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Soliton solutions of Heisenberg spin chain equation with parabolic law nonlinearity

Selvi Altun[1] · **Neslihan Ozdemir**[2] · **Muslum Ozisik**[3] · **Aydin Secer**[4] · **Mustafa Bayram**[4]

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Abstract
In this article, we presented and investigated a model of the (2+1)-dimensional Heisenberg ferromagnetic spin chain (HFSC) equation with parabolic law nonlinearity, for the first time. This study intends on two main elements. The first is to construct the parabolic law nonlinearity form of the HFSC model, and the second is to peruse the effect of this form on the soliton wave behavior. Two different analytical techniques, which are a subversion of the new extended auxiliary equation method and the improved generalized Kudryashov method (iGKM), are applicated to construct the soliton solutions of investigated HFSC form. With the help of these methods, bright, singular, and kink soliton forms have been obtained and their characteristics have been depicted by 2D and 3D diagrams. In accordance with our litterateur investigation, the subject of the study and the results obtained have not been reported before and we can say that the results we have attained will be a guide for researchers who want to study in this field.

Keywords Ferromagnet Heisenberg · Spin chain · Parabolic law · SAEM246 · Improved generalized Kudryashov method

✉ Mustafa Bayram
mustafabayram@biruni.edu.tr

Selvi Altun
selvialtun89@gmail.com

Neslihan Ozdemir
neozdemir@gelisim.edu.tr

Muslum Ozisik
ozisik@yildiz.edu.tr

Aydin Secer
asecer@biruni.edu.tr

1 Suleymaniye Mh. Destan Sk. Inegol, Bursa, Turkey

2 Software Engineering Department, Istanbul Gelisim University, Istanbul, Turkey

3 Mathematical Engineering Department, Yildiz Technical University, Istanbul, Turkey

4 Computer Engineering Department, Biruni University, Istanbul, Turkey

1 Introduction

The term ”soliton” has been a magic word that has been the focus of many researchers in the last quarter century. One of the main reasons for this has been the success of soliton solutions in obtaining analytical solutions to many engineering problems. Therefore, in the last twenty years, especially water waves, ocean engineering, earth science (earthquake, seismology), quantum mechanics, medicine (genetics, virus and cancer researches and models), chemistry, statistics, nonlinear optics, heat, and mass transmission has been the first fields of application that come to mind. Huge advancements in electronics, computers, and software have contributed positively to research in these areas. Various models have been made about these fields by many researchers, and nonlinear partial differential equations have been introduced to the literature with different names as a representation of these models. For example, the magical and productive Kadomtsev-Petviashvili (KP) and Korteweg-de Vries (KdV) equations and hundreds of models, equations, and systems which are related to KdV and KP (Darvishi et al. 2007, 2007; Darvishi and Najafi 2012; Darvishi et al. 2012, 2016, 2016; Wazwaz 2021; Cinar et al. 2022; Wazwaz 2020; Hon and Fan 2004). Introducing the stated models to the literature is one phase of the process, and the other phase is the development of solution techniques to obtain analytical, semi-analytical, and numerical solutions to these models and equations. New solution algorithms and techniques have been introduced and performed by many investigators. The exp-function method (Khani et al. 2010; Rani et al. 2022), the modified exp-function method (Shakeel et al. 2023; Attaullah et al. 2022), the generalized ex-function method (Shakeel et al. 2022), the semi-analytical solution (Arbabi et al. 2016), exact solutions (Darvishi et al. 2018), closed forms (Shakeel et al. 2022, 2022), Kudryashov's based methods (Ozisik et al. 2022), Jacobi elliptic function expansion method (JEFEM) (Kumar 2018), auxiliary methods (Zayed and Alurrfi 2016; Khater et al. 2019; Yomba 2008; Sirendaoreji and Sun 2006), modified exp-function method (Shakeel et al. 2023).

Soliton and its utilization in ferromagnetic spin chains have been examined densely in recent years (Ma et al. 2018; Hosseini et al. 2021; Bulut et al. 2018, 2018; Bashar and Islam 2020; Seadawy et al. 2020; Osman et al. 2020; Kang and Yang 2020; Ozkan 2021; Daniel 1995). In favor of the HFSC model, semiclassical nonlinear spin dynamics are identified for Heisenberg ferromagnetic spin chains. The HFSC models are prototypes with particular magnetized interactive relations, which many researchers have particularly drawn attention to and focused on. There are a lot of efficient methods that have been worked by many researchers to obtain analytical soliton solutions for some forms of HFSC. Some of these techniques are the generalized tanh methods and the complex envelope function Ansatz method (Inc et al. 2017), Bernoulli's equation approach and sin-cosine method (Ali et al. 2019), Bernoulli subequation method, a novel Kudryashov method and He's homotopy perturbation (Attia et al. 2022), the simplest equation method and Nucci's reduction method (Hashemi 2018), the extended $\left(G'/G^2\right)$-expansion function (Nisar et al. 2022), the extended rational sine-cosine and sinh-cosh methods (Cinar et al. 2021).

In this paper, the techniques implemented in the presented mathematical model, which are the improved generalized Kudryashov method and SAEM246. We take an interest in the (2+1)-dimensional HFSC equation which models the wave propagation in the Heisenberg ferromagnetic spin chain is reads (Bulut et al. 2018):

$$i\frac{\partial u}{\partial t}+\mu\frac{\partial^2 u}{\partial x^2}+\lambda\frac{\partial^2 u}{\partial y^2}+\eta\frac{\partial^2 u}{\partial x\partial y}-\gamma|u|^2u=0, \tag{1}$$

$u = u(x, y, t)$ means the dynamic attitude of complex wave envelope in ferromagnetic supplies, and x, y are the spatial coordinates while t depicts temporal coordinate.

$$\mu = \Gamma^4\left(\Omega + \Omega_2\right), \quad \lambda = \Gamma^4\left(\Omega_1 + \Omega_2\right), \quad \eta = 2\Gamma^4\Omega_2, \quad \gamma = 2\Gamma^4 Z,$$

where $\lambda, \Omega, \Omega_1, \Omega_2$, and Z stem from the lattice parameter, the coefficient of bilinear exchange interactions along the x-direction, the coefficient of bilinear exchange interactions along the y-direction, the neighboring interaction along the diagonal and the uni-axial crystal field anisotropy parameter, respectively.

