

Combination of Graph Reduction and Tie-set Techniques for Network Reliability Assessment

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Abstract—The reliability of the communication network is used as one of the quality of service factors defining the availability and the resilience of the operating networks. Many classical algorithms exist but most of them fail to be applied for real-time reliability assessment in complex networks where the calculation load is increased in function of the complexity. In the present work, a new algorithm is proposed to resolve the problem of the reliability evaluation for all kinds of networks (simple and complex). The algorithm is based on the combination of two classical reliability methods giving an efficient hybrid algorithm. Graph transformation method and tie sets methods are fused into multi stages algorithm. The network passes through many simplification layers based on series, parallel, and edge factoring graph transformation to transform a complex network into a simpler network. The simplified topology is then used as input for a classical algorithm; here the tie sets for the two-terminal reliability evaluation. A simulation is performed on a 11-node random complex network to compare the performances of the new algorithm to the existing algorithms. The results of the proposed algorithm are compared to those resulting from the use of tie sets algorithm in term of computing time. The implementation of the algorithms by MATLAB shows a noticeable improvement in the time for reliability calculation. The improvement depends on the selected commodity as source-destination pair. For example, the network under simulation shows an improvement in time computing compared to classical tie-set algorithm of 550% for the commodity (2-3). Also, there is no limitation on the size and complexity of the network.

Keywords— *Network Reliability, Graph Reduction, tie-set*

I. INTRODUCTION

The reliability is the probability that a system will function properly within a specified period of time under a number of constraints like failure rate, system complexity, and operating conditions [1]. This means reliability is a quality measurement over time, affected by the system performances and its environment. The communication networks reliability is an important Quality of Service (QoS) factor in the study of the system performances especially those based on heterogeneous networks. It checks how long the infrastructure of a network is operational without interruption providing a functionality capability indication. Many technologies rely on reliability calculations in communication networks to ensure that information is sent and received with good reliability. A reliable network contains redundant path between the source node and destination node, which means that in the case of any link failure, it can be replaced by a backup link giving an alternate source-destination path. Network topological

presentation using graph theory is the main factor in the network reliability researches.

Communications networks are becoming more and more complex because of the development of modern technologies, applications, and services. Various classical probabilistic methods are used for network reliability estimation such as Graph Reduction Method (GRM) [2], State Space Enumeration Method (SSEM) [3], Tie-Sets [4], Cut-Sets [5], and approximation methods [6, 7]. The reliability assessment for complex networks requires a complicated computing operation due to the complexity and availability of many possible paths between various nodes. Therefore, the classical computational methods fail to evaluate the reliability under multiple operational conditions.

New researches based on the use of Artificial Neural Network [8], and clustering technique [9] have been proposed to simplify the computation for complex networks. The clustering of communication nodes provides a mean of simplification by partitioning one complex network into many smaller, interconnected sub-network where the application of classical algorithms can be performed. Hybrid algorithms composed of selected classical methods have been proposed for complex communication reliability assessment. Usually, the algorithm starts by simplifying the original network by clustering or graph transformation before applying a second classical algorithm on the simplified analogous network [10]. The two-terminal reliability evaluation between a source node n_s and a destination node n_d is presented in [11]. The general algorithm is presented as a holistic technique based on two sub-algorithms that provide a simple calculation of the minimum cut-sets for complex networks. Other researches are interested in the evaluation of all-terminal (overall network) reliability. The two-terminal algorithm is modified to consider all terminal pairs such that a set of operational edges provides communication paths between every pair of nodes (n_i, n_j with $n_i \neq n_j$) [12]. The problem of k-terminal reliability, where only part of the network is considered, is presented in [13]. The k-terminal reliability is the general method varying between two-terminal to all-terminal methods.

The proposed algorithm is implemented using three simplification stages based on graph reduction technique followed by the application of tie set algorithm. The article is organized as follows: the theoretical backgrounds of network modeling, tie-set method and GRM are presented in Section II; the proposed algorithms are presented in Section III; a case study of simple network is implemented, and simulation results from the developed algorithm is compared to those

from classical algorithm in Section IV; Section V concludes the article.

