



Performance analysis of majority-based transmit antenna selection and maximal ratio combining in MIMO-NOMA networks

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Abstract

Non-orthogonal multiple access (NOMA) is paramount in modern wireless communication systems since it enables efficient multiple access schemes, allowing multiple users to share the same spectrum resources and thus improving overall network capacity. Multiple-input multiple-output (MIMO) technology is crucial in wireless communication as it leverages multiple antennas to enhance data throughput, increase link reliability, and mitigate signal interference, resulting in improved communication performance. The combination of MIMO and NOMA represents a transformative synergy that harnesses the benefits of both technologies, facilitating efficient spectrum utilization, higher data rates, and improved reliability in wireless networks. This makes it particularly valuable in the fifth-generation (5G) era and beyond. This paper investigates the performance of majority-based transmit antenna selection and maximal ratio combining (TAS-maj/MRC) in MIMO-NOMA networks. We derive a closed-form expression for the exact bit error rate (BER) for binary phase shift keying (BPSK) modulation in Nakagami- m fading channels. Moreover, asymptotic expressions are obtained in the high signal-to-noise ratio (SNR) region to get further insight into the BER behavior of the system. Finally, we verify the analytical results' accuracy through simulations. The results demonstrate that diversity and code gains are achieved. In addition, the BER performance is significantly improved as the number of receive antennas increases or channel condition enhances.

Keywords Non-orthogonal multiple access (NOMA) · Multiple-input multiple-output (MIMO) · Majority-based transmit antenna selection and maximal ratio combining (TAS-maj/MRC) · Bit error rate (BER) · Binary phase shift keying (BPSK) · Nakagami- m

1 Introduction

Over the past few years, non-orthogonal multiple access (NOMA) has garnered substantial interest for its potential to enhance the next generation of wireless communications (WCs) due to its ability to provide greater spectral efficiency, connectivity, and reduced latency [1]. In NOMA, multiple users can simultaneously use the same spectrum resources,

with power-domain NOMA taking advantage of superposition coding (SC) at the transmitter and successive interference cancellation (SIC) at the receiver to multiplex users in the power domain [2].

Multiple-input multiple-output (MIMO) technology can potentially increase wireless system capacity while decreasing error probability [3]. MIMO systems employ antenna diversity techniques to mitigate the effects of fading channels. Although these techniques can improve data capacity and system performance, they also increase hardware complexity and power consumption due to using multiple radio frequency (RF) chains for antennas. Nevertheless, antenna selection (AS) schemes such as receive antenna selection (RAS), transmit antenna selection (TAS), joint transmit and receive antenna selection (JTRAS), and TAS/maximal ratio combining (TAS/MRC) can overcome these limitations while maintaining the benefits of multi-antenna systems with fewer RF chains [4].

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1.1 Related works

Recently, various works have studied the diversity techniques in the MIMO-NOMA networks. Particularly, the authors in [2] examined the outage probability (OP) and ergodic sum-rate performance for the NOMA network with the maximal ratio combining (MRC). The authors in [5–10] and [11–15] studied the performance of the NOMA networks, in which maximum ratio transmission (MRT), Alamouti space-time block coding (STBC), orthogonal STBC, and transmit antenna selection (TAS) are employed, respectively. The selection criteria of the TAS scheme were applied differently in [11–15]. Specifically, in [11], the ergodic performance is investigated for the NOMA network with the TAS that offers the highest sum rate. However, in [12], the sum rate of the NOMA network is optimized with the TAS that considers the fairness between the users. On the other hand, in [13–15], the secrecy OP performance of the NOMA networks is examined with TAS considering the security between users. The OP or sum rate or bit error rate (BER) performance is analyzed for the NOMA network with the suboptimal hybrid AS schemes such as majority-based TAS (TAS-maj)/MRC and majority-based joint transmit and receive antenna selection (JTRAS-maj) in [16] and joint AS, max-max-max (A^3), and max-min-max (AIA) in [19–21]. The authors of [22] investigated the performance of an amplify-and-forward (AF) relay-aided coordinated direct and relay transmission (CDRT) protocol in a downlink NOMA-based Internet-of-Things (IoT) network, in which the source shares the direct as well as the relayed links to the IoT near-user (NU) and the far-user (FU). In [23], the authors investigated the performance of a new framework for low-outage downlink NOMA using a CDRT scheme with direct links to the NU and FU. The authors of [24] demonstrated that despite the power constraints on all transmit nodes caused by the interference temperature limit (ITL) imposed by the primary user, power-domain cooperative NOMA can ensure much higher throughput in underlay cognitive radio networks than conventional OMA. In [25], the authors showed that combining the signals at NU and FU can significantly enhance the throughput of the CDRT-based NOMA network. In recent years, limited studies have concentrated on examining the error rate performance [26–38]. On the other hand, there are also many works using the diversity techniques in the cooperative MIMO-NOMA networks [39–51].