Previously, semi-classical approximation, because of its compatibility and consistency with the problem, has been considered a suitable method to investigate soliton excitations in quantum spin systems (Huang et al. 1991; Lu et al. 2009). Considering the semiclassical approach in the long wavelength and the low heat limits, soliton stimulations have been classified in ferromagnets in Glauber (1963); Holstein and Primakoff (1940); Kapor et al. (1991); Tosizumi (1996); Douglass (1971); Freitas and Dodonov (2022). All of these studies are in one dimension, and Lakshmanan and Myrzakulov have developed these studies as the (2+1)-dimensional class of the NLSE (Myrzakulov et al. 1997; Lakshmanan et al. 1998). After these studies, many important researches have been made about the ferromagnetic Heisenberg chain equation and the results have been reported. Myrzakul et al. studied the NLSEs and their spin systems in Myrzakul et al. (2021). Bashar and Islam investigated the exact solutions to the (2 + 1)-dimensional HFSC equation by the modified simple equation and improve F-expansion (Bashar and Islam 2020), nonautonomous complex wave solutions for the (2+1)-dimensional HFSC equation with variable coefficients studied by Peng (2019), He's homotopy perturbation and the Bernoulli sub-equation (Raghda et al. 2022), the extended simple equation, the extended direct algebraic and the Paul-Painleve approach methods in Zahran and Beki (2022), the fractional form of (2+1)-dimensional HFSC equation under the effect of Wiener process with Jacobi elliptic function method (Mohammed et al. 2022), Lie symmetry in Kumar et al. (2022).

In this study, we present the (2+1)-dimensional HFSC equation which models the wave propagation in the Heisenberg ferromagnetic spin chain with parabolic law nonlinearity follow as:

$$i\frac{\partial u}{\partial t} + \mu\frac{\partial^2 u}{\partial x^2} + \lambda\frac{\partial^2 u}{\partial y^2} + \eta\frac{\partial^2 u}{\partial x \partial y} - \left(\gamma_1|u|^2 + \gamma_2|u|^4\right)u = 0, \tag{2}$$

as it is presented with Eq. (1),

$$\mu = \Gamma^4\left(\Omega + \Omega_2\right), \quad \lambda = \Gamma^4\left(\Omega_1 + \Omega_2\right), \quad \eta = 2\Gamma^4\Omega_2, \quad \gamma = 2\Gamma^4 Z,$$

in which $\lambda, \Omega, \Omega_1, \Omega_2$, correspond to the lattice parameter, the coefficient of bilinear exchange interactions along the x-direction, the coefficient of bilinear exchange interactions along the y-direction, the neighboring interaction along the diagonal and the uni-axial crystal field anisotropy parameter, respectively. Besides, the parameters γ_1, γ_2 come from parabolic law nonlinearity.

The concept of nonlinearity plays a crucial role in NLPDEs. Although this concept mostly appears in forms such as Kerr, parabolic, power, dual-power, quadratic-cubic, generalized quadratic-cubic, cubic-quintic, cubic-quintic-septic, triple power, parabolic with weak non-local, anti-cubic- generalized anti-cubic, log law in problems related to nonlinear optics in connection with self-phase modulation, nonlinearity is of great importance as a general phenomenon that occurs in the modeling of many optical problems. Although Kerr and power law nonlinearity terms are predominantly included in the models developed for the modeling of physical phenomena such as water waves, solid-state physics, quantum mechanics, and heat problems, recently, studies on the effect of parabolic law nonlinearity have started to be reported and there are also experimental studies on them. To the best of our knowledge, the parabolic law nonlinearity form of the (2+1)-HFSC equation has not been introduced to the literature, and the soliton productivity of this form has not been examined before. However, we would like to emphasize the following point. As it is known, the modeling of physical phenomena of nonlinear partial differential equations generally started as (1+1)-dimensional (spatial and temporal) and then it was possible to transform these models into higher-dimensional problems. Problems designed mathematically in this way should also have a physical equivalent, that is, they should represent a physical event. For the concept of nonlinearity, the nonlinearity terms included in the equation must correspond to the purposed physical phenomenon. In this context, the parabolic law nonlinearity form introduced for the (2+1)-HFSC equation within the scope of the article is not only presented as a mathematical equation but the soliton solutions of this form are obtained, furthermore, the effect of the parabolic law nonlinearity form on the soliton dynamics is examined. In this respect, we need to emphasize that the study differs from other studies in its field.

Ferromagnetism is one of the most important topics for magnetism, and in general, the spins of all atoms in the ground state are oriented in one direction. This parallel alignment of spins takes place within the framework of the theory proposed by Heisenberg, called exchange interaction. The HFSC model has an important place in contemporary magnet doctrine. Equation (1) identifies the non-linear properties of magnets and semiclassical nonlinear spin dynamics. So, soliton-like excitations in a spin chain for the ferromagnetic system are a special and important research area. The HFSC model with Kerr law has been investigated by many researchers before, but in this study, we investigated the HFSC equation with parabolic law nonlinearity for the first time.

The continuation of the research work is planned as follows: Sect. 2 contains the mathematical examination of the equation under consideration. Section 3, covers the main characteristics and utilization of the iGKM. In Sect. 4, the properties of SAEM246 are given and its application to the introduced mathematical model is tested. Section 5 and Sect. 6 cover the result and discussion and conclusion, respectively.