II. THEORITICAL BACKGROUND

A. Network Topology Modeling

A Communication network is modeled by a graph $G = (N, L)$, where N is the set of communication nodes while L is the set of undirected/ bi-directed links. Node and link individual reliabilities obey to the binomial distribution law where each element has two possible states, either success (working) with probability p or failure with the complement probability $q = 1 - p$. The event-state of a network component (working or failed) is considered independent from the other components event-state. The probability describes the evaluation of functionality and performance of links and communication nodes depending on the manufacturing technology and are included in the production specifications.

A 3-dimension matrix $M[N, N, K]$ describing the network topology where K is the maximum number of parallel links between two nodes. The matrix element $M[i, j, x]$ represents the probability of the x^{ieme} unidirectional link connecting node-pair (n_i, n_j) . M is a multi-layer matrix where the first layer shows the basic network connectivity and the x^{ieme} layer describes the network topology composed of the connectivity between node-pair having x parallel links between them with $1 \leq x \leq K$. If there is no link between two nodes, the corresponding element in M is set to zero.

B. Graph Transformation Technique

Complex networks can be simplified into a simpler analogous network by a series of graph transformations. Series/Parallel [1], delta-to-star [14], and edge factoring [15] are considered among the most popular transformations. Series transformation is the most widely used due to its frequent presence in the functional networks. If three nodes are connected in series as shown in fig. 1-a, then the node 2 can be removed, and the new link reliability is calculated by:

$$R_{13} = P_{12} \times P_{23}, \quad (1)$$

Parallel simplification is also a frequent simplification when there are two or more links in parallels between two nodes as in fig. 1-b. Consider $q_i = 1 - p_i$, as the complement of the probability, then these links are replaced by one link with reliability equal to:

$$R_{12} = 1 - \prod_{i=1}^n (1 - p_i) = 1 - \prod_{i=1}^n q_i \quad (2)$$

The edge factoring is more complex than the previous ones since it demands to consider two cases, then summing them in the final equation to evaluate the network reliability. This graph partitioned into two subgraphs as shown in fig. 2 according to the state of the link between the nodes i_1 and i_2 . The application of the law of probability twice for the A-graph and B-graph and integrating the solutions in one equation gives the final solution. If the link $i_1 - i_2$ is up ($p_5 = 1$), then the topology is reduced to A-graph:

$$P(A) = (1 - q_1 q_2)(1 - q_3 q_4) \quad (3)$$

Considering link $i_1 - i_2$ is down ($p_5 = 0$), B-graph gives:

$$P(B) = 1 - (1 - p_1 p_3)(1 - p_2 p_4) \quad (4)$$

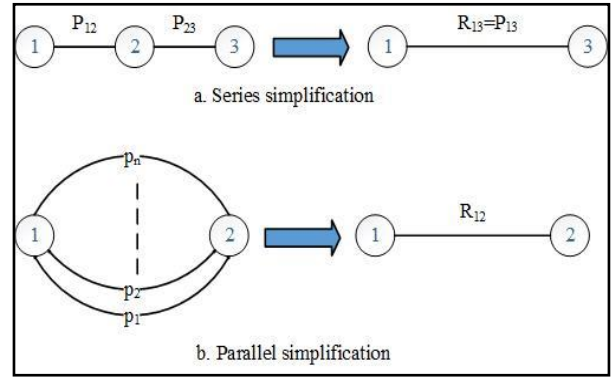


Fig. 1 Series-parallel simplification

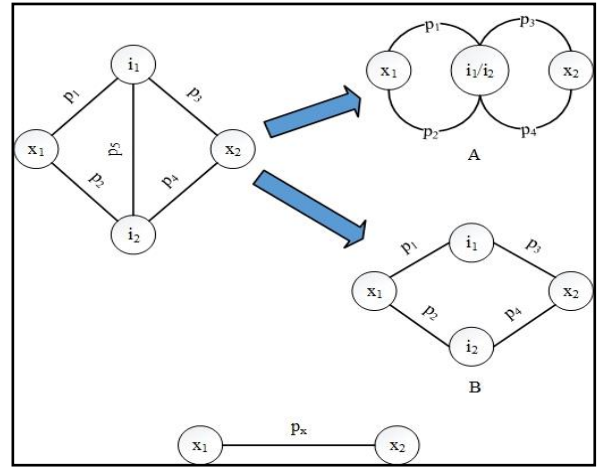


Fig. 2. Edge factoring simplification

Using probability theory and combining “(3)” and “(4)” the equivalent reliability between nodes $x_1 - x_2$ becomes:

$$R_{x_1-x_2} = p_5 P(A) + q_5 P(B) \quad (5)$$

C. Tie-set Technique

The tie-set technique is based on enumerating all the links groups forming a loop-free path between source nodes n_s and the destination node n_d called tie sets. Therefore, this method is applied only to simple and medium networks because the enumeration process increases exponentially with the network complexity. Tie-set algorithm is an exact solution used to evaluate two-terminal, K-terminal, and all-terminal network reliability. It starts by finding all tie sets $(T_1, T_2, \dots, T_i)$ between a source-destination pair and applying the expansion equation (Poincare):

$$R_{sd} = P(T_1 + T_2 + \dots + T_i) \quad (6)$$

Where R_{sd} is the reliability between n_s and n_d .

Tie sets $(T_1, T_2, \dots, T_i)$ are not disjoint link groups because they have common links. Thus, it requires the expansion of “(6)” into:

$$R_{sd} = P[(T_1) + P(T_2) + \dots P(T_i)] - [P(T_1 T_2) + P(T_1 T_3) + \dots P(T_r T_k)_{r \neq k}] + [P(T_1 T_2 T_3) + \dots]$$

$$P(T_1 T_2 T_4) + P(T_r T_k T_j)_{r \neq k \neq j} + \dots + (-1)^{i-1} [P(T_1 T_2 T_3 \dots T_i)] \quad (7)$$

III. THE PROPOSED ALGORITHM

The developed algorithm consists of two stages: the first is the simplification using GRM, and the second is the application of the tie-set. The application of the algorithm starts by the input network topology presented by the matrix (M). GRM stage begins by the application of the parallel simplification, then the series reduction, and finishes by applying the edge factoring sub-algorithm. The last stage is the evaluate the reliability evaluation using tie-set algorithm.

A. Initialization

The network topology is presented as 3D M-matrix describing the connectivity state of the network. The elements of M are the working probability of an edge or a vertex defined by their corresponding reliability:

$$M_{ijk} = \begin{cases} M[i, j, k] & \text{link between } i - j \\ 0 & \text{No link between } i - j \\ M[i, i] & \text{Probability of node } (i) \end{cases} \quad (8)$$

Where the third dimension defining the link in parallel between i - j nodes.

B. Parallel Reduction Algorithm

The developed algorithm starts by removing links in parallel as presented in fig 3. The step begins with a consideration of all links in parallel by checking the values of the element $M[i, j, k]$ for $k > 1$ only. After localization of parallel form, “(2)” is applied for simplification. The simplification continues until all links in parallel are removed. M becomes with all elements null for $k > 1$, however, it is kept as 3D matrix for future simplification (new born parallel links after series simplification).

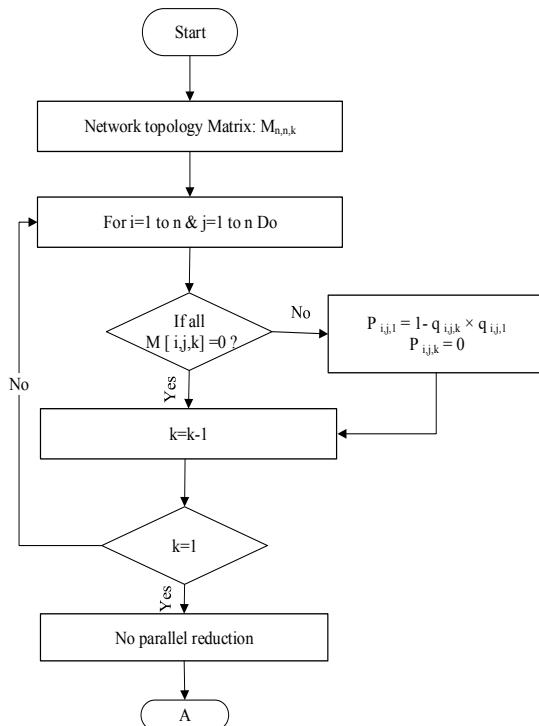


Fig. 3 Parallel reduction sub-algorithm

C. Series Reduction Algorithm

Series node reduction begins after the parallel sub-algorithm by the recognition of nodes in series, considering cases where a node is connected to only two other nodes as depicted in fig. 4. This is carried out by check node connectivity from M, and then the application of “(1)” for removing the redundant nodes. This process is applied to all nodes except n_s and n_d which are unreducible.

D. Edge Factoring Algorithm

As in the parallel and series sub-algorithm, the edge factoring algorithm starts by the recognition of the topological form and then applying “(5)” for simplification as presented in fig. 5. Nodes n_s and n_d are excluded from this simplification as for series simplification. The recognition of the edge form is accomplished through many steps by verifying the connectivity as in M. After confirmation of the topology-shape searched for, the adjoining nodes are subject to node simplification procedures by removing two mid nodes n_i and n_j updating the M elements values and setting the diagonal elements $M_{ii,1}$ and $M_{jj,1}$ to zero.

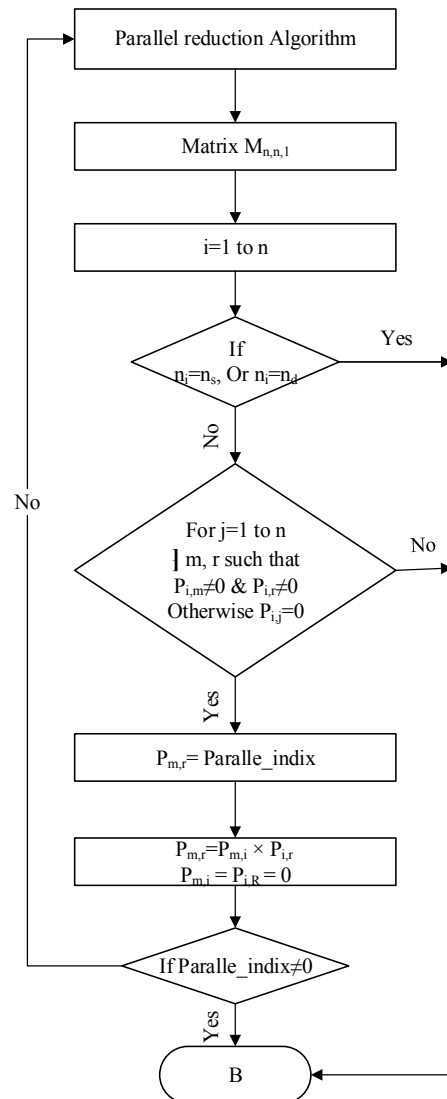


Fig. 4 Series reduction sub-algorithm

E. Algorithm Application

The application of the GRM is performed by step starting from parallel reduction, series, checking again for possible newly born parallel form, edge factoring. The process is repeated till a simplified analogous network topology is reached (fixed by program as links or nodes number). The Matrix M is reduced in dimensions beginning from its original 3D form with size $(N \times N \times K)$ to a 2D matrix with size $(R \times R)$ with $R \ll N$. The original M is transferred by preserving its main ingredients which make the two matrices exactly analogous but given in two forms. The well-known tie-set algorithm is then applied for two-terminal

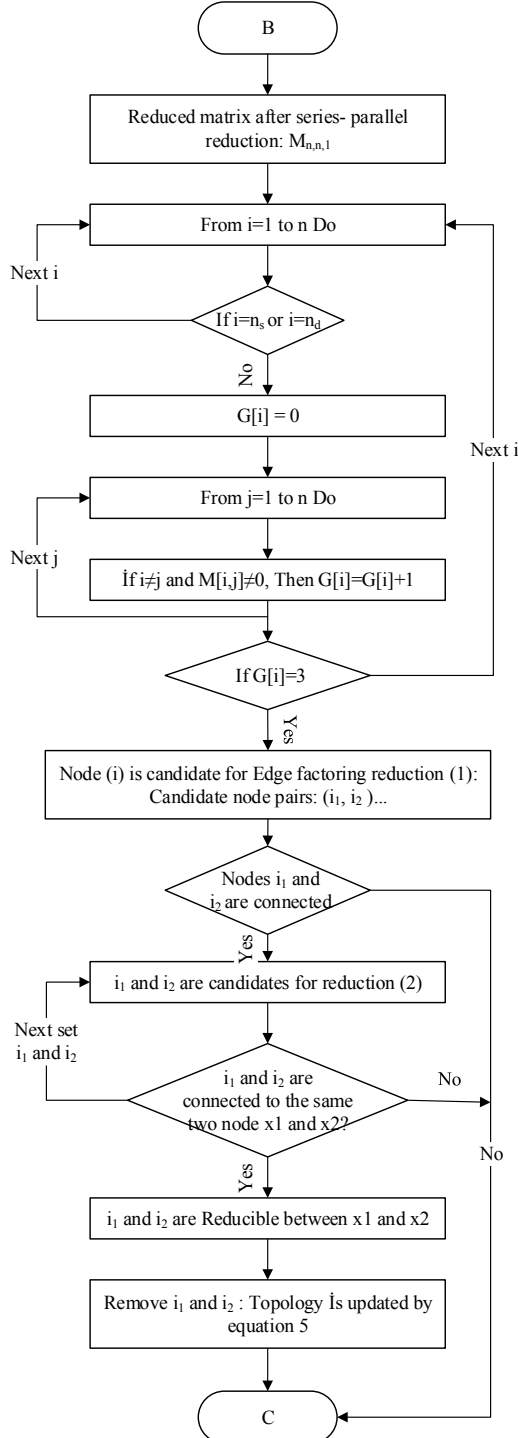


Fig. 5 Edge factoring sub-algorithm

reliability calculation. All loop-free paths between n_s and n_d are found which represent the tie set link-group, and “(7)” is applied to determine the exact value of the probability.

IV. RESULTS AND DISCUSSION

To validate the new algorithm, the 11-node network in the fig. 6 is implemented by MATLAB. Nodes are considered as perfects (reliability= 100%), and all links are bidirectional with probability ($P_{ik} = 0.9$). The two-terminal reliability is calculated by both the classical tie-set and the developed algorithm. Results in the table I shows that the reliability values, taken as output, are the same for both algorithms which verify the exactness of the new algorithm. The computing time shows a noticeable improvement of the new algorithm compared to the classical tie-set algorithm. This can be explained by the topology simplification performed by the developed GRM before the application of tie-set. The superiority of the developed algorithm is expected to be more clear for complicated networks. Also by the GRM simplification, the new algorithm can be applied to all network topology while the classical tie-set can be applied only for simple networks with $N \leq 20$ nodes.

V. CONCLUSION

The difficulty of network reliability evaluation depends directly on the its complexity. Classical evaluation algorithms like tie-set give exact solution but can require long computing time for complex network. In this work, a hybrid technique for reliability evaluation is proposed. It is based on a combined successive procedures using GRM and tie-set algorithms. GRM starts by simplifying the topology by a structured application of parallel, series, and edge factoring techniques. The topology is transformed to simple one regardless of complexity of the original network. Finally, an exact solution is found by the application of the tie-set algorithm to the simplified topology. As a final stage of calculation, any classical exact method can be used as part of this hybrid algorithm at the place of tie-set algorithm. A case study of a 11-node random network is used to validate the developed algorithm. By reducing the complexity of the network, the same reliability evaluation value is obtained with a much better computing time. Depending on the source-destination nodes, the evaluation time is decreased to a value equal to 20% of the time required for tie-set method. This improvement can be huge for more complicated case studies.

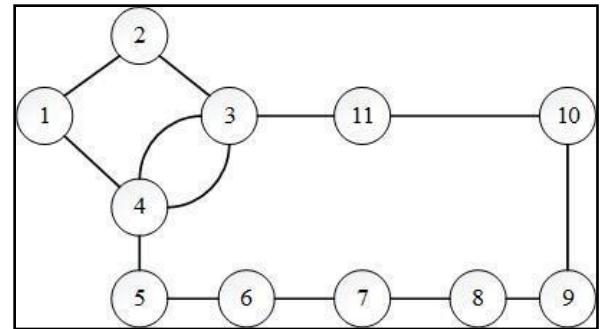


Fig 6. Example: 11-node network

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Source/ Destination		New Algorithm		Classical tie-set algorithm	
n_s	n_d	R_{sd}	$T(sec)$	R_{sd}	$T(sec)$
1	2	0.9877	0.278000	0.9877	0.386711
1	3	0.9876	0.360643	0.9876	0.337407
1	4	0.9877	0.070094	0.9877	0.312645
1	5	0.9582	0.165634	0.9582	0.395695
1	6	0.9462	0.171653	0.9462	0.355118
1	7	0.9460	0.157285	0.9460	0.339759
1	8	0.9531	0.163187	0.9531	0.340648
1	9	0.9547	0.152618	0.9547	0.365134
1	10	0.9810	0.141991	0.9810	0.341206
1	11	0.9747	0.152074	0.9747	0.360114
2	3	0.9957	0.054248	0.9957	0.301098
2	4	0.9952	0.053762	0.9952	0.272399
2	5	0.9657	0.161163	0.9657	0.295380
2	6	0.9539	0.143847	0.9539	0.304391
2	7	0.9537	0.136448	0.9537	0.291275
2	8	0.9612	0.174797	0.9612	0.299286
2	9	0.9629	0.134323	0.9629	0.287770
2	10	0.9897	0.059673	0.9897	0.277919
2	11	0.9851	0.069975	0.9851	0.270368
3	4	0.9992	0.054306	0.9992	0.332738
3	5	0.9693	0.139933	0.9693	0.357934
3	6	0.9573	0.135766	0.9573	0.317370
3	7	0.9571	0.134853	0.9571	0.334702
3	8	0.9643	0.139210	0.9643	0.330254
3	9	0.9659	0.158596	0.9659	0.333649
3	10	0.9926	0.058455	0.9926	0.333547
3	11	0.9844	0.132820	0.9844	0.347534
4	5	0.9698	0.072435	0.9698	0.281501
4	6	0.9574	0.084149	0.9574	0.290915
4	7	0.9572	0.067408	0.9572	0.284159
4	8	0.9641	0.071134	0.9641	0.293724
4	9	0.9657	0.073020	0.9657	0.327185
4	10	0.9920	0.051816	0.9920	0.283865
4	11	0.9838	0.147329	0.9838	0.290767
5	6	0.9698	0.206428	0.9698	0.236969
5	7	0.9679	0.085324	0.9679	0.234377
5	8	0.9572	0.088975	0.9572	0.228494
5	9	0.9571	0.110107	0.9571	0.223072
5	10	0.9657	0.213340	0.9657	0.226845
5	11	0.9562	0.234195	0.9562	0.246202
6	7	0.9900	0.063522	0.9963	0.202574
6	8	0.9679	0.085406	0.9679	0.197952
6	9	0.9661	0.089928	0.9661	0.209847
6	10	0.9571	0.067579	0.9571	0.200371
6	11	0.9462	0.161715	0.9462	0.241970
7	8	0.9698	0.082619	0.9698	0.192564
7	9	0.9679	0.084962	0.9679	0.201259
7	10	0.9572	0.065704	0.9572	0.200632
7	11	0.9461	0.160539	0.9461	0.221014
8	9	0.9900	0.060175	0.9963	0.204609
8	10	0.9679	0.069408	0.9679	0.214387
8	11	0.9551	0.140828	0.9551	0.230198
9	10	0.9698	0.065825	0.9698	0.234026
9	11	0.9569	0.142071	0.9569	0.255761
10	11	0.9851	0.070863	0.9851	0.312101

TABLE I. SIMULATION RESULTS