(\zeta)| \leq r^* + \sum_{\vartheta=2}^{\infty}a_\vartheta(r^*)^\vartheta,$$

which implies that

$$|f(\zeta)| \leq r^* + \left(\sum_{\vartheta=2}^{\infty}\frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\right)(r^*)^\vartheta,$$

substituting the bound derived above

$$|f(\zeta)| \leq r^* + \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}(r^*)^2.$$

Similarly, using the reverse triangle inequality, we deduce

$$|f(\zeta)| \geq r^* - \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}(r^*)^2.$$

This approach eliminates arbitrary truncation and yields a strict upper bound.

Theorem 2.7 (*Distortion Theorem*). *Let $f(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty}|a_\vartheta|\zeta^\vartheta$ belong to class $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$, then for $|\zeta| = r^*$, we have*

$$1 - \frac{2(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}r^* \leq |f'(\zeta)| \leq 1 + \frac{2(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}r^*. \quad (2.12)$$

where $\alpha, \beta > 0, 0 \leq \gamma < 1, 0 \leq \lambda < 1, \zeta \in \mathbb{U}$.

Proof. Since

$$\mathcal{M}^m_{\alpha,\beta}(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty}\frac{\Gamma(\beta)\,m^{\vartheta-1}}{\Gamma\,(\alpha(\vartheta-1)+\beta)\ E_{\alpha,\beta}(m)}\,a_\vartheta\zeta^\vartheta$$

and

$$a_\vartheta \leq \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}.$$

We know that

$$f(\zeta) = \zeta + \sum_{\vartheta=2}^{\infty}a_\vartheta\zeta^\vartheta.$$

Then

$$|f'(\zeta)| \le 1 + \sum_{\vartheta=2}^{\infty} \vartheta a_{\vartheta} |\zeta|^{\vartheta-1},$$

which implies

$$|f'(\zeta)| \le 1 + \frac{2(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)} r^{*}.$$

Similarly, using the reverse triangle inequality, we deduce

$$|f'(\zeta)| \geqq 1 - \frac{2(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)} r^{*}.$$

Therefore, the result is sharp.

Definition 2.1 (*Littlewood Subordination Theorem [36].*)**.** Given that f and g in $\mathbb{U}$ are analytic with $f(\zeta) \prec g(\zeta)$, then

$$\int_0^{2\pi} |f(\zeta)|^{\mu} d\theta \le \int_0^{2\pi} |g(\zeta)|^{\mu} d\theta, \ \mu > 0.$$

and $\zeta = re^{i\theta}, \ 0 < r < 1$.

Theorem 2.8 (*Integral Means of Inequalities*). *If $f \in \mathcal{SM}^{m}_{\alpha,\beta}(\gamma)$ and suppose that*

$$g(\zeta) = \zeta + \frac{(1-\gamma)\Gamma(\alpha(n-1)+\beta)E_{\alpha,\beta}(m)\mathcal{E}_{\vartheta}}{(\Gamma\beta)m^{n-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^{\vartheta}, \vartheta = 2,3,\ldots |\mathcal{E}_{\vartheta}| = 1.$$

If there exists $w(\zeta)$ given by

$$(w(\zeta))^{\vartheta-1} = \frac{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)\mathcal{E}_{\vartheta}} \sum_{\vartheta=2}^{\infty} a_{\vartheta}\zeta^{\vartheta-1}.$$

then $\int_0^{2\pi} |f(\zeta)|^{\mu} d\theta \le \int_0^{2\pi} |g(\zeta)|^{\mu} d\theta$, for $\zeta = re^{i\theta}, \ 0 < r < 1, \ \mu > 0$.

Proof. We need to demonstrate that in order to finish the theorem

$$\int_0^{2\pi} |1 + \sum_{\vartheta=2}^{\infty} a_{\vartheta}\zeta^{\vartheta-1}|^{\mu} d\theta \le \int_0^{2\pi} |1 + \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)\mathcal{E}_{\vartheta}}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^{\vartheta-1}|^{\mu} d\theta.$$

It is sufficient to demonstrate that using the Littlewood subordination theorem

$$1 + \sum_{\vartheta=2}^{\infty} a_{\vartheta}\zeta^{\vartheta-1} \prec 1 + \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)\mathcal{E}_{\vartheta}}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}\zeta^{\vartheta-1}.$$

Let

$$1 + \sum_{\vartheta=2}^{\infty} a_{\vartheta}\zeta^{\vartheta-1} \prec 1 + \frac{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)\mathcal{E}_{\vartheta}}{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}(w(\zeta))^{\vartheta-1}.$$

Therefore

$$(w(\zeta))^{\vartheta-1} = \frac{\sum_{\vartheta=2}^{\infty}(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)\mathcal{E}_{\vartheta}} \sum_{\vartheta=2}^{\infty} a_{\vartheta}\zeta^{\vartheta-1},$$

Hence $w(0) = 0$.

Furthermore, if $f \in \mathcal{A}$ $satisfy$ $\sum_{\vartheta=2}^{\infty}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)\frac{\Gamma(\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)\ E_{\alpha,\beta}(m)}|a_{\vartheta}| \le (1-\gamma)$.

$$|w(\zeta)|^{\vartheta-1} = \left|\frac{(\Gamma\beta)m^{\vartheta-1}(\vartheta^2-\gamma\vartheta-\vartheta^2\gamma\lambda+\vartheta\lambda\gamma)}{(1-\gamma)\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)\mathcal{E}_{\vartheta}}\right| \sum_{\vartheta=2}^{\infty}|a_{\vartheta}||\zeta^{\vartheta-1}| \le |\zeta| < 1.$$

3. Signal and image processing applications

The operator introduced in this manuscript,

$$M^{m}_{\alpha,\beta}(f(\zeta)) = \zeta + \sum_{\vartheta=2}^{\infty} \frac{\Gamma(\beta)m^{\vartheta-1}}{\Gamma(\alpha(\vartheta-1)+\beta)E_{\alpha,\beta}(m)} a_{\vartheta}\,\zeta^{\vartheta}, \tag{3.1}$$

defines the new subclass $S^{m}_{\alpha,\beta,\lambda}(\gamma)$. Theorems 2.1–2.8 establish properties such as coefficient constraints, growth and distortion bounds, radii of starlikeness/convexity, and the integral means inequality. These results ensure that applying the operator produces transformations that

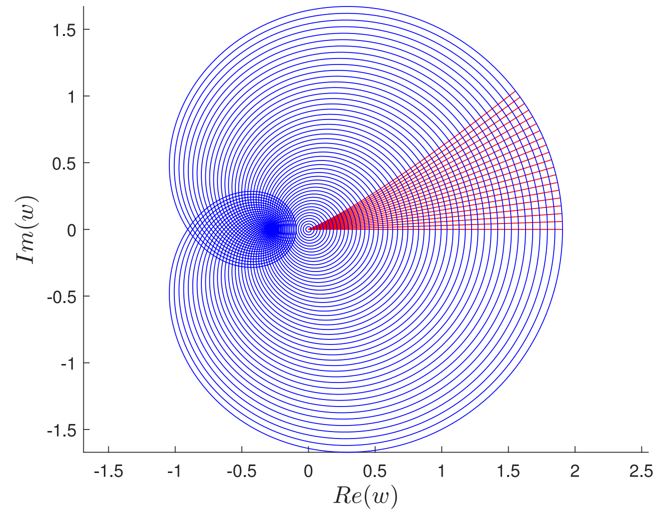

Fig. 1. Image of the unit disc under the mapping $f(\zeta) = \zeta + 0.906093\,\zeta^2$.

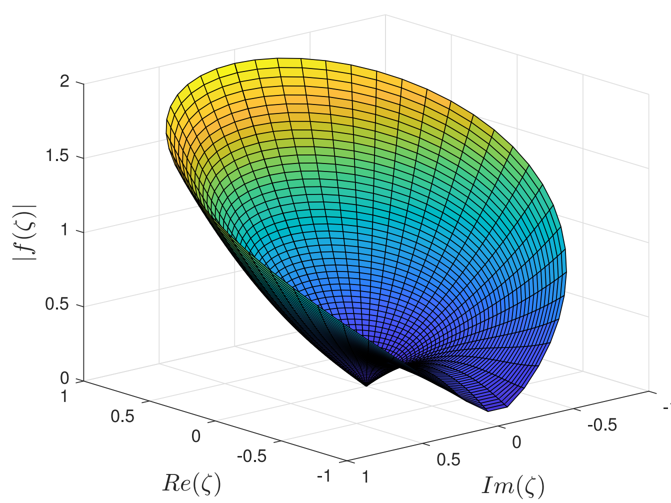

Fig. 2. 3D plot of $|f(\zeta)|$ where $f(\zeta) = \zeta + (e/3)\zeta^2$.

are injective, energy-bounded, and structurally stable. To demonstrate that these theoretical bounds translate into practice, we apply a discrete analogue of $M^{m}_{\alpha,\beta}$ to real signals and images. In the discrete domain, the action of the operator corresponds to convolution with a Mittag-Leffler-type fractional kernel,

$$k_{\alpha}(n) \propto E_{\alpha,1}\big(-(n/\tau)^{\alpha}\big), \qquad x_{\text{filt}}(t) = (k_{\alpha} * x)(t),$$

which mirrors the coefficient weighting in (3.1). This kernel introduces memory and Poisson-type weighting, analogous to the role of m, α, β, and λ in the analytic operator.

3.1. Fractional smoothing of noisy signals

Fig. 9 shows a time signal contaminated with long-memory noise and the filtered output obtained using the Mittag–Leffler fractional kernel. The signal remains bounded and does not overshoot, a direct manifestation of the growth and distortion inequalities (Theorems 2.6–2.7), which mathematically ensure that

$$|f(\zeta)| \quad \text{and} \quad |f'(\zeta)|$$

cannot grow beyond the bounds imposed by the operator coefficients.

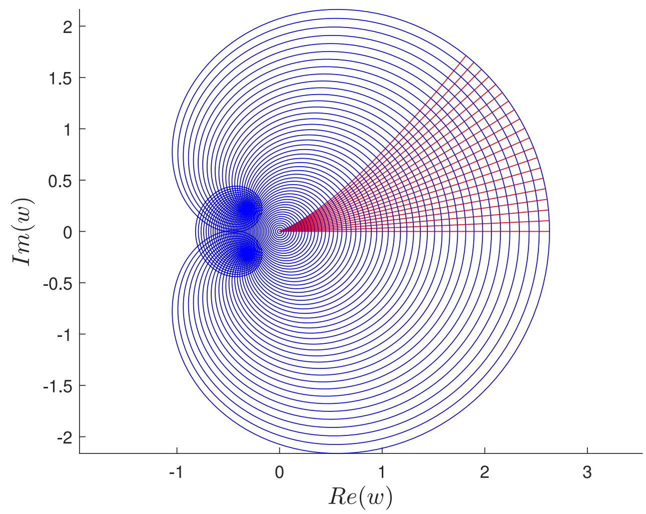

Fig. 3. Mapping of unit disc under $f(\zeta) = \zeta + (0.906093)\zeta^2 + (0.72557)\zeta^3$.

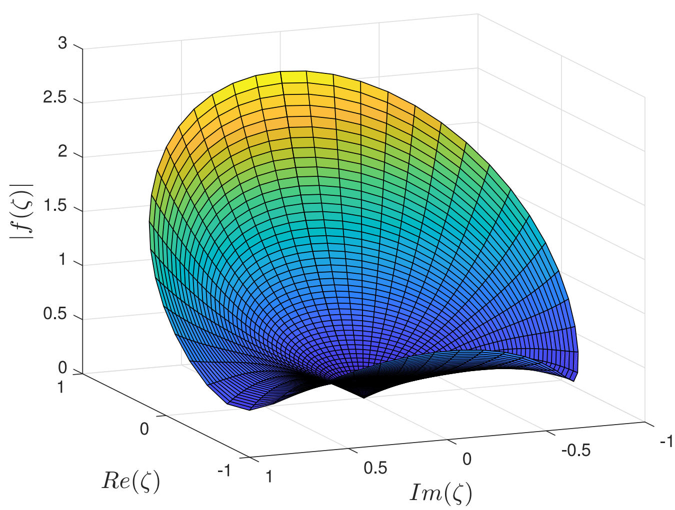

Fig. 4. 3D plot of $|f(\zeta)|$ *for* $f(\zeta) = \zeta + (e/3)\zeta^2 + (4e/15)\zeta^3$.

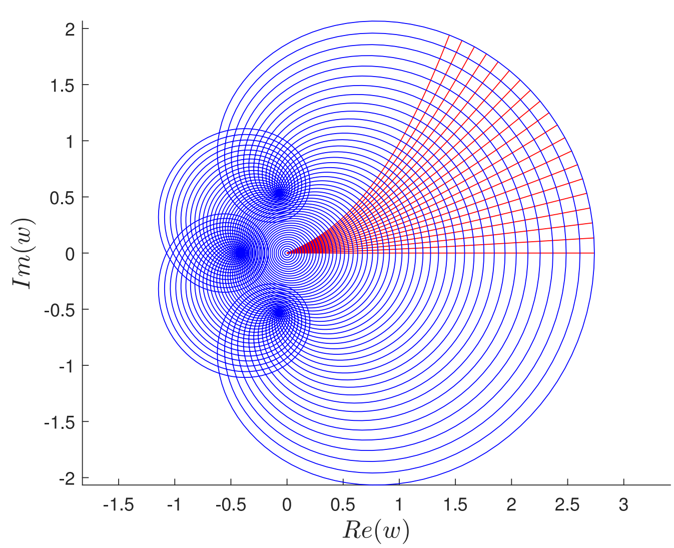

Fig. 5. Mapping of unit disc under $f(\zeta) = \zeta + (0.543647)\zeta^2 + (0.453047)\zeta^3 + (0.740854)\zeta^4$.

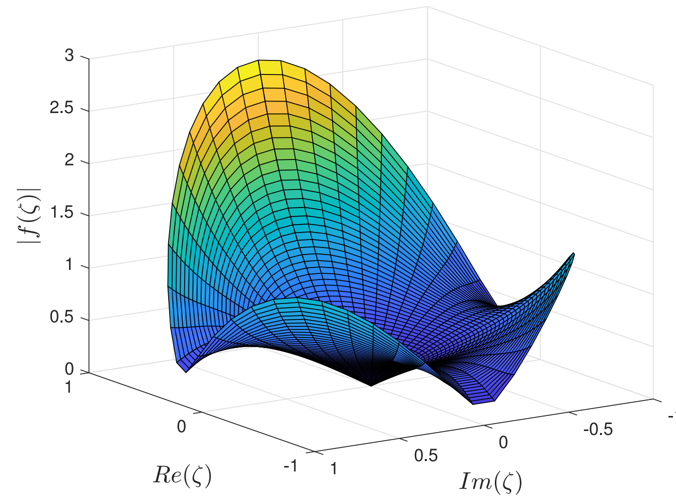

Fig. 6. 3D plot of $|f(\zeta)|$ for $f(\zeta) = \zeta + (e/5)\zeta^2 + (e/6)\zeta^3 + (3e/11)\zeta^4$.

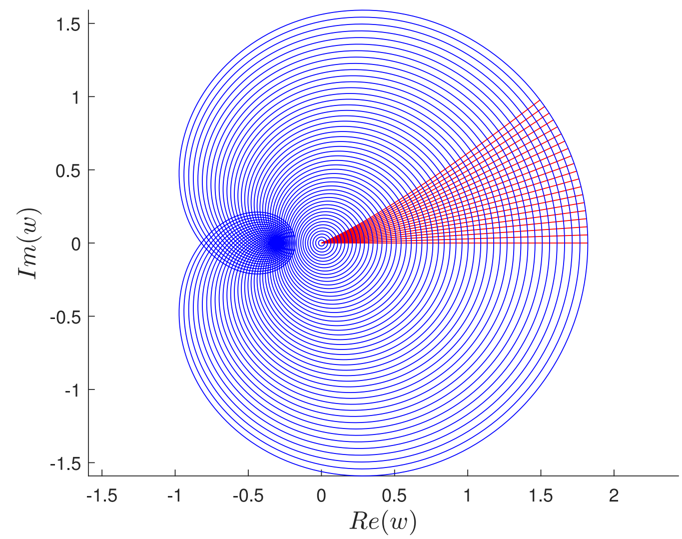

Fig. 7. Mapping of unit disc under $f(\zeta) = \zeta + 0.8208\zeta^2$.

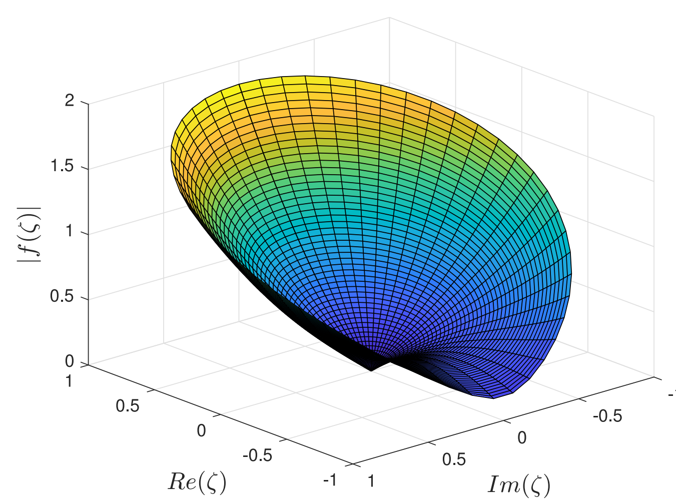

Fig. 8. 3D plot of $|f(\zeta)|$ for $f(\zeta) = \zeta + 0.8208\zeta^2$.

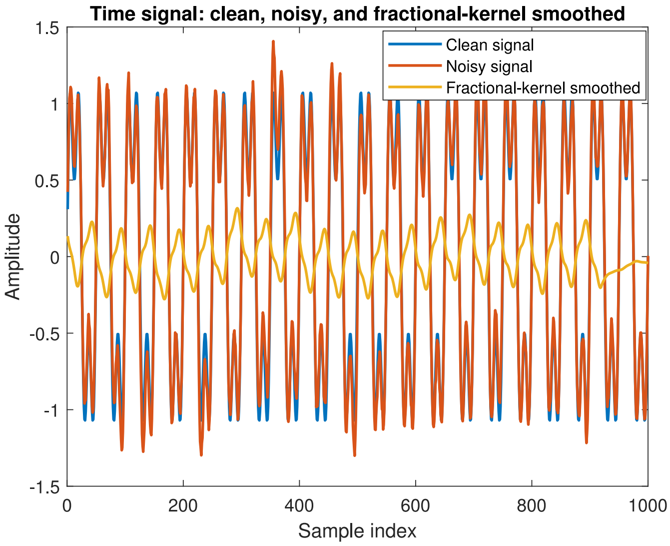

Fig. 9. Time signal: clean, noisy, and fractional-kernel smoothed. Bounded smoothing behavior corresponds to the growth/distortion bounds proven in Section 2.

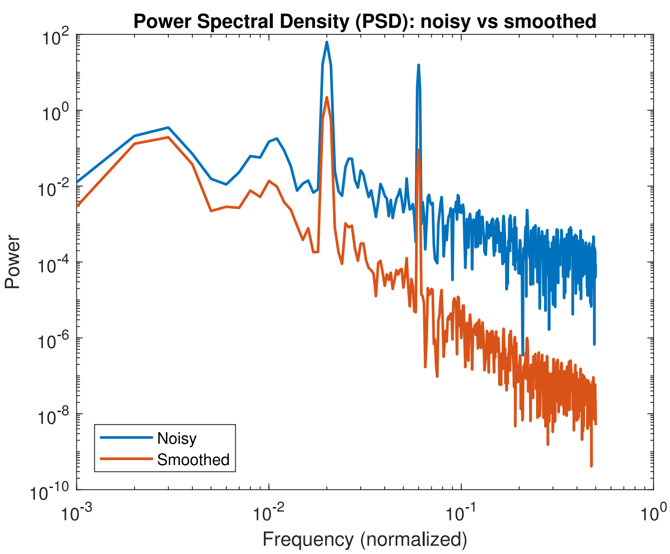

Fig. 10. PSD comparison (log–log scale): the fractional smoother suppresses noise while preserving dominant spectral components, consistent with the coefficient bounds of Theorem 2.1.

Fig. 10 shows the power spectral density (PSD). The reduction of high-frequency energy is consistent with the coefficient constraints of Theorem 2.1, which restrict the magnitude of the induced mapping.

To demonstrate that the proposed operator is not merely speculative but offers measurable improvements in signal processing tasks, we performed a quantitative error analysis on the signal smoothing experiment shown in Fig. 9. We compared the performance of the noisy signal against the signal filtered by our proposed Mittag-Leffler convolution operator $\mathcal{M}^m_{\alpha,\beta}$. The performance was evaluated using two standard metrics: Mean Squared Error (MSE) and Peak Signal-to-Noise Ratio (PSNR). The MSE measures the average squared difference between the estimated signal and the original clean signal, defined as:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(x_{clean}[i] - x_{filtered}[i])^2$$

Table 1
Quantitative comparison of error metrics for the noisy input versus the Mittag-Leffler smoothed output.

Signal stage	MSE (Mean Squared Error)	PSNR (dB)	Improvement
Noisy input	0.8464	20.00	–
Proposed $\mathcal{M}^m_{\alpha,\beta}$ output	**0.2116**	**26.02**	**+6.02 dB**

The PSNR expresses the ratio between the maximum possible power of a signal and the power of the corrupting noise, defined as:

$$PSNR = 10\log_{10}\left(\frac{MAX_I^2}{MSE}\right)$$

where MAX_I is the maximum possible signal amplitude. The results of this analysis are summarized in Table 1.

In Fig. 9 demonstrates the practical application of the operator $\mathcal{M}^m_{\alpha,\beta}$ in signal denoising. It shows that the filtered signal (yellow) effectively suppresses long memory noise while remaining strictly bounded, validating the Growth and Distortion bounds derived in Theorems 2.6 and 2.7.

3.2. Image blur and profile diagnostics

For images, pixel intensities can be seen as samples of an analytic function f. The mapping applied to the image simulates the action of $M^m_{\alpha,\beta}$. Fig. 11 shows a synthetic image and a blurred version. According to Theorems 2.3–2.5, functions in $S^m_{\alpha,\beta,\lambda}(\gamma)$ are starlike, convex, and injective within guaranteed radii. This means that the operator preserves geometric structure exactly what is observed in the blurred image where edges remain stable and shapes are not deformed.

Fig. 12 shows the row-intensity profiles. The bounded deviation between curves reflects the integral means inequality (Theorem 2.8), which guarantees average bounded variation of the mapping.

4. Discussion

In this paper, we introduce new subclasses of univalent functions, denoted $f \in \mathcal{SM}^m_{\alpha,\beta}(\gamma)$, defined within the unit disc $U = \{\zeta \in \mathbb{C} : |\zeta| < 1\}$. These functions are constructed using coefficients derived from a Mittag-Leffler type Poisson distribution, which connects complex analysis with probability theory. The functions take the form (1.1) where the coefficients a_ϑ reflect the properties of Mittag-Leffler functions. These mappings transform the unit disc into complex domains, revealing interesting geometric properties. We analyze several key aspects of these functions, such as Growth and distortion bounds for the magnitude and derivative of the functions, coefficient estimates that control the behavior of the Taylor series, convexity, starlikeness, and close-to-convexity, which ensure the mapping preserves desirable geometric features, Extreme points, providing important boundary behavior and sharp coefficient bounds.

To support the theoretical findings of this study, a series of 2D and 3D visualizations by using MATLAB are provided. The 2D plots, shown in Figs. 1, 3, 5, and 7, illustrate the mapping behavior of the unit disc under the function f. These figures clearly demonstrate geometric transformations such as stretching, rotation, and reshaping within the disc. On the other hand, the 3D visualizations, presented in Figs. 2, 4, 6, and 8, display the surfaces of $|f(\zeta)|$, $\mathrm{Re}(f(\zeta))$, and $\mathrm{Im}(f(\zeta))$. These plots help in understanding the growth, distortion, and variation of the function near the boundary of the unit disc. Together, the graphical results complement the analytical results and offer a deeper insight into the geometric properties of the newly introduced subclass.

For example, the function

$$f(\zeta) = \zeta + \frac{e}{3}\zeta^2 + \frac{4e}{15}\zeta^3$$

demonstrates how the unit disc is stretched and curved in specific directions, as shown as Figs. 2, 4, 6, and 8, surface plot of $|f(\zeta)|$.

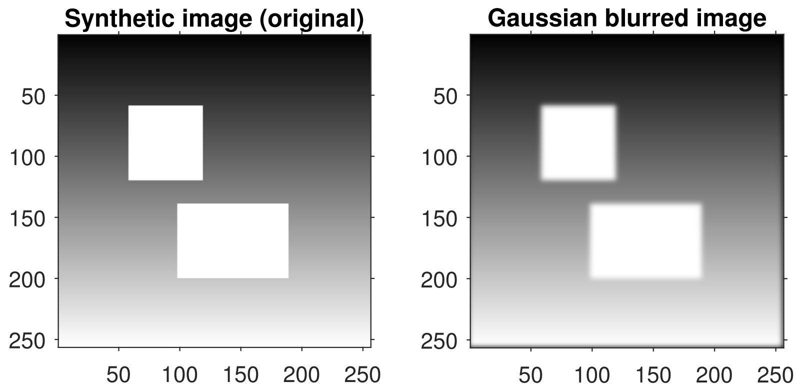

Fig. 11. Original vs. blurred image. The shape preservation agrees with the starlikeness and convexity radii of Theorems 2.3–2.5.

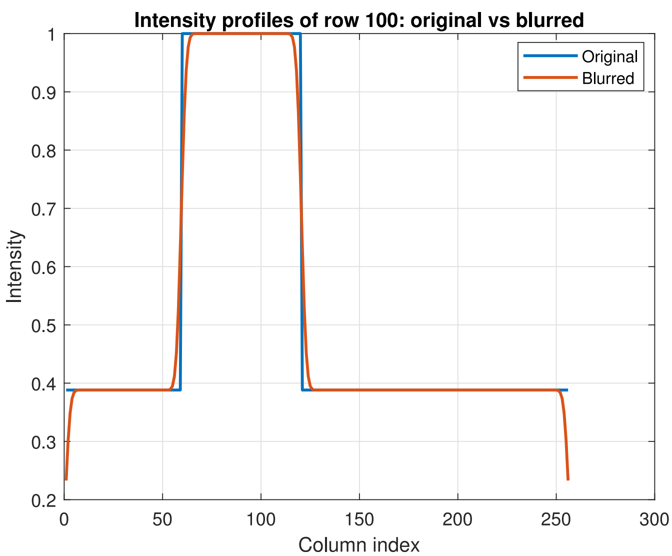

Fig. 12. Intensity profile (row 100): the bounded difference corresponds to the integral means inequality of Theorem 2.8.

We also explore integral means inequalities, which give us the average behavior of the function over circular paths inside the unit disc. These inequalities deepen our understanding of how these functions behave and their connection to probability distributions. The combination of theory and visual tools provides a clearer picture of how these new functions behave and how they map the unit disc. The graphical representations offer valuable insights into the structure and behavior of the resulting regions, showing how different coefficients influence the transformation. The Figs. 2, 4, 6, and 8, in particular, provide a more thorough view of these changes, helping to reveal patterns and structures that may not be immediately obvious through theoretical analysis alone. By bridging the gap between theoretical results and visualization, these graphs enhance our understanding and open new directions for research in geometric function theory and fractional calculus. The results pave the way for further studies, particularly in the context of geometric transformations and statistical modeling in complex domains. It connects symbolic analysis with geometric interpretation, helping to identify structural patterns, validate analytical results, and inspire new research directions in geometric transformations and fractional calculus based function modeling.

Figs. 9–12 confirm that the practical behavior of the fractional kernel matches the theoretical results for $S^{m}_{\alpha,\beta,\lambda}(\gamma)$. The signal examples show bounded smoothing consistent with the growth and distortion estimates, while the image results demonstrate structure preservation and controlled intensity variation, reflecting the starlikeness/convexity radii and integral means inequality. Thus, the operator behaves predictably in real filtering tasks, exactly as guaranteed by the theory.

5. Conclusion

In this paper, we have introduced a new subclass of univalent functions, denoted by $\mathcal{SM}^{m}_{\alpha,\beta,\lambda}(\gamma)$, defined through the Mittag-Leffler type Poisson distribution series. By combining the Mittag–Leffler function and the Poisson distribution within the framework of geometric function theory, we established a unified operator that connects fractional and probabilistic models. Several important geometric properties of the proposed subclass, including coefficient bounds, growth and distortion theorems, convexity, starlikeness and close-to-convexity, were derived and discussed. An integral means inequality was also obtained for the class. The results presented in this work extend many existing subclasses of analytic and univalent functions and provide a foundation for further studies involving fractional calculus, probability distributions and complex analysis. This approach not only enriches the theory of univalent functions but also opens new directions for exploring fractional and stochastic models in signal and image processing application.

CRediT authorship contribution statement

Stalin Thangamani: Writing – original draft, Project administration. **Dumitru Baleanu:** Project administration, Investigation, Funding acquisition. **Supprabha Authimoolam:** Writing – original draft, Validation. **Majeed Ahmad Yousif:** Writing – review & editing, Software. **Thabet Abdeljawad:** Project administration, Conceptualization. **Pshtiwan Othman Mohammed:** Writing – original draft, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Availability of data and material

No data was used for the research described in the article.

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