2 Mathematical analysis of Eq. (2)

Let us pay regard to Eq. (2) for obtaining nonlinear ordinary differential equation (NODE) form. Primarily, the wave transformation is given as:

$$u(x,y,t)=U(\xi)e^{i\theta(x,y,t)},\quad \theta(x,y,t)=-\beta_1x-\beta_2y+\rho t+\varphi_0,\quad \xi=\alpha_1x+\alpha_2y-\omega t, \tag{3}$$

where $U(\xi)$ comes to mean the shape of the pulse and $\theta(x,y,t)$ indicates the phase portion of the soliton, ρ is the frequency of the soliton, the parameters β_1 and β_2 are the wave numbers, ω is the velocity of the soliton and the parameters α_1 and α_2 mean the inverse width of the soliton, while φ_0 is the center of the phase.

If we place Eq. (3) in Eq. (2), then separating the real and imaginary parts, we procure the following equations, respectively:

$$\left(\left(-\eta\beta_2-2\mu\beta_1\right)\alpha_1+\left(-\eta\beta_1-2\lambda\beta_2\right)\alpha_2-\omega\right)\left(\frac{dU}{d\xi}\right)=0, \tag{4}$$

$$\left(\eta\alpha_1\alpha_2+\lambda\alpha_2^2+\mu\alpha_1^2\right)\left(\frac{d^2U}{d\xi^2}\right)-U\left(U^4\gamma_2+U^2\gamma_1+\mu\beta_1^2+\eta\beta_2\beta_1+\lambda\beta_2^2+\rho\right)=0, \tag{5}$$

where $U=U(\xi)$ and since $U'(\xi)\neq 0$, Eq. (4) serves the following constraint condition:

$$\omega=\left(-\alpha_1\beta_2-\alpha_2\beta_1\right)\eta-2\mu\alpha_1\beta_1-2\lambda\alpha_2\beta_2, \tag{6}$$

which is the velocity.

3 Definition and implementation of iGKM to Eq. (2)

In this section, we put forward the particulars of iGKM (Ozisik et al. 2022). In the literature, there is a mixture of development and methods that have been the subject of many articles for solving NLEEs equations. Such as the generalized Riccati mapping method (Kumar and Niwas 2023), Kudryashov's simplest equation approach (Kumar and Niwas 2022), the generalized exponential rational function approach (Kumar et al. 2023), Hirota's method [63], Lie symmetry method (Kumar and Rani 2020), Painlevé analysis (Kumar et al. 2022), the one-parameter Lie group of transformations method (Kumar and Rani 2021), the extended sub-equation method (Ouahid et al. 2022). The proposed methods in the article are analytically based and both methods do not require many mathematical operations, are easy to apply, and give effective results. As compared with other Kudryashov-based methods, these methods do not focus on obtaining a great number of solution functions. In this respect, it provides great advantages in obtaining dark, bright, kink, and singular soliton types, which are considered basic.

In regard to the utilized method [20], the solution is given as:

$$U(\xi)=\frac{\sum_{i=0}^{n}a_iR^i(\xi)}{\sum_{j=0}^{m}b_jR^j(\xi)}, \tag{7}$$

where a_i, b_j are real coefficients to be determined later, such that a_n, b_m together can not take the value zero and m, n are called positive integer balance numbers. Besides $R(\xi)$ is the following function:

$$R(\xi)=\frac{1}{\sqrt{\chi-\kappa e^{2\sigma\xi}}}, \tag{8}$$

where $R(\xi)$ implements the following structure:

$$\frac{dR(\xi)}{d\xi}=\sigma\left(\chi R(\xi)^3-R(\xi)\right). \tag{9}$$

In Eqs. (8), (9), χ,κ and σ are arbitrary real values. If we pay attention to Eqs. (7), (9) and take advantage of the balance rule between the terms U'', U^5 in Eq. (5), we attain $n=m+1$. For ease of operation, by choosing $m=1$ we get $n=2$ and Eq. (7) reduces to

$$U(\xi) = \frac{a_0 + a_1 R(\xi) + a_2 (R(\xi))^2}{b_0 + b_1 R(\xi)}. \tag{10}$$

If Eqs. (10), (6) and (9) are substituted into Eq. (5), we gain a polynomial form which consists various power of $R(\xi)$. The solution of this structure requires that the coefficient of each power of R be set to zero. The result of this gives the following system:

R^0 **coefficient:**

$$a_0\left(\Omega_1 b_0^4 + \gamma_1 a_0^2 b_0^2 + \gamma_2 a_0^4\right) = 0.$$

R^1 **coefficient:**

$$\left(\Omega_2\sigma^2 - \Omega_1\right)a_1 b_0^4 - b_1 a_0\left(\Omega_2\sigma^2 + 4\Omega_1\right)b_0^3 - 3a_0^2 a_1 b_0^2\gamma_1 - 2a_0^3 b_0 b_1\gamma_1 - 5\gamma_2 a_0^4 a_1 = 0.$$

R^2 **coefficient:**

$$\begin{aligned}&\left(4\Omega_2\sigma^2 - \Omega_1\right)a_2 b_0^4 + \left(\Omega_2\sigma^2 - 4\Omega_1\right)b_1 a_1 b_0^3 - a_0\left(\left(\Omega_2\sigma^2 + 6_1\right)b_1^2 + 3\left(\gamma_1\left(a_0 a_2 + a_1^2\right)\right)\right)b_0^2\\&-6a_0^2 a_1 b_0 b_1\gamma_1 - \left(b_1^2\gamma_1 + 5\gamma_2\left(a_0 a_2 + 2a_1^2\right)\right)a_0^3 = 0.\end{aligned}$$