1.2 Motivations

While prior research has made valuable contributions to NOMA systems, we have identified some noteworthy observations that we would like to highlight.

- It is important to note that some studies mentioned in [26–38] evaluate networks under Rayleigh fading. However, the Nakagami- m fading model is more suitable for fitting empirical data than the Rayleigh fading model and can model a wide range of fading channel conditions that include Rayleigh fading as a particular case when $m = 1$.
- It should be highlighted that the studies discussed in [26–38] mainly focus on examining the error rate performance of single-input single-output (SISO)-NOMA networks and do not explore the error rate performance of MIMO-NOMA networks. However, it is essential to note that the performance of NOMA can be significantly improved by leveraging MIMO technology and AS schemes.
- Despite applying some AS schemes in NOMA networks [11–20], finding an optimal transmit AS solution remains challenging. For instance, the TAS scheme used in [11–15] does not select the best transmit antenna that provides the maximum channel gain between the base station and users. Instead, it selects the best antenna based on the highest sum rate [11], fairness between users [12], or security between users [13–15]. As a result, the optimal TAS solution has not yet been achieved in NOMA networks. Since finding an optimal AS solution may not be feasible in NOMA networks, some researchers suggest sub-optimal AS schemes [16–18], [47], and [19, 20]. However, the majority-based AS schemes proposed in [16–18] and [47] improve the performance for more than half of the users. In contrast, the sub-optimal AS schemes proposed in [19, 20] enhance the performance of only the NU or FU.
- The MRC scheme is a well-known diversity technique that can provide both diversity and array gains and can be employed on the receiver side. Despite [37] studies the BER for MIMO-NOMA network with JTRAS-maj, combining the TAS-maj scheme with the MRC scheme can achieve higher performance [16–18] and [47]^{1,2}. Thus, in this paper, we adopt the TAS-maj/MRC scheme.

¹ In [16–18] and [47], the authors investigated the OP performance for single-hop and dual-hop MIMO-NOMA networks, respectively. Nevertheless, in our work, we investigate the BER performance for single-hop MIMO-NOMA networks.

² Note that the OP focuses on the reliability of the communication link, addressing the probability of service falling below a certain quality threshold. At the same time, the BER is concerned with the accuracy of data transmission, quantifying the probability of bit errors in the received data. These metrics are used in different contexts and are important for different communication system design and evaluation aspects. In practice, both metrics may be considered together to assess the overall performance of a communication system, especially in scenarios where both reliability and data accuracy are critical.

1.3 Contributions

This paper aims to address the issues mentioned above and explore the BER performance of the downlink MIMO-NOMA network using the TAS-maj/MRC scheme. The following are the main contributions of this study:

- We investigate the BER performance for binary phase shift keying (BPSK) in the downlink MIMO-NOMA network with the TAS-maj/MRC scheme under the Nakagami- m fading channel model.
- We derive an exact expression for the BER in closed form.
- To get further insight into the BER behavior of the system, we also conduct an asymptotic analysis in high SNR regions to evaluate the diversity and array gains achieved by the TAS-maj/MRC scheme.
- Finally, we verify the accuracy of the theoretical results by conducting Monte Carlo simulations. The results show that the BER performance is significantly improved as the number of receive antennas increases or channel condition enhances.

2 System model

In Fig. 1, we have a downlink MIMO-NOMA network with one transmitter T_x and multiple receivers R_l ($1 \leq l \leq L$). The T_x and R_l have N_t transmit and N_r receive antennas, respectively. The TAS-maj/MRC scheme is applied between the T_x and R_l such that the transmit antenna of the T_x with the maximum sum of squared channel gains for more than half of the receivers is selected. This can be realized by Algorithm 1.

In this paper, we assume that the T_x obtains the knowledge of necessary channel state information (CSI) and order of channel quality for the links between the T_x and receivers. During the training period, by assuming the reciprocity of

the downlink and uplink channels, each receiver sends pilot symbols to the T_x , and then the T_x estimates the channel gains and determines the index of the majority transmit antenna of the T_x , and order of the channel quality of the links. The T_x also sends the channel gains to the receivers via the feedback channel. However, for simplicity, we assume the perfect CSI case.

Algorithm 1 TAS-maj/MRC scheme.

- The conventional TAS/MRC scheme is applied individually for R_l . The transmit antenna $i_l^{\max}$ is selected according to the maximum total of squared channel gains at R_l , i.e.,

$$i_l^{\max} = \arg \max_{i=1, \dots, N_t} \left\{ \varphi_l^i = \sum_{j=1}^{N_r} |h_l^{(i,j)}|^2 \right\}. \quad (1)$$

In Eq. 1, $i_l^{\max} \in \{1, \dots, N_t\}$ and $h_l^{(i,j)}$ ($1 \leq i \leq N_t$ and $1 \leq j \leq N_r$) denotes the fading channel coