Journal of Intelligent & Fuzzy Systems 38 (2020) 1649–1662
DOI:10.3233/JIFS-182733
IOS Press

CODAS method using Z-fuzzy numbers

Nurdan Tüysüz[a,b] and Cengiz Kahraman[b,*]
[a]*Istanbul Gelisim University, Faculty of Engineering and Architecture, Industrial Engineering Department, Avcilar, Istanbul, Turkey*
[b]*Istanbul Technical University, Industrial Engineering Department, Besiktas, Istanbul, Turkey*

Abstract. CODAS is a new multicriteria decision making method based on the maximum distances to negative ideal solutions, which are obtained from Euclidean and Taxicab distances. This paper develops a Z-fuzzy CODAS method based on restriction and reliability functions under uncertainty. We obtain the criteria weights from the Z-fuzzy pairwise comparison matrix. An illustrative application to supplier selection problem is also given. A comparative analysis is presented with ordinary fuzzy simple additive weighting (SAW) method.

Keywords: CODAS, Z-fuzzy, restriction function, reliability, pairwise comparison, supplier selection, SAW

1. Introduction

COmbinative Distance-based ASsessment (CODAS) method was proposed by Keshavarz Ghorabaee et al. [15] to be used for MCDM problems. According to this method, the best alternative is assessed by considering Euclidean and Taxicab distances from the negative ideal solution. If the incremental Euclidean distances between two alternatives are sufficiently large, a total distance is calculated taking into account the Taxicab distances. The best alternative is the alternative which has farthest total distance from the negative ideal solution.

There are few publications on CODAS method in the literature. Fuzzy CODAS methods can consider linguistic evaluations by transforming them into numerical values. Table 1 lists the crisp CODAS papers published in the literature.

Each of the MCDM methods in the literature has their own advantages and disadvantages. The CODAS uses the Euclidean distance as the primary measure of assessment while Taxicab distance is used as the secondary measure of assessment when Euclidean distances of two alternatives are very close to each other. The closeness degree between two Euclidean distances is determined by a threshold parameter. CODAS has been integrated with different fuzzy extensions in a few fields of application in recent years, as given in Table 2. Few publications on fuzzy CODAS method are summarized in Table 2.

In Fig. 1, the subject areas of the papers on CODAS method are illustrated. Business, management and accounting, computer science, and economics, econometrics and finance are the top three areas with 75% while mathematics and decision sciences share the second and third ranks with 16.7% and 8.3%, respectively.

As it is seen from Table 2, the crisp CODAS method has been extended to several extensions of ordinary fuzzy sets under uncertainty. One of the possible extensions is to extend the crisp CODAS method by using Z-fuzzy numbers under uncertainty. Z-fuzzy numbers are defined by both a restriction function and a reliability function, each having its own membership function. Z-fuzzy numbers have been employed in the development of fuzzy extensions of several MCDM methods such as Z-fuzzy AHP and Z-fuzzy TOPSIS.

Kang et al. [13] proposed a linguistic MCDM method using Z-fuzzy numbers and applied it for

*Corresponding author. Cengiz Kahraman, Istanbul Technical University, Industrial Engineering Department, Besiktas, Istanbul, Turkey. E-mail: kahramanc@itu.edu.tr.

ISSN 1064-1246/20/$35.00 © 2020 – IOS Press and the authors. All rights reserved

Table 1
A literature review on crisp CODAS

Year	Authors	Application area
2016	Keshavarz Ghorabaee, Zavadskas, Turskis, & Antucheviciene [15]	Most appropriate robot selection
2018	Badi, Abdulshahed, & Shetwan [4]	Supplier selection in Libia
2018	Mathew and Sahu [19]	Material handling equipment selection
2018	Badi, Ballem, Shetwan, [5]	Site selection of desalination plant in Libya

Table 2
A literature review on fuzzy CODAS

Year	Authors	Extension of CODAS	Application area
2017	Keshavarz Ghorabaee, Amiri, E.K., R., & Antucheviciene [11]	Fuzzy CODAS	Market segment evaluation
2018	Panchal, Chatterjee, Shukla, Choudhury, & Tamosaitiene [9]	Fuzzy CODAS	Maintenance decision problem
2018	Ren [24]	IF-CODAS	Energy storage technologies for promoting the development of renewable energy
2018	Peng & Garg [29]	Interval valued fuzzy soft CODAS	A case study in mine emergency decision making
2018	Bolturk [6]	Pythagorean fuzzy CODAS	Supplier selection
2018	Bolturk & Kahraman [7]	Interval-valued intuitionistic fuzzy CODAS	Wave energy facility location selection
2018	Pamucar, D., Badi, I., Sanja, K., Obradović, R. [20]	Linguistic Neutrosophic CODAS	Optimal Power-Generation Technology Selection (PGT) in Libya.

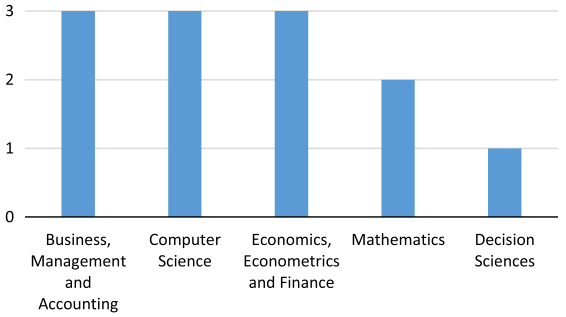


Fig. 1. Subject areas of the CODAS papers.

a vehicle selection problem. Azadeh et al. [3] proposed an AHP method using Z-fuzzy numbers. They determined the weights of criteria to assess the performance of universities. Sahrom and Dom [24] integrated AHP and DEA method for the risk assesment problem. AHP method is used to determine the weights of criteria and Z-fuzzy DEA method is used for ranking the risk priority of 20 bridge structures. They used Kang et al. (2012)'s approach for converting Z-fuzzy numbers to classical fuzzy numbers in the Z-fuzzy DEA method. Yaakob and Gegov [29] modificated TOPSIS method using Z-fuzzy numbers by expanding a fuzzy rule based approach in MCDM. They showed that the proposed approach is a successfully applicable method to express vagueness in decision making. Azadeh and Kokabi [2] proposed a new DEA method using Z- fuzzy numbers for the portfolio selection problem. They transformed Z-fuzzy DEA method to linear possibility programming and obtained a crisp linear programming model using α-cut approach. Sadi-Nezhad and Sotoudeh-Anvari [23] proposed a new DEA using Z-fuzzy numbers. Decision makers indicate the opinion with linguistic terms. They used trapezoidal and triangular fuzzy numbers for the first and second components of Z-fuzzy numbers, respectively. They indicated that the DEA with Z-fuzzy data can be effectively used for solution of real-world problems. Yaakob and Gegov [28] presented a modified TOPSIS method using Z-fuzzy numbers which is called Z-TOPSIS. They applied Z-TOPSIS algorithm for stock selection problem and showed its effectiveness. Peng and Wang [20] introduced hesitant uncertain linguistic Z-fuzzy numbers (HULZNs) for MCDM problems under uncertainty. They extended VIKOR method using HULZNs and applied for ERP selection problem. Khalif et al. [16] presented a fuzzy similarity based TOPSIS method using Z-fuzzy numbers and applied it for performance assessment problem.

Khalif et al. [17] presented a hybrid fuzzy MCDM model using z-fuzzy numbers and applied it to select

the most appropriate staff in recruitment. Wang et al. [26] extended TODIM method with Choquet integral using Z-fuzzy numbers. They used it in the evaluation of medical inquiry applications. Karthika and Sudha [14] applied F-AHP method using Z-fuzzy numbers for risk assessment and they decided the best safety measure for the disease. They used triangular fuzzy numbers for the components of Z-fuzzy numbers. Then they added the second component to first component using centroid method. Forghani et al. [10] proposed a supplier selection model for pharmaceutical companies using Principal component analysis (PCA), Z-TOPSIS and MILP. PCA method is used to reduce the number of supplier selection criteria. Importance value of each supplier is obtained using Z-TOPSIS method. They finally used these values for the mixed integer linear programming (MILP) model. Chatterjee and Kar [8] proposed COPRAS method using Z-fuzzy numbers for renewable energy selection. Aboutorab et al. [1] improved the best-worst method using Z-fuzzy numbers in order to overcome the uncertain expressions. Peng and Wang [21] extended Z-fuzzy MULTIMOORA method to handle multi criteria group decision making problems. They used it in the evaluation of potential areas of air pollution. Shen and Wang [25] proposed a modified VIKOR method using Z-fuzzy numbers. They applied it in the selection of economic development plan.

In the extensions of ordinary fuzzy sets such as type-2 fuzzy sets, hesitant fuzzy sets and intuitionistic fuzzy sets, decision makers try to reflect the uncertainty in their mind through membership functions. In type-2 fuzzy sets, three dimensional membership functions are used. In hesitant fuzzy sets, more than one membership degrees can be assigned for a certain x value. In intuitionistic fuzzy sets, decision makers' hesitancy depends on the sum of membership and non-membership degrees, providing that this sum is equal to at most 1.

Alternatively, Z-fuzzy numbers can take into account the uncertainty in decision makers' mind through a reliability function, which express how confident they are about their evaluations. Z-fuzzy numbers have been very popular after they are introduced by Zadeh [18]. Figure 2 illustrates the frequencies of Z-fuzzy number publications with respect to the years. There is a clear acceleration in the frequencies of Z-fuzzy number publications after the year 2014.

Figure 3 shows the frequencies of Z-fuzzy number publications with respect to their subject areas. The top three subject areas that Z-fuzzy numbers are used are computer science, mathematics, and engineering, respectively.

The organization of the paper is as follows. In Section 2, Z-fuzzy numbers are explained in detail. In Section 3, the proposed Z-fuzzy CODAS method is presented. In Section 4, an application is presented for a supplier selection problem. In Section 5, a comparative analysis is performed with fuzzy simple additive weighting method. Finally, conclusions are given in the last section.

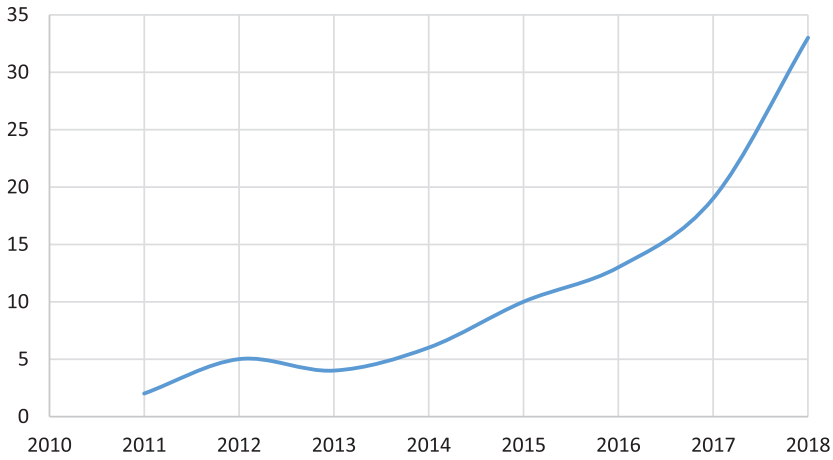


Fig. 2. Frequencies of Z-fuzzy publications with respect to the years.

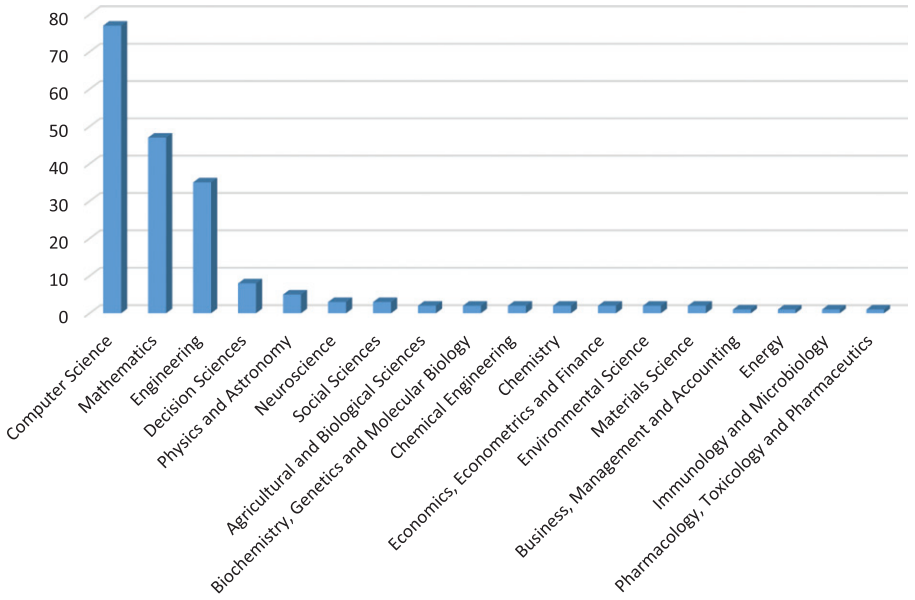


Fig. 3. Frequencies of Z-fuzzy number publications with respect to their subject areas.

2. Z-Fuzzy numbers

Zadeh [18] introduced the Z-fuzzy numbers to the literature in 2011. A Z-fuzzy number is an ordered pair of fuzzy numbers, $Z(\tilde{A}, \tilde{B})$ as given in Fig. 4. The first component $\tilde{A}$ is a restriction function whereas the second component $\tilde{B}$ is a measure of reliability for the first component.

The concept of a Z-fuzzy number is intended to provide a basis for computation with ordinary fuzzy numbers which are not reliable.

Definition 1. Let an ordinary fuzzy set $\tilde{A}$ be defined on a universe X. Then, $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | x \in X\}$ where $\mu_{\tilde{A}} : X \to [0, 1]$ is the membership function $\tilde{A}$. The membership function $\mu_{\tilde{A}}(x)$ describes the degree of belongingness of $x \in X$ in $\tilde{A}$. The fuzzy expectation of a fuzzy set is denoted as [22]:

$$E_{\tilde{A}}(x) = \int_x x\mu_{\tilde{A}}(x)dx \tag{1}$$

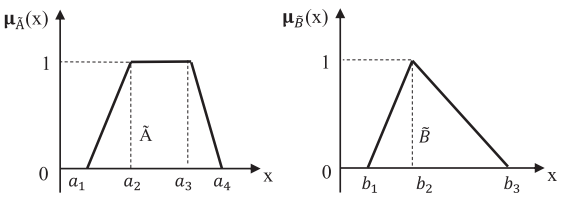


Fig. 4. A simple Z-fuzzy number, $Z(\tilde{A}, \tilde{B})$.

The fuzzy expectation of a fuzzy set is not the same as the meaning of the probabilistic expectation of a classical set.

Definition 2: Conversion of a Z-fuzzy number to an Ordinary Fuzzy Number

Consider a simple Z-fuzzy number $Z = (\tilde{A}, \tilde{R})$, which is illustrated in Fig. 4. The figure on the left represents the restriction function, and the figure on the right represents the reliability function. Let $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | \mu(x) \in [0, 1]\}$ and $\tilde{R} = \{\langle x, \mu_{\tilde{R}}(x)\rangle | \mu(x) \in [0, 1]\}$, where $\mu_{\tilde{A}}(x)$ is a trapezoidal membership function and $\mu_{\tilde{R}}(x)$ is a triangular membership function [12].

(1) Convert the second part (reliability) into a crisp number.

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)dx}{\int \mu_{\tilde{R}}(x)dx} \tag{2}$$

where $\int$ denotes an algebraic integration.

Alternatively, the defuzzification equation $(a_1 + 2 * a_2 + 2 * a_3 + a_4)/6$ for symmetrical trapezoidal fuzzy numbers and $(a_1 + 2 * a_2 + a_3)/4$ for symmetrical triangular fuzzy numbers can be used.

(2) Add the weight of the second part (reliability) to the first part (restriction). The weighted Z-fuzzy number can be denoted as

$$\tilde{Z}^{\alpha} = \{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle | \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x), \mu(x) \in [0, 1]\} \tag{3}$$

(3) Convert the Z fuzzy number (weighted restriction) to ordinary fuzzy number. The ordinary fuzzy set can be denoted as in Equation (4)

$$\tilde{Z}' = \{\langle x, \mu_{\tilde{Z}'}(x)\rangle | \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), \mu(x) \in [0,1]\} \tag{4}$$

$\tilde{Z}'$ has the same Fuzzy Expectation with $\tilde{Z}^{\alpha}$, and they are equal with respect to Fuzzy Expectation, which can be denoted by Fig. 5.

(4) If the restriction function and reliability function are defined as in Fig. 6 (their heights may be any value between 0 and 1), the calculations are modified as follows:

Let $\tilde{A}_{\delta} = \{\langle x, (\mu_{\tilde{A}}(x); \delta)\rangle | \mu(x) \in [0,1]\}$ and $\tilde{R}_{\beta} = \{\langle x, (\mu_{\tilde{R}}(x); \beta)\rangle | \mu(x) \in [0,1]\}$, $\mu^{\delta}_{\tilde{A}}(x)$ is a trapezoidal membership function, $\mu^{\beta}_{\tilde{R}}(x)$ is a triangular membership function.

In this case, restriction and reliability functions are defined as in Equations (5, 6), respectively. The reliability membership function in Equation (6) is substituted into the defuzzification formula (Equation (2)); so that, Equation (7) is obtained.

$$\mu^{\delta}_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}\delta, & if\ a_1 \le x \le a_2 \\ \delta, & if\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}\delta, & if\ a_3 \le x \le a_4 \\ 0, & Otherwise \end{cases} \tag{5}$$

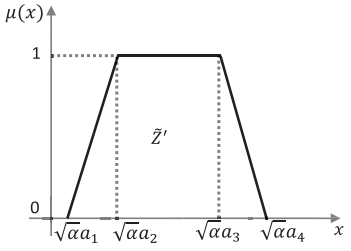


Fig. 5. Ordinary fuzzy number converted from Z-fuzzy number.

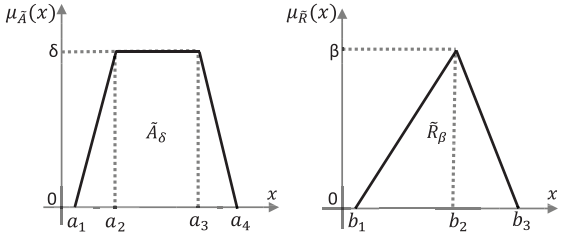


Fig. 6. A simple $\tilde{Z}_{\delta,\beta}$ number, $\tilde{Z}_{\delta,\beta} = (\tilde{A}_{\delta}, \tilde{R}_{\beta})$.

$$\mu^{\beta}_{\tilde{R}}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1}\beta, & if\ b_1 \le x \le b_2 \\ \frac{b_3-x}{b_3-b_2}\beta, & if\ b_2 \le x \le b_3 \\ 0, & Otherwise \end{cases} \tag{6}$$

Thus, we have

$$\sqrt{\alpha} = \sqrt{\frac{\int x\mu^{\beta}_{\tilde{R}}(x)dx}{\int \mu^{\beta}_{\tilde{R}}(x)dx}} \tag{7}$$

Then, the weighted $\tilde{Z}_{\delta,\beta}$ number can be denoted as in Equation (8).

$$\tilde{Z}^{\alpha}_{\delta,\beta} = \left\{\langle x, \mu^{\delta}_{\tilde{A}^{\alpha}}(x)\rangle \,\middle|\, \mu^{\delta}_{\tilde{A}^{\alpha}}(x) = \frac{\int x\mu^{\beta}_{\tilde{R}}(x)dx}{\int \mu^{\beta}_{\tilde{R}}(x)dx}\mu^{\delta}_{\tilde{A}}(x), \mu(x) \in [0,1]\right\} \tag{8}$$

The ordinary fuzzy number converted from Z-fuzzy number can be given as in Equation (9)

$$\tilde{Z}'_{\delta,\beta} = \left\{\langle x, \mu^{\delta}_{\tilde{z}'}(x)\rangle \,\middle|\, \mu^{\delta}_{\tilde{z}'}(x) = \mu^{\delta}_{\tilde{A}}\left(x\frac{\int \mu^{\beta}_{\tilde{R}}(x)dx}{\int x\mu^{\beta}_{\tilde{R}}(x)dx}\right), \mu(x) \in [0,1]\right\} \tag{9}$$

3. Classical CODAS and Z-Fuzzy CODAS

3.1. Classical CODAS [15]

Step 1. Construct the decision-making matrix (X), shown as in Equation (10).

$$X = [x_{ij}]_{n\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} C1\ \ C2\ \ C3\ \cdots\ C_m \\ x_{11}\ x_{12}\ x_{13}\ \cdots\ x_{1_m} \\ x_{21}\ x_{22}\ x_{23}\ \cdots\ x_{2m} \\ \vdots\ \ \vdots\ \ \vdots\ \ \ddots\ \ \vdots \\ x_{n1}\ x_{n2}\ x_{n3}\ \cdots\ x_{nm} \end{array} \tag{10}$$

where $x_{ij}(x_{ij} \ge 0)$ denotes the performance value of ith alternative on jth criterion ($i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, m\}$)

Step 2. Calculate the normalized decision matrix. Performance values are calculated using linear normalization as in Equation (11).

$$n_{ij} = \begin{cases} \frac{x_{ij}}{\max_i x_{ij}} & if\ j \in N_b \\ \frac{\min_i x_{ij}}{x_{ij}} & if\ j \in N_c \end{cases} \tag{11}$$

where N_b and N_c represent the benefit and cost criteria, respectively.

Step 3. Calculate the weights of the criteria by using Saaty's pairwise comparison matrix as in Equation (12).

$$W = [w]_{m\times m} = \begin{bmatrix} 1 & \frac{w_1}{w_2} & \frac{w_1}{w_3} & \frac{w_1}{w_4} & \cdots & \frac{w_1}{w_m} \\ \frac{w_2}{w_1} & 1 & \frac{w_2}{w_3} & \frac{w_2}{w_4} & \cdots & \frac{w_2}{w_m} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & 1 & \frac{w_3}{w_3} & \cdots & \frac{w_3}{w_m} \\ \frac{w_4}{w_1} & \frac{w_4}{w_2} & \frac{w_4}{w_3} & 1 & \cdots & \frac{w_4}{w_m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{w_m}{w_1} & \frac{w_m}{w_2} & \frac{w_m}{w_3} & \frac{w_m}{w_4} & \cdots & 1 \end{bmatrix} \quad (12)$$

From Equation (12), we obtain $W = (w_{C1}, w_{C2}, w_{C3}, \ldots, w_{Cm},)$.

Step 4. Calculate the weighted normalized decision matrix as in Equation (13).

$$r_{ij} = w_j n_{ij} \quad (13)$$

where w_j $(0 < w_j < 1)$ denotes the weight of jth criterion, and $\sum_{j=1}^{m} w_j = 1$

Step 5. Determine the negative-ideal solution as in Equations (14) and (15).

$$ns = \left[ns_j\right]_{1\times m} \quad (14)$$

$$ns_j = \min_i r_{ij} \quad (15)$$

Step 6. Calculate the Euclidean and Taxicab distances of alternatives from the negative-ideal solution, shown as in Equations (16) and (17).

$$E_i = \sqrt{\sum_{j=1}^{m} \left(r_{ij} - ns_j\right)^2} \quad (16)$$

$$T_i = \sum_{j=1}^{m} \left|r_{ij} - ns_j\right| \quad (17)$$

Step 7. Construct the relative assessment matrix, shown as in Equations (18) and (19).

$$Ra = [h_{ik}]_{n\times n} \quad (18)$$

$$h_{ik} = (E_i - E_k) + (\psi(E_i - E_k) \times (T_i - T_k)) \quad (19)$$

where $k \in \{1, 2, \ldots, n\}$ and ψ denotes a threshold function to recognize the equality of the Euclidean distances of two alternatives and is defined as in Equation (20).

$$\psi(x) = \begin{cases} 1 & if \quad |x| \geq \tau \\ 0 & if \quad |x| < \tau \end{cases} \quad (20)$$

where τ is a threshold parameter set by decision-makers. It is recommended to set this parameter at a value between 0.01 and 0.05. If the difference between Euclidean distances of two alternatives is larger than τ, these two alternatives are also compared by Taxicab distance. In this study, we set $\tau = 0.02$ in the application section.

Step 8. Calculate the assessment score of each alternative, shown as in Equation (21).

$$H_i = \sum_{k=1}^{n} h_{ik} \quad (21)$$

Step 9. Rank the alternatives according to assessment score (H_i). The alternative, which has the highest H_i is the best choice among the alternatives.

3.2. Proposed Z-Fuzzy CODAS

Step 1. Construct the Z-fuzzy decision matrices as given by Equation (22) and aggregate them using the geometric mean operation.

$$\tilde{D}_Z = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} C1 \\ Z_{11}\left(\tilde{A}, \tilde{B}\right) \\ Z_{21}\left(\tilde{A}, \tilde{B}\right) \\ \vdots \\ Z_{n1}\left(\tilde{A}, \tilde{B}\right) \end{array} \begin{array}{c} C2 \\ Z_{12}\left(\tilde{A}, \tilde{B}\right) \\ Z_{22}\left(\tilde{A}, \tilde{B}\right) \\ \vdots \\ Z_{n2}\left(\tilde{A}, \tilde{B}\right) \end{array} \begin{array}{c} C3 \\ Z_{13}\left(\tilde{A}, \tilde{B}\right) \\ Z_{23}\left(\tilde{A}, \tilde{B}\right) \\ \vdots \\ Z_{n3}\left(\tilde{A}, \tilde{B}\right) \end{array} \begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \ddots \\ \cdots \end{array} \begin{array}{c} Cm \\ Z_{1m}\left(\tilde{A}, \tilde{B}\right) \\ Z_{2m}\left(\tilde{A}, \tilde{B}\right) \\ \vdots \\ Z_{nm}\left(\tilde{A}, \tilde{B}\right) \end{array} \quad (22)$$

where $Z_{11}\left(\tilde{A}, \tilde{B}\right) \geq 0$ and it denotes the Z-fuzzy performance value of ith alternative on jth criterion ($i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, m\}$)

Tables 3 and 4 can be used for the restriction scale and reliability scale, respectively.

Step 2. Convert Z-fuzzy performance values to ordinary fuzzy numbers $\tilde{Z}'$ using Equation (4).

Table 3
Restriction scale

Linguistic Terms	Abbr.	Z-fuzzy restriction function
Very Poor	VP	(1/4,1/2,1/2,1;1)
Poor	P	(1/2,1,1,3;1)
Medium Poor	MP	(1,3,3,5;1)
Fair	F	(3,5,5,7;1)
Medium good	MG	(5,7,7,9;1)
Good	G	(7,9,9,10;1)
Very good	VG	(9,10,10,10;1)

Table 4
Reliability scale

Linguistic Reliability	Abbr.	Triangular Z-fuzzy reliability function
Certainly Reliable	CR	(1, 1, 1; 1)
Very Strongly Reliable	VSR	(0.8, 0.9, 1; 1)
Strongly Reliable	SR	(0.7, 0.8, 0.9; 1)
Very Highly Reliable	VHR	(0.6, 0.7, 0.8; 1)
Highly Reliable	HR	(0.5, 0.6, 0.7; 1)
Fairly Reliable	FR	(0.4, 0.5, 0.6; 1)
Weakly Reliable	WR	(0.3, 0.4, 0.5; 1)
Very Weakly Reliable	VWR	(0.2, 0.3, 0.4; 1)
Strongly Unreliable	SU	(0.1, 0.2, 0.3; 1)
Absolutely Unreliable	AU	(0, 0.1, 0.2; 1)

Then, Equation (22) becomes Equation (23):

$$\tilde{D}_{\tilde{Z}'} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccccc} C1 & C2 & C3 & \cdots & Cm \end{array} \\ \begin{array}{cccccc} \tilde{Z}'_{11} & \tilde{Z}_{12}{}' & \tilde{Z}_{13}{}' & \cdots & \tilde{Z}_{1m}{}' \end{array} \\ \begin{array}{cccccc} \tilde{Z}_{21}{}' & \tilde{Z}_{22}{}' & \tilde{Z}_{23}{}' & \cdots & \tilde{Z}_{2m}{}' \end{array} \\ \begin{array}{cccccc} \vdots & \vdots & \vdots & \ddots & \vdots \end{array} \\ \begin{array}{cccccc} \tilde{Z}_{n1}{}' & \tilde{Z}_{n2}{}' & \tilde{Z}_{n3}{}' & \cdots & \tilde{Z}_{nm}{}' \end{array} \end{array} \quad (23)$$

where $\tilde{Z}'_{ij} = (z'_{ija}, z'_{ijb}, z'_{ijc}, z'_{ijd})$, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

Step 3. Calculate the normalized $\widetilde{Z}'$-fuzzy decision matrix. Performance values are calculated using linear normalization as in Equations (24–27).

$$\tilde{n}_{ij} = \begin{cases} \frac{z'_{ij}}{z'^{*}_{ijd}} & if\ j \in N_b \\ \frac{z'_{ija^-}}{z'_{ij}} & if\ j \in N_c \end{cases} \quad (24)$$

where

$$\tilde{n}_{ij} = \begin{array}{l} \left(\frac{z'_{ija}}{z'_{ijd*}}, \frac{z'_{ijb}}{z'_{ijd*}}, \frac{z'_{ijc}}{z'_{ijd*}}, \frac{z'_{ijd}}{z'_{ijd*}} \right) if\ j \in N_b \\ \left(\frac{z'_{ija^-}}{z'_{ijd}}, \frac{z'_{ija^-}}{z'_{ijc}}, \frac{z'_{ija^-}}{z'_{ijb}}, \frac{z'_{ija^-}}{z'_{ija}} \right) if\ j \in N_c \end{array} \quad (25)$$

$$z'_{ijd*} = \max_i z'_{ijd}\ of\ Z'_{ij}s\ j \in N_b \quad (26)$$

Table 5
Linguistic scale for weighing the criteria

Linguistic Terms	Abbr.	Restriction function
Absolutely Important	AI	(7,9,9)
Strongly Important	SI	(5,7,9)
Highly Important	HI	(3,5,7)
Weakly Important	WI	(1,3,5)
Equally Important	EI	(1,1,1)

$$z'_{ija^-} = \min_i z'_{ija}\ of\ \tilde{Z}'_{ij}s\ j \in N_c \quad (27)$$

where N_b and N_c represent the sets of benefit and cost criteria, respectively.

Step 3. Calculate the criteria weights by applying Buckley's (1985) fuzzy pairwise comparison procedure as follows:

Step 3.1. Fill in the pairwise comparison matrix for calculating the Z-fuzzy weights of the criteria by using the scales given in Tables 4 and 5.

Step 3.2. Convert Z-fuzzy evaluations to ordinary fuzzy evaluations by using Equation (4).

Step 3.3. Calculate the geometric mean for each parameter of $\tilde{a}_{ij}$ in the m dimensional pairwise comparison matrix. Thus, $m \times m$ matrix is converted to $m \times 1$ matrix.

Step 3.4. Sum the values of each parameter in the column to normalize the values in $m \times 1$ matrix.

Step 3.5. Apply fuzzy division operation to get the normalized weights vector.

Step 3.6. Defuzzify the normalized weights vector using the center of gravity method given by Equation (2).

Step 3.7. Normalize the weights so that their sum is equal to 1.

Step 4. Calculate the weighted normalized decision matrix by using Equation (28).

$$\tilde{r}_{ij} = w_j \tilde{n}_{ij} \quad (28)$$

where w_j $(0 < w_j < 1)$ denotes the weight of jth criterion, and $\sum_{j=1}^{m} w_j = 1$.

Step 5. Determine the negative-ideal solution as in Equations (29) and (30). The negative ideal solutions of each criterion are determined based on the defuzzification equations mentioned in Definition (3).

$$\widetilde{ns} = [\widetilde{ns}_j]_{1\times m} \tag{29}$$

$$\widetilde{ns}_j = \min_i \tilde{r}_{ij} \tag{30}$$

Step 6. Calculate the Euclidean and Taxicab distances of alternatives from the negative-ideal solution, shown as in Equations (31) and (32).

$$E_i = \sqrt{\sum_{j=1}^{m} \frac{1}{4}\left[(\tilde{r}_{ija} - \widetilde{ns}_{jd})^2 + (\tilde{r}_{ijb} - \widetilde{ns}_{jc})^2 + (\tilde{r}_{ijc} - \widetilde{ns}_{jb})^2 + (\tilde{r}_{ijd} - \widetilde{ns}_{ja})^2\right]} \tag{31}$$

$$T_i = \sum_{j=1}^{m} \left\{ \frac{1}{4}(|\tilde{r}_{ija} - \widetilde{ns}_{ja}| + |\tilde{r}_{ijb} - \widetilde{ns}_{jb}| + |\tilde{r}_{ijc} - \widetilde{ns}_{jc}| + |\tilde{r}_{ijd} - \widetilde{ns}_{jd}|) \right\} \tag{32}$$

Step 7. Construct the relative assessment matrix, shown as in Equations (33) and (34):

$$Ra = [h_{ik}]_{n\times n} \tag{33}$$

$$h_{ik} = (E_i - E_k) + (\psi(E_i - E_k) \times (T_i - T_k)) \tag{34}$$

where $k \in \{1, 2, \ldots, n\}$ and ψ denotes a threshold function to recognize the equality of the Euclidean distances of two alternatives, and is defined as in Equation (35):

$$\psi(x) = \begin{cases} 1 & if \quad |x| \geq \tau \\ 0 & if \quad |x| < \tau \end{cases} \tag{35}$$

where τ is a threshold parameter set by decision-makers. It is recommended to set this parameter at a value between 0.01 and 0.05. If the difference between Euclidean distances of two alternatives is larger than τ, these two alternatives are also compared by Taxicab distance. In this study, we set $\tau = 0.02$ in the application section.

Step 8. Calculate the assessment score of each alternative, shown as in Equation (36):

$$H_i = \sum\nolimits_{k=1}^{n} h_i k \tag{36}$$

Step 9. Rank the alternatives according to assessment score (H_i). The alternative which has the highest H_i is the best choice among the alternatives.

4. Application

Let us consider a supplier selection problem with three criteria (C1, C2 and C3) and three alternatives (A1, A2 and A3). Three decision makers (DMs) evaluated the alternatives using the criteria *quality* (C1), *price* (C2), and *delivery time* (C3). We now apply the steps of the model as described in Section 3.2.

Step 1. First, we collect Z-fuzzy evaluations for alternatives from decision makers. Tables 6–8 present Z-fuzzy evaluation matrices for alternatives according to the determined criteria.

The aggregated decision matrix is obtained as in Table 9 using the geometric mean operation.a

Step 2. We convert Z-fuzzy numbers to ordinary fuzzy numbers using Equation (3). The obtained ordinary fuzzy numbers are shown in Table 10.

Step 3. We normalize ordinary fuzzy evaluation values of alternatives by using Equations (24–27). Thus, Table 11 is obtained. In this application, price (C2) and delivery time (C3) are cost criteria while quality (C1) is benefit criterion. However, the assigned scores

Table 6
DM 1's evaluations

Criteria	Alternatives	Evaluations	Restriction	Reliability
C1	A1	G,SR	(7, 9, 9, 10)	(0.7, 0.8, 0.9)
	A2	P,WR	(0.5, 1, 1, 3)	(0.3, 0.4, 0.5)
	A3	MP,HR	(1, 3, 3, 5)	(0.5, 0.6, 0.7)
C2	A1	P,VHR	(0.5, 1, 1, 3)	(0.6, 0.7, 0.8)
	A2	VG,SU	(9, 10, 10, 10)	(0.1, 0.2, 0.3)
	A3	G,AR	(7, 9, 9, 10)	(0.8, 0.9, 1)
C3	A1	MP,AU	(1, 3, 3, 5)	(0, 0.1, 0.2)
	A2	G,FR	(7, 9, 9, 10)	(0.4, 0.5, 0.6)
	A3	VG,SR	(9, 10, 10, 10)	(0.7, 0.8, 0.9)

Table 7
DM 2's evaluations

Criteria	Alternatives	Evaluations	Restriction	Reliability
C1	A1	P, HR	(0.5, 1, 1, 3)	(0.5, 0.6, 0.7)
	A2	G, VWR	(7, 9, 9, 10)	(0.2, 0.3, 0.4)
	A3	G, SR	(7, 9, 9, 10)	(0.7, 0.8, 0.9)
C2	A1	MG, FR	(5, 7, 7, 9)	(0.4, 0.5, 0.6)
	A2	F, HR	(3, 5, 5, 7)	(0.5, 0.6, 0.7)
	A3	MG, AU	(5, 7, 7, 9)	(0, 0.1, 0.2)
C3	A1	P, WR	(0.5, 1, 1, 3)	(0.3, 0.4, 0.5)
	A2	VG, HR	(9, 10, 10, 10)	(0.5, 0.6, 0.7)
	A3	VP, FR	(0.25, 0.5, 0.5, 1)	(0.4, 0.5, 0.6)

Table 8
DM 3's evaluations

Criteria	Alternatives	Evaluations	Restriction	Reliability
C1	A1	F, SR	(3, 5, 5, 7)	(0.7, 0.8, 0.9)
	A2	MP, HR	(1, 3, 3, 5)	(0.5, 0.6, 0.7)
	A3	MG, SR	(5, 7, 7, 9)	(0.7, 0.8, 0.9)
C2	A1	MG, HR	(5, 7, 7, 9)	(0.5, 0.6, 0.7)
	A2	G, WR	(7, 9, 9, 10)	(0.3, 0.4, 0.5)
	A3	VP, VHR	(0.25, 0.5, 0.5, 1)	(0.6, 0.7, 0.8)
C3	A1	F, HR	(3, 5, 5, 7)	(0.5, 0.6, 0.7)
	A2	MG, HR	(5, 7, 7, 9)	(0.5, 0.6, 0.7)
	A3	P, SR	(0.5, 1, 1, 3)	(0.7, 0.8, 0.9)

Table 9
Aggregated decision matrix

Criteria	Alternatives	Restriction	Reliability
C1	A1	(2.190, 3.557, 3.557, 5.944)	(0.626, 0.727, 0.828)
	A2	(1.518, 3.000, 3.000, 5.313)	(0.311, 0.416, 0.519)
	A3	(3.271, 5.739, 5.739, 7.663)	(0.626, 0.727, 0.828)
C2	A1	(2.321, 3.659, 3.659, 6.240)	(0.493, 0.594, 0.695)
	A2	(5.739, 7.663, 7.663, 8.879)	(0.247, 0.363, 0.472)
	A3	(2.061, 3.158, 3.158, 4.481)	(0.000, 0.398, 0.543)
C3	A1	(1.145, 2.466, 2.466, 4.718)	(0.000, 0.288, 0.412)
	A2	(6.804, 8.573, 8.573, 9.655)	(0.464, 0.565, 0.665)
	A3	(1.040, 1.710, 1.710, 3.107)	(0.581, 0.684, 0.786)

Table 10
Ordinary fuzzy numbers converted from Z-fuzzy numbers

	C1	C2	C3
A1	(1.867, 3.032, 3.032, 5.068)	(1.789, 2.821, 2.821, 4.811)	(0.615, 1.325, 1.325, 2.534)
A2	(0.979, 1.935, 1.935, 3.427)	(3.460, 4.620, 4.620, 5.353)	(5.113, 6.442, 6.442, 7.255)
A3	(2.789, 4.893, 4.893, 6.533)	(1.300, 1.992, 1.992, 2.827)	(0.860, 1.414, 1.414, 2.570)

Table 11
Normalized decision matrix

	C1	C2	C3
A1	(0.286, 0.464, 0.464, 0.776)	(0.334, 0.527, 0.527, 0.899)	(0.085, 0.183, 0.183, 0.349)
A2	(0.150, 0.296, 0.296, 0.525)	(0.646, 0.863, 0.863, 1.000)	(0.705, 0.888, 0.888, 1.000)
A3	(0.427, 0.749, 0.749, 1.000)	(0.243, 0.372, 0.372, 0.528)	(0.119, 0.195, 0.195, 0.354)

are in terms of benefit. Hence, the normalization type of benefit criteria is applied to all the criteria.

For weighing criteria, we apply steps 3.1–3.7.

Step 3.1. We construct Z-fuzzy pairwise comparison matrix for the criteria. Three decision makers compromised on the evaluation of the criteria weights as given in Table 12. The corresponding numerical scale is used to construct numerical Z-fuzzy pairwise comparison matrix as given in Table 13.

Step 3.2. We convert Z-fuzzy evaluation values of criteria to ordinary fuzzy numbers by using Equation (3). Table 14 shows the converted ordinary fuzzy values of criteria.

Step 3.3. We calculate the geometric mean for each parameter of Z-fuzzy numbers. The obtained results are given in Table 15.

For instance, the weight of C1 is calculated as follows.

$$w_1 = [(1.301/5.139, 2.226/3.619, 2.952/2.319)]$$
$$= [0.253, 0.615, 1.273]$$

Table 12
Pairwise comparisons of the criteria using Z-fuzzy numbers

	C1	C2	C3
C1	EI, CR	HI, HR	WI, VSR
C2	1/HI, HR	EI, CR	1/HI, VHR
C3	1/WI, VSR	HI, VHR	EI, CR

Table 13
Z-fuzzy pairwise comparison matrix for criteria

	C1		C2		C3	
	Restriction	Reliability	Restriction	Reliability	Restriction	Reliability
C1	(1, 1, 1)	(1, 1, 1)	(3, 5, 7)	(0.5, 0.6, 0.7)	(1, 3, 5)	(0.8, 0.9, 1.0)
C2	(0.1, 0.2, 0.3)	(0.5, 0.6, 0.7)	(1, 1, 1)	(1, 1, 1)	(0.1, 0.2, 0.3)	(0.6, 0.7, 0.8)
C3	(0.2, 0.3, 1.0)	(0.8, 0.9, 1.0)	(3, 5, 7)	(0.6, 0.7, 0.8)	(1, 1, 1)	(1, 1, 1)

Table 14
Ordinary fuzzy pairwise comparison matrix for criteria

	C1	C2	C3
C1	(1, 1, 1)	(2.32, 3.87, 5.42)	(0.95, 2.85, 4.74)
C2	(0.11, 0.15, 0.26)	(1, 1, 1)	(0.12, 0.17, 0.28)
C3	(0.19, 0.32, 0.95)	(2.51, 4.18, 5.86)	(1, 1, 1)

Table 15
Geometric mean of ordinary fuzzy pairwise comparison matrices

Criteria	Geometric means of each parameter of a_{ij}
C1	(1.301, 2.226, 2.952)
C2	(0.236, 0.296, 0.416)
C3	(0.781, 1.098, 1.771)
Total	(2.319, 3.619, 5.139)

Table 16
Relative fuzzy weights and crisp weights of each criterion

Criteria	Fuzzy	Defuzzification	Crisp weights
C1	(0.253, 0.615, 1.273)	0.689	0.590
C2	(0.046, 0.082, 0.179)	0.097	0.083
C3	(0.152, 0.303, 0.764)	0.381	0.326
	Total:	1.167	1.000

Table 17
Weighted normalized decision matrix and negative ideal solutions

	C1	C2	C3
A1	(0.169, 0.274, 0.274, 0.458)	(0.028, 0.044, 0.044, 0.075)	(0.028, 0.060, 0.060, 0.114)
A2	(0.089, 0.175, 0.175, 0.310)	(0.054, 0.072, 0.072, 0.083)	(0.230, 0.290, 0.290, 0.326)
A3	(0.252, 0.442, 0.442, 0.590)	(0.020, 0.031, 0.031, 0.044)	(0.039, 0.064, 0.064, 0.116)
NIS	(0.089, 0.175, 0.175, 0.310)	(0.020, 0.031, 0.031, 0.044)	(0.028, 0.060, 0.060, 0.114)

Table 18
Euclidean and Taxicab distances of alternatives

	Euclidean Distance	Rank	Taxicab Distance	Rank
A1	0.2205	3	0.2445	3
A2	0.2801	2	0.3680	1
A3	0.3212	1	0.3283	2

Table 19
The relative assessment matrix and the assessment scores of alternatives

	A1	A2	A3	Total H_i	Rank
A1	0	–0.183	–0.184	–0.367	3
A2	0.183	0	–0.001	0.181	2
A3	0.184	0.001	0	0.186	1

Table 20
Expected values of DMs' evaluations

Criteria	Alternatives	DM 1	DM 2	DM 3
C1	A1	(4.9, 7.2, 7.2, 9.0)	(0.3, 0.6, 0.6, 2.1)	(2.1, 4.0, 4.0, 6.3)
	A2	(0.2, 0.4, 0.4, 1.5)	(1.4, 2.7, 2.7, 4.0)	(0.5, 1.8, 1.8, 3.5)
	A3	(0.5, 1.8, 1.8, 3.5)	(4.9, 7.2, 7.2, 9.0)	(3.5, 5.6, 5.6, 8.1)
C2	A1	(0.3, 0.7, 0.7, 2.4)	(2.0, 3.5, 3.5, 5.4)	(2.5, 4.2, 4.2, 6.3)
	A2	(0.9, 2.0, 2.0, 3.0)	(1.5, 3.0, 3.0, 4.9)	(2.1, 3.6, 3.6, 5.0)
	A3	(5.6, 8.1, 8.1, 10.0)	(0.0, 0.7, 0.7, 1.8)	(0.2, 0.4, 0.4, 0.8)
C3	A1	(0.0, 0.3, 0.3, 1.0)	(0.2, 0.4, 0.4, 1.5)	(1.5, 3.0, 3.0, 4.9)
	A2	(2.8, 4.5, 4.5, 6.0)	(4.5, 6.0, 6.0, 7.0)	(2.5, 4.2, 4.2, 6.3)
	A3	(6.3, 8.0, 8.0, 9.0)	(0.1, 0.3, 0.3, 0.6)	(0.4, 0.8, 0.8, 2.7)

Steps 3.4–3.7. After step 3.3, we obtain m^*1 fuzzy matrix. We calculate the sum of values and normalized according to fuzzy division rules. Then, we defuzzify the fuzzy numbers and normalize them to obtain the weights of criteria. The obtained values are given in Table 16.

Step 4 and Step 5. We calculate weighted normalized decision matrix as in Equation (28) and determine the negative ideal solutions by applying Equations (29 and 30). In this step, the weights obtained at the end of Step 3 are used, then Table 17 is obtained.

Step 6. Euclidean and Taxicab distances are calculated by using Equations (31 and 32). Table 18 presents the calculated values.

Step 7–Step 9. In these steps we calculate the relative assessment matrix and the assessment scores of alternatives by using Equations (33–36). Finally, we rank the alternatives. Table 19 presents the calculated results.

Supplier A3 is determined as the best alternative. A1 is the worst supplier among three alternatives. A3 is the best alternative according to Euclidean distance while A2 is the best alternative according to taxicab distance (see Table 18). CODAS method considers both distances to rank the alternatives. This shows the advantage of the CODAS method, which allows both distances to be considered.

5. Comparative analysis using fuzzy simple additive weighting

We compared the proposed method with the fuzzy simple additive weighting (fuzzy SAW) method. Initial decision matrix of fuzzy SAW method is constructed by multiplying restriction and reliability values of DMs' evaluations. Thus, the expected values of each evaluation are obtained and shown in Table 20.

Table 21
Average of the DMs' evaluations

Alternative	C1	C2	C3
A1	(2.417, 3.933, 3.933, 5.800)	(1.600, 2.800, 2.800, 4.700)	(0.550, 1.233, 1.233, 2.467)
A2	(0.683, 1.633, 1.633, 3.000)	(1.500, 2.867, 2.867, 4.300)	(3.267, 4.900, 4.900, 6.433)
A3	(2.967, 4.867, 4.867, 6.867)	(1.917, 3.050, 3.050, 4.200)	(2.250, 3.017, 3.017, 4.100)

Table 22
Normalized decision matrix

Alternative	C1	C2	C3
A1	(0.352, 0.573, 0.573, 0.845)	(0.340, 0.596, 0.596, 1.000)	(0.085, 0.192, 0.192, 0.383)
A2	(0.100, 0.238, 0.238, 0.437)	(0.319, 0.610, 0.610, 0.915)	(0.508, 0.762, 0.762, 1.000)
A3	(0.432, 0.709, 0.709, 1.000)	(0.408, 0.649, 0.649, 0.894)	(0.350, 0.469, 0.469, 0.637)

Table 23
Weighted normalized decision matrix

Alternative	C1	C2	C3
A1	(0.208, 0.338, 0.338, 0.499)	(0.028, 0.050, 0.050, 0.083)	(0.028, 0.063, 0.063, 0.125)
A2	(0.059, 0.140, 0.140, 0.258)	(0.027, 0.051, 0.051, 0.076)	(0.166, 0.248, 0.248, 0.326)
A3	(0.255, 0.419, 0.419, 0.590)	(0.034, 0.054, 0.054, 0.074)	(0.114, 0.153, 0.153, 0.208)

Table 24
Final score, defuzzified score and rank of alternatives

Alternative	Final score	Defuzzified score	Rank
A1	(0.264, 0.450, 0.450, 0.707)	0.344	2
A2	(0.251, 0.440, 0.440, 0.660)	0.335	3
A3	(0.403, 0.626, 0.626, 0.873)	0.484	1

Table 25
Comparison of fuzzy SAW and Z-CODAS methods

Alternative	Fuzzy SAW	Z-fuzzy CODAS
A1	2	3
A2	3	2
A3	1	1

We calculate the average of the expected values of evaluations to aggregate DMs' decision. Table 21 shows the obtained values.

We normalize the values of Table 21 using Equations (24–27) and obtain the normalized decision matrix as given in Table 22.

We calculate weighted normalized decision matrix by multiplying the normalized values with the weights of criteria and Table 23 is obtained.

We obtain the final score of each alternative by summing the values in each row. Then we defuzzify final scores to obtain crisp values as mentioned in Definition 2. Finally, we rank the alternatives according to the defuzzified values as seen in Table 24.

Comparison of two methods is given in Table 25. A3 is the first ranking among the alternatives in both methods. The ranking of the A1 and A2 alternatives in two methods differs. Thus, we are more confident that A3 is the best alternative.

6. Conclusions

CODAS method has been very popular after its introduction to the literature. After ordinary fuzzy CODAS method was developed by Ghorabaee et al. [15], its fuzzy extensions such as type-2 fuzzy CODAS, intuitionistic fuzzy CODAS, hesitant fuzzy CODAS, Pythagorean fuzzy CODAS, and neutrosphic CODAS have been proposed by several researchers. Extensions of multicriteria decision making methods by using Z-fuzzy numbers are

relatively new when compared with these MCDM methods. CODAS method has been extended by using Z-fuzzy numbers and applied to a supplier selection problem. A comparative analysis has also been presented. The application and comparative analysis showed that the proposed Z-fuzzy CODAS method yields meaningful results. DMs could incorporate their opinions related to the reliability of a determined membership function.

For further research, we suggest LR type Z-fuzzy numbers to be used in CODAS method for comparative analyses. Nonlinear membership functions may provide a more realistic approach to the method.

References

[1] H. Aboutorab, M. Saberi, M.R. Asadabadi, O. Hussain and E. Chang, ZBWM: The Z-number extension of Best Worst Method and its application for supplier development, *Expert Systems with Applications* **107** (2018), 115–125.

[2] A. Azadeh and R. Kokabi, Z-number DEA: A new possibilistic DEA in the context of Z-numbers, *Advanced Engineering Informatics* **30**(3) (2016), 604–617.

[3] A. Azadeh, M. Saberi, N.Z. Atashbar, E. Chang and P. Pazhoheshfar, Z-AHP: A Z-number extension of fuzzy analytical hierarchy process, In *Digital Ecosystems and Technologies (DEST), 2013 7th IEEE International Conference on*, 2013, pp. 141–147. IEEE.

[4] I. Badi, A. Abdulshahed and A. Shetwan, A case study of supplier selection for steelmaking company in Libya by using Combinative Distance-based ASsessemnt (CODAS) model, *Decision Making: Applications in Management and Engineering* **1**(1) (2018), 1–12.

[5] I. Badi, M. Ballem and A. Shetwan, Site selection of desalination plant in Libya by using combinative distance-based assessment (CODAS) method, *International Journal for Quality Research* **12**(3) (2018).

[6] E. Bolturk, Pythagorean fuzzy CODAS and its application to supplier selection in a manufacturing firm, *Journal of Enterprise Information Management* **31**(4) (2018), 550–564.

[7] E. Bolturk and C. Kahraman, Interval-valued intuitionistic fuzzy CODAS method and its application to wave energy facility location selection problem, *Journal of Intelligent & Fuzzy Systems* **35**(4) (2018), 4865–4877.

[8] K. Chatterjee and S. Kar, A multi-criteria decision making for renewable energy selection using Z-numbers in uncertain environment, *Technological and Economic Development of Economy* **24**(2) (2018), 739–764.

[9] D. Panchal, P. Chatterjee, R.K. Shukla, T. Choudhury and J. Tamosaitiene, Integrated fuzzy AHP-CODAS framework for maintenance decision in urea fertilizer industry, *Economic Computation and Economic Cybernetics Studies and Research* **51**(3) (2017), 179–196.

[10] A. Forghani, S.J. Sadjadi and B.F. Moghadam, A supplier selection model in pharmaceutical supply chain using PCA, Z-TOPSIS and MILP: A case study, *PloS one* **13**(8) (2018), e0201604.

[11] M. Keshavarz Ghorabaee, M. Amiri, E.K. Zavadskas, R. Hooshmand and J. Antuchevičcienė, Fuzzy extension of the CODAS method for multi-criteria market segment evaluation, *Journal of Business Economics and Management* **18**(1) (2017), 1–19.

[12] B. Kang, D. Wei, Y. Li and Y. Deng, A method of converting Z-number to classical fuzzy number, *Journal of Information & Computational Science* **9**(3) (2012a), 703–709.

[13] B. Kang, D. Wei, Y. Li and Y. Deng, Decision making using Z-numbers under uncertain environment, *Journal of Computational Information Systems* **8**(7) (2012b), 2807–2814.

[14] R. Karthika and A.S. Sudha, A Decision. Making Problem on FAHP with Z-Numbers.

[15] M. Keshavarz Ghorabaee, E.K. Zavadskas, Z. Turskis and J. Antucheviciene, A new combinative distance-based assessment (CODAS) method for multi-criteria decision-making, *Economic Computation & Economic Cybernetics Studies & Research* **50**(3) (2016).

[16] K. Khalif, K.M. Naim, A. Gegov, A. Bakar and A. Syafadhli, Z-TOPSIS approach for performance assessment using fuzzy similarity, In *2017 IEEE International Conference on Fuzzy Systems IEEE*, 2017a.

[17] K. Khalif, K.M. Naim, A. Gegov, A. Bakar and A. Syafadhli, Hybrid fuzzy MCDM model for Z-numbers using intuitive vectorial centroid, *Journal of Intelligent & Fuzzy Systems* **33**(2) (2017b), 791–805.

[18] L.A. Zadeh, A note on Z-numbers, *Information Science* **181** (2011), 2923–2932.

[19] M. Mathew and S. Sahu, Comparison of new multi-criteria decision making methods for material handling equipment selection, *Management Science Letters* **8**(3) (2018), 139–150.

[20] D. Pamučar, I. Badi, K. Sanja and R. Obradović, A novel approach for the selection of power-generation technology using a linguistic neutrosophic CODAS method: A case study in Libya, *Energies* **11**(9) (2018), 2489.

[21] H.G. Peng and J.Q. Wang, Hesitant uncertain linguistic Z-numbers and their application in multi-criteria group decision-making problems, *International Journal of Fuzzy Systems* **19**(5) (2017), 1300–1316.

[22] H.G. Peng and J.Q. Wang, A multicriteria group decision-making method based on the normal cloud model with Zadeh's Z-numbers, *IEEE Transactions on Fuzzy Systems* (2018).

[23] M.L. Puri and D.A. Ralescu, Fuzzy random variables, *J Math Anal Appl* **114** (1986), 409–422.

[24] J. Ren, Sustainability prioritization of energy storage technologies for promoting the development of renewable energy: A novel intuitionistic fuzzy combinative distance-based assessment approach, *Renewable Energy* **121** (2018), 666–676.

[25] S. Sadi-Nezhad and A. Sotoudeh-Anvari, A new Data Envelopment Analysis under uncertain environment with respect to fuzziness and an estimation of reliability, *Opsearch* **53**(1) (2016), 103–115.

[26] N.A. Sahrom and R.M. Dom, A Z-number extension of the hybrid Analytic Hierarchy Process-Fuzzy Data Envelopment Analysis for risk assessment, In *Research and Education in Mathematics (ICREM7), 2015 International Conference on*, IEEE, 2015, pp. 19–24.

[27] K.W. Shen and J.Q. Wang, Z-VIKOR method based on a new comprehensive weighted distance measure of Z-number and its application, *IEEE Trans Fuzzy Syst https://doi.org/10.1109/TFUZZ, 581* (2018).

[28] J.Q. Wang, Y.X. Cao and H.Y. Zhang, Multi-criteria decision-making method based on distance measure and choquet integral for linguistic Z-numbers, *Cognitive Computation* **9**(6) (2017), 827–842.

[29] X.-D. Peng and H. Garg, Algorithms for interval-valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with new information measure, *Computers & Industrial Engineering* **119** (2018), 439–452.

[30] A.M. Yaakob and A. Gegov, Interactive TOPSIS based group decision making methodology using Z-numbers, *International Journal of Computational Intelligence Systems* **9**(2) (2016), 311–324.

[31] A. Yaakob and A.E. Gegov, Fuzzy rule based approach with z-numbers for selection of alternatives using TOPSIS, In *2015 IEEE International Conference on Fuzzy Systems*, Istanbul, Turkey, 2015.

Copyright of Journal of Intelligent & Fuzzy Systems is the property of IOS Press and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.