R^3 **coefficient:**

$$\begin{aligned}&\left(4\left(\chi\sigma^2\Omega_2 a_0\right)\right)b_1 b_0^3 - a_1\left(\left(\Omega_2\sigma^2 + 6\Omega_1\right)b_1^2 + \gamma_1\left(6a_0 a_2 + a_1^2\right)\right)b_0^2 - \left(11\Omega_2\sigma^2 - 4\Omega_1\right)a_2 b_1 b_0^3\\&-4a_1\chi\sigma^2\Omega_2 b_0^4 + b_1\left(\left(\Omega_2\sigma^2 - 4_1\right)b_1^2 + 6\gamma_1\left(a_0 a_2 + a_1^2\right)\right)a_0 b_0 - a_0^2 a_1\left(3b_1^2\gamma_1 + 10\gamma_2\left(2a_0 a_2 + a_1^2\right)\right) = 0.\end{aligned}$$

R^4 **coefficient:**

$$\begin{aligned}&a_0\left(\Omega_2\sigma^2 - \Omega_1\right)b_1^4 + \left(\left(\Omega_2\left(8\chi a_0 + 11a_2\right)\sigma^2 - 6a_2\Omega_1\right)b_0^2 - 3\gamma_1 a_0\left(a_0 a_2 + a_1^2\right)\right)b_1^2\\&-\left(\Omega_2\sigma^2 + 4\Omega_1\right)a_1 b_0 b_1^3 - \left(8\left(\chi\sigma^2\Omega_2 b_0^2\right) + \gamma_1\left(12a_0 a_2 + 2a_1^2\right)\right)a_1 b_0 b_1 - 12a_2\chi\sigma^2\Omega_2 b_0^4\\&-3a_2\gamma_1\left(a_0 a_2 + a_1^2\right)b_0^2 - 5a_0\gamma_2\left(2a_2^2 a_0^2 + 6a_2 a_0 a_1^2 + a_1^4\right) = 0.\end{aligned}$$

R^5 **coefficient:**

$$\begin{aligned}&-a_1\Omega_1 b_1^4 + \left(\left(4\chi a_0 + 5a_2\right)\Omega_2\sigma^2\right)b_0 b_1^3 - \left(4a_2\Omega_1\right)b_0 b_1^3 - \left(4\chi\sigma^2\Omega_2 b_0^2 + \gamma_1\left(6a_0 a_2 + a_1^2\right)\right)a_1 b_1^2\\&-3b_0\left(\sigma^2\chi\Omega_2\left(\chi a_0 + 12a_2\right)b_0^2\right)b_1 + \left(6a_2\gamma_1\left(a_0 a_2 + a_1^2\right)\right)b_1\\&+\left(3\chi^2\sigma^2\Omega_2 b_0^4 - 3a_2^2\gamma_1 b_0^2 - \left(30a_2^2 a_0^2 + 20a_2 a_0 a_1^2 + a_1^4\right)\gamma_2\right)a_1 = 0.\end{aligned}$$

R^6 **coefficient:**

$$\begin{aligned}&\left(-10a_0^2\gamma_2 - b_0^2\gamma_1\right)a_2^3 + \left(-30a_0 a_1^2\gamma_2 - 3a_0 b_1^2\gamma_1 - 6a_1 b_0 b_1\gamma_1\right)a_2^2 + \left(\left(\Omega_2\sigma^2 - \Omega_1\right)b_1^4\right)a_2\\&\left(\left(-40\chi\sigma^2\Omega_2 b_0^2 - 3\gamma_1 a_1^2\right)b_1^2 + 8\chi^2\sigma^2\Omega_2 b_0^4 - 5\gamma_2 a_1^4\right)a_2 - 7b_1\chi^2\sigma^2 b_0^2\left(a_0 b_1 - a_1 b_0\right)\Omega_2 = 0.\end{aligned}$$

R^7 **coefficient:**

$$\begin{aligned}&-5\chi\sigma^2 b_0\left(\chi a_0 + 4a_2\right)\Omega_2 b_1^3 + \left(5\chi^2\sigma^2\Omega_2 b_0^2 - 3a_2^2\gamma_1\right)a_1 b_1^2\\&+b_0\left(25\chi^2\sigma^2\Omega_2 b_0^2 - 2a_2^2\gamma_1\right)a_2 b_1 - 10a_2^2 a_1\left(2a_0 a_2 + a_1^2\right)\gamma_2 = 0.\end{aligned}$$

R^8 **coefficient:**

$$-\chi\sigma^2\left(\chi a_0 + 4a_2\right)\Omega_2 b_1^4 + a_1 b_1^3\chi^2\sigma^2\Omega_2 b_0 + \left(29\chi^2\sigma^2\Omega_2 b_0^2 - a_2^2\gamma_1\right)a_2 b_1^2 - 5\gamma_2 a_2^3\left(a_0 a_2 + 2a_1^2\right) = 0.$$

R^9 **coefficient:**

$$\left(15b_1^3\chi^2\sigma^2\Omega_2 b_0 - 5\left(a_1 a_2^3\gamma_2\right)\right)a_2 = 0.$$

R^{10} **coefficient:**

$$\left(3\chi^2\sigma^2\Omega_2 b_1^4 - a_2^4\gamma_2\right)a_2 = 0.$$

where $\Omega_1 = \eta\beta_1\beta_2 + \lambda\beta_2^2 + \mu\beta_1^2 + \rho$, $\Omega_2 = \mu\alpha_1^2 + \eta\alpha_2\alpha_1 + \lambda\alpha_2^2$. Eq. (11) is one of the solution of above system:

$$Set_1 : \left\{ \begin{array}{l} \alpha_1 = \alpha_1, a_0 = 0, a_1 = -\frac{\gamma_1\chi\sqrt{3}b_0}{2\sqrt{-\gamma_2\chi\gamma_1}}, a_2 = \frac{\sqrt{-3\gamma_2\chi\gamma_1}b_1}{2\gamma_2}, b_0 = b_0, \\ b_1 = b_1, \rho = \frac{-16(\Omega_1-\rho)\gamma_2+3\gamma_1^2}{16\gamma_2}, \sigma = -\frac{\sqrt{3}\gamma_1}{4\sqrt{\gamma_2\Omega_2}} \end{array} \right\}, \tag{11}$$

in which $\gamma_1, \gamma_2, \chi, \Omega_2 \neq 0$. If we pay regard to the Eq. (10) with Eqs. (3), (8) and Set_1 in Eq. (11), together, the following formula is procured:

$$u_1(x,y,t) = \frac{\frac{a_1}{\sqrt{\chi-\kappa e^{2\sigma(\alpha_1 x+\alpha_2 y-\omega t)}}} + \frac{a_2}{\chi-\kappa e^{2\sigma(\alpha_1 x+\alpha_2 y-\omega t)}}}{b_0 + \frac{b_1}{\sqrt{\chi-\kappa e^{2\sigma(\alpha_1 x+\alpha_2 y-\omega t)}}}} e^{i(-\beta_1 x-\beta_2 y+\rho t+\varphi_0)}, \tag{12}$$

where $\omega = \left(-\alpha_1\beta_2 - \alpha_2\beta_1\right)\eta - 2\mu\alpha_1\beta_1 - 2\lambda\alpha_2\beta_2$ and $\alpha_1, a_1, a_2, b_0, b_1, \rho$ and σ are given in Eq. (11).

4 Description and application of SAEM246 for Eq. (2)

In this section, we give the characteristics and application of SAEM246 (Ozisik et al. 2022). In accordance with the introduced method, we peruse the solution of Eq. (2) as:

$$U(\xi) = \sum_{i=0}^{m} a_i(R(\xi))^i, \quad a_m \neq 0, \tag{13}$$

in which $a_0, a_1, ..., a_m$ are unspecified constants, m refers to balance number and $R(\xi)$ is:

$$R(\xi) = \sqrt{\frac{16e^{2\delta\xi}}{\left(e^{2\delta\xi} + 4\chi_1\right)^2 + 64\chi_2}}, \tag{14}$$

where δ, χ_1 and χ_2 are nonzero real values and $R(\xi)$ obeys to Eq. (15):

$$\left[\frac{dR(\xi)}{d\xi}\right]^2 = \delta^2 R(\xi)^2\left(1 - \chi_1 R(\xi)^2 - \chi_2 R(\xi)^4\right). \tag{15}$$

When utilizing the homogeneous balancing principle between the terms U'', U^5 in Eq. (5) and taking into account Eqs. (13), (15), we obtain $m = 1$. Consequently, Eq. (13) reduces to

$$U(\xi) = a_0 + a_1 R(\xi). \tag{16}$$

By combining Eq. (16) with Eqs. (5), (6) and (15), the following algebraic form is obtained as a result of some mathematical operations:

$$
\begin{aligned}
R^0(\xi) &: a_0\left(\Omega_1 + a_0^4\gamma_2 + a_0^2\gamma_1\right) = 0,\\
R^1(\xi) &: \left(\Omega_2\delta^2 - \Omega_2 - 5a_0^4\gamma_2 - 3a_0^2\gamma_1\right)a_1 = 0,\\
R^2(\xi) &: a_0a_1^2\left(10a_0^2\gamma_2 + 3\gamma_1\right) = 0,\\
R^3(\xi) &: a_1\left(2\delta^2\Omega_2\chi_1 + a_1^2\left(10a_0^2\gamma_2 + \gamma_1\right)\right) = 0,\\
R^4(\xi) &: -5a_0a_1^4\gamma_2 = 0,\\
R^5(\xi) &: -\left(3\delta^2\Omega_2\chi_2 + a_1^4\gamma_2\right)a_1 = 0,
\end{aligned}
$$

in which $\Omega_1 = \eta\beta_1\beta_2 + \lambda\beta_2^2 + \mu\beta_1^2 + \rho,\ \ \Omega_2 = \mu\alpha_1^2 + \eta\alpha_2\alpha_1 + \lambda\alpha_2^2$.

The above system yields Eq. (17):

$$
Set_{2,3} : \left\{ \begin{array}{l} \alpha_1 = \frac{-\delta\eta\alpha_2\chi_1\gamma_2 + \sqrt{-\left(\delta^2\chi_1^2\alpha_2^2\left(4\mu\lambda-\eta^2\right)\gamma_2 + 3\mu\chi_2\gamma_1^2\right)\gamma_2}}{2\mu\gamma_2\chi_1\delta}, \\ a_0 = 0, a_1 = \pm\frac{\sqrt{6\chi_1\gamma_2\chi_2\gamma_1}}{2\chi_1\gamma_2},\ \ \rho = \frac{-4\chi_1^2(\Omega_1-\rho)\gamma_2 - 3\chi_2\gamma_1^2}{4\gamma_2\chi_1^2} \end{array} \right\}, \tag{17}
$$

where $\mu, \gamma_2, \chi_1, \delta \neq 0$. Taking into consideration Eq. (16) with Eqs. (3), (6), (14) and $Set_{2,3}$, together, the following soliton solutions are obtained:

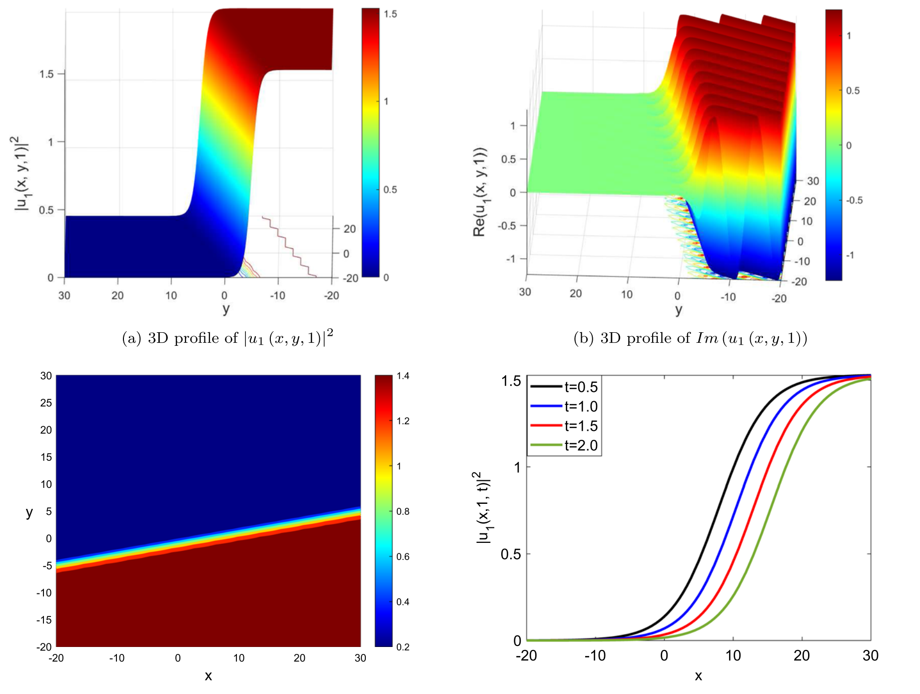
$$
u_{2,3}(x,y,t) = \pm\frac{2}{\chi_1\gamma_2}\left(\frac{6\chi_1\gamma_2\chi_2\gamma_1 e^{2\delta\left(\alpha_1x+\alpha_2y-\omega t\right)}}{\left(e^{2\delta\left(\alpha_1x+\alpha_2y-\omega t\right)} + 4\chi_1\right)^2 + 64\chi_2}\right)^{\frac{1}{2}} e^{i\left(-\beta_1x-\beta_2y+\rho t+\varphi_0\right)}, \tag{18}
$$

herein are the solutions of Eq. (2) and α_1 is defined as in Eq. (17).

5 Result and discussion

In this part, we accomplishedly fulfilled various soliton solutions for the (2+1)-dimensional HFSC model with the parabolic law nonlinearity using Matlab and Maple symbolic computation softwares. 2D, contour and 3D graphics of some solutions have been visualized to depict the soliton dynamics of the obtained solution with Matlab. Figure 1 pertains to the solution function gained as a consequence of the perusal of the Eq. (2) by using the improved generalized Kudryashov method. In order to make the graphical representation of the solution function given in Eq. (12), Set_1 and the appropriate parameter values determined as $\gamma_1 = 1.98$, $\gamma_2 = 0.97$, $\chi = -0.99$, $\alpha_1 = 0.39$, $\alpha_2 = -1.97$, $\beta_1 = -1.28$, $\beta_2 = 0.76$, $\kappa = 2.1$, $\lambda = 1.62$, $\eta = 0.92$, $\mu = -1.17$, $\varphi_0 = 0.41$, $b_0 = 0.75$, $b_1 = 0.78$, $t = 1$ have been chosen. Figure 1a-d represent the 3D diagram of the square modulus of $u_1(x,y,t)$, 3D graph of the imaginary part of $u_1(x,y,1)$, contour view of $\left|u_1(x,y,1)\right|^2$ and 2D representation of $\left|u_1(x,1,t)\right|^2$ for $t = 0.5, 1, 1.5, 2$. On the other hand, Fig. 1a, b and d reflect the kink soliton character which has the wave feature traveling towards the right (from black line to green line) for $\left|u_1(x,y,1)\right|^2$.

In order to graphically express the solution function presented in Eq. (18), Set_2 in Eq. (17) have been used and the parameter selection has been made which is determined as $\gamma_1 = 0.35$, $\gamma_2 = 0.28$, $\chi_1 = 1$, $\chi_2 = 1.4$, $\alpha_1 = 1.3$, $\alpha_2 = 0.37$, $\beta_1 = 0.28$, $\beta_2 = 0.69$, $\lambda = 4.3$, $\eta = 0.2$, $\mu = -1.1$, $\varphi_0 = -0.1$, $\delta = 0.56$, $y = 1$. 3D graphs of the square modulus solution function of $u_2(x,y,t)$ and the imaginary parts of $u_2(x,1,t)$ are indicated by Fig. 2a and



(a) 3D profile of $|u_1(x, y, 1)|^2$

(b) 3D profile of $Im(u_1(x, y, 1))$

(c) $|u_1(x, y, 1)|^2$ in contour

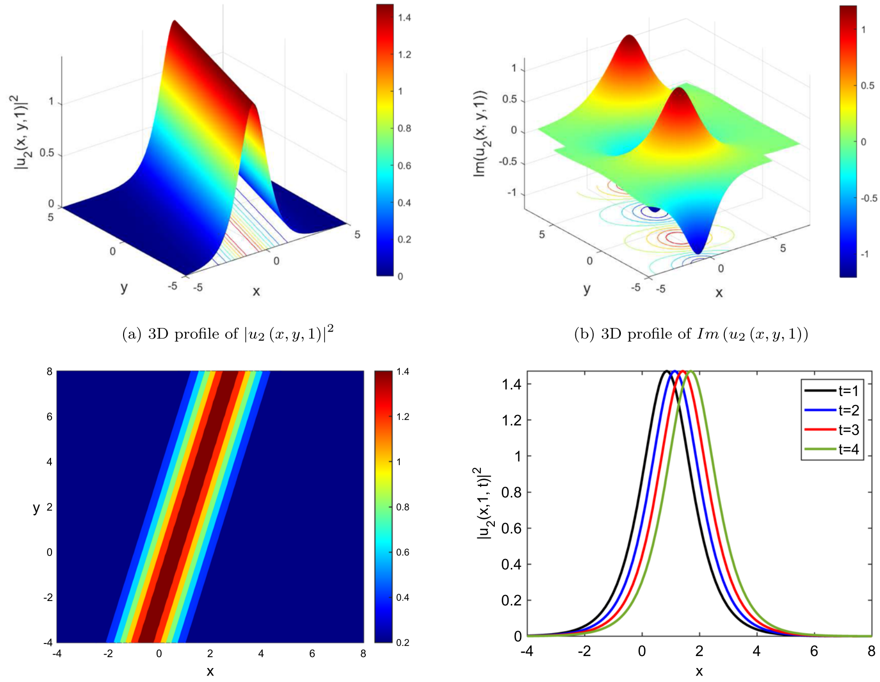
(d) 2D views of $|u_1(x, 1, t)|^2$

Fig. 1 The some profiles of $u_1(x, y, t)$ in Eq. (12) by use of Set_1 in Eq. (11) and $\gamma_1 = 1.98$, $\gamma_2 = 0.97$, $\chi = -0.99$, $\alpha_1 = 0.39$, $\alpha_2 = -1.97$, $\beta_1 = -1.28$, $\beta_2 = 0.76$, $\kappa = 2.1$, $\lambda = 1.62$, $\eta = 0.92$, $\mu = -1.17$, $\varphi_0 = 0.41$, $b_0 = 0.75$, $b_1 = 0.78$, $t = 1$.

b, respectively. Figure 2c and d represent the contour plot of $\left|u_2(x, 1, t)\right|^2$ and 2D view of $\left|u_2(x, 1, t)\right|^2$ for $t = 1, 2, 3, 4$, respectively. On the other hand, Fig. 2a-d represent the bright soliton character which has the wave feature traveling towards the right (from black line to green line) for $\left|u_2(x, 1, t)\right|^2$.

In order to make the graphical representation of the solution function given in Eq. (18), Set_3 in Eq. (17) and the appropriate parameter values determined as $\gamma_1 = -0.35$, $\gamma_2 = 2.28$, $\chi_1 = 14$, $\chi_2 = -1.4$, $\alpha_1 = 1.3$, $\alpha_2 = 0.37$, $\beta_1 = 0.28$, $\beta_2 = -0.69$, $\lambda = -0.3$, $\eta = 0.2$, $\mu = 1.1$, $\varphi_0 = -0.25$, $\delta = 2.56$, $y = 1$ have been chosen. Figure 3a-d portrait the 3D diagram of the square modulus solution of $u_3(x, y, t)$, 3D graph of the imaginary part of $u_3(x, 1, t)$, real part of $u_3(x, 1, t)$ and 2D representation of $\left|u_3(x, 1, t)\right|^2$ for $t = 0.5, 1.5, 2.5, 3.5$, respectively. On the other hand, Fig. 3a-d reflect the singular soliton character which has the wave feature traveling towards the left (from black line to green line) for $\left|u_3(x, 1, t)\right|^2$.

It was previously emphasized that the focal point of the study is the examination of the parabolic form of the HFSC equation. Therefore, Fig. 4 has been generated for the effect of γ_2 in parallel with this purpose. In accordance with this goal, the soliton graphs previously obtained in Figs. 1a, 2a, and 3a have been examined under different γ_2 values. The parameter selections made for the graphs Figs. 1a, 2a, and 3a of the graphs given in Fig. 4 are exactly the same, the only difference is that γ_2 values are selected differently to take the specified values. If Eqs. (1) and (2) are reviewed again, it will be seen that the only



(a) 3D profile of $|u_2(x, y, 1)|^2$

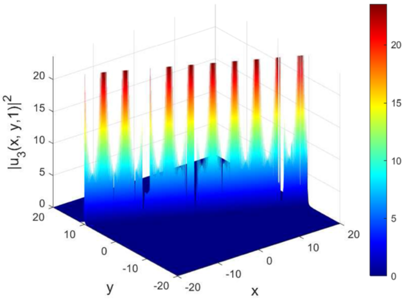
(b) 3D profile of $Im(u_2(x, y, 1))$

(c) $|u_2(x, y, 1)|^2$ in contour

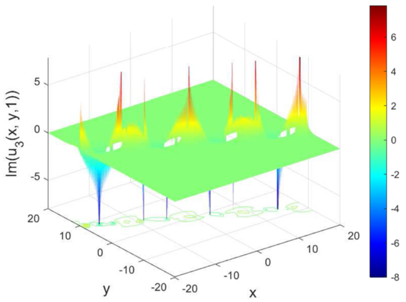
(d) 2D views of $|u_2(x, 1, t)|^2$

Fig. 2 The some diagrammatic presentments of $u_2(x, y, t)$ in Eq. (18) by using Set_2 in Eq. (17) and $\gamma_1 = 0.35$, $\gamma_2 = 0.28$, $\chi_1 = 1$, $\chi_2 = 1.4$, $\alpha_1 = 1.3$, $\alpha_2 = 0.37$, $\beta_1 = 0.28$, $\beta_2 = 0.69$, $\lambda = 4.3$, $\eta = 0.2$, $\mu = -1.1$, $\varphi_0 = -0.1$, $\delta = 0.56$, $y = 1$.

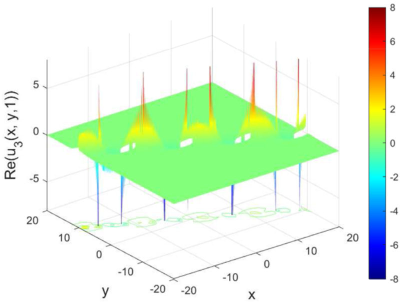
difference is the term with the coefficient γ_2. In this context, if Fig. 4 is interpreted in more detail, it is observed that the kink soliton character of the soliton is preserved due to the changing γ_2 values in Fig. 4a, but the decline in the vertical amplitude due to the increasing γ_2 values, in a sense, maintains the lower flatness of the soliton, while the upper flatness of the soliton is in the downward direction. This movement does not show an increase evenly as the proportionally increasing γ_2 (for example, dark to blue displacement and red to green displacement). A similar examination has been made for the bright soliton character in Fig. 4b, which was previously obtained with Fig. 2a. Here, the same parameter values obtained in Fig. 2a drawings were chosen, but the values 0.25, 0.5, 0.75, and 1.0 were chosen for γ_2, respectively. Based on the heightening values of γ_2, the soliton still continues its bright soliton form, no horizontal movement is observed, but the peak (vertical amplitude) of the soliton shows a movement tendency (decrease) towards the skirts of the soliton. It can be clearly observed from the graph that this amount of movement (decrease) is not proportional to the increasing γ_2 values. A similar situation exists for the graph given in Fig. 4c. Depending on the singularity of the soliton type represented by Fig. 4c, it can also be seen that there is a decline in the horizontal amplitude (it is clearly visible from the throat part). Therefore, it is clear that the obtained form (parabolic law form) added to the Eq. (1) (Kerr law form) with a γ_2 coefficient has a remarkable impact on the soliton behavior.



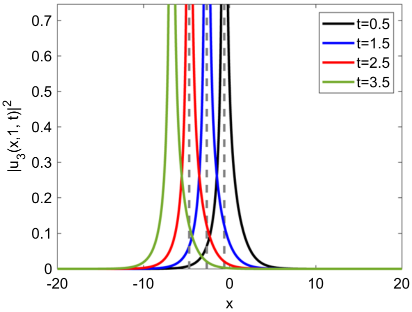
(a) 3D profile of $|u_3\,(x, y, 1)|^2$



(b) 3D profile of $Im\,(u_3\,(x, y, 1))$

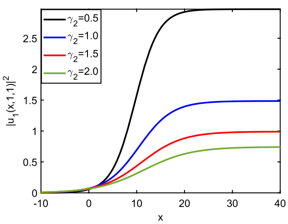


(c) 3D profile of $Re\,(u_3\,(x, y, 1))$

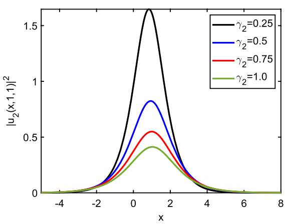


(d) 2D views of $|u_3(x, 1, t)|^2$

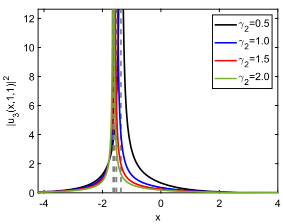
Fig. 3 The some charts of $u_3(x, y, t)$ in Eq. (18) by taking into consideration Set_3 and $\gamma_1 = -0.35$, $\gamma_2 = 2.28$, $\chi_1 = 1$, $\chi_2 = -1.4$, $\alpha_1 = 1.3$, $\alpha_2 = 0.37$, $\beta_1 = 0.28$, $\beta_2 = -0.69$, $\lambda = -0.3$, $\eta = 0.2$, $\mu = 1.1$, $\varphi_0 = -0.25$, $\delta = 2.56$, $y = 1$.



(a) 2D of eq. (12) for γ_2



(b) 2D of eq. (18) for γ_2



(c) 2D of eq. (18) for γ_2

Fig. 4 2D simulations of Eqs. (12), (18) under γ_2 influence, previously presented in Figs. 1a, 2a, 3a

At the end of this section, we would like to add a few more sentences. First point, the main focus of the article is not to acquire particular types of solitons. Our priority is to present the parabolic law nonlinearity form of the (2+1)-dimensional HFSC equation for the first time and to examine this form's capability to produce solitons. In this context, it has been shown that the presented form has the capability to produce bright, kink, singular soliton types. Naturally, the capability to produce other types of solitons can be demonstrated by using other methods. The second priority is to observe the parabolic law nonlinearity effect on soliton dynamics, which is the focus of the presented form.

During the examination of the introduced problem in the article, Maple and Mathematica have been used for all kinds of calculus operations, and Matlab has been used effectively for graphical presentations. The solution functions are given by Eqs. (12) and (18) have been controlled for providing the main equation given by Eq. (2) every time. Besides the parameter choosings made for the graphical presentations have been made by considering the limitations that must be provided as a requirement of both the equation and the method.

6 Conclusion

In this study, by presenting (2+1)-dimensional HFSC model with the parabolic law nonlinearity for the first time, we obtained analytical soliton solutions of this form and examined its effect on the soliton solutions we obtained. Kink, bright, and singular soliton solutions are gained based on SAEM246. In the meantime, the kink soliton solution is derived by the help of the improved generalized Kudryashov method (iGKM). However, in line with the methods preferred within the scope of the article, this requirement has been removed and the applied methods have yielded effective results. In order that interprets the physical structure of the attained soliton solutions, we displayed a variety of graphs with appropriate parameters. Also, as shown for the first time by this study, the parabolic form has a significant effect on soliton dynamics. Obtaining other types of solitons that the form presented in the article can produce, the interaction of these solitons, working by constituting fractional forms of the presented equation, and working on models with similar structures are the topics that can be studied in the future. We believe that the results of the problem examined in this study and the applied methods will be useful for all researchers related to nonlinear wave dynamics without being limited to this discipline.

Author Contributions All parts contained in the research carried out by the authors through hard work and a review of the various references and contributions in the field of mathematics and Applied physics.

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Data availibility Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest This research received no specific grant from any funding agency in the public, commercial or not-for-profit sectors. The authors did not have any competing interests in this research.

Ethical approval The Corresponding Author, declare that this manuscript is original, has not been published before, and is not currently being considered for publication elsewhere. The Corresponding Author confirm that the manuscript has been read and approved by all the named authors and there are no other persons who satisfied the criteria for authorship but are not listed. I further confirm that the order of authors listed in the manuscript has been approved by all of us. we understand that the Corresponding Author is the sole contact for the Editorial process and is responsible for communicating with the other authors about progress, submissions of revisions, and final approval of proofs.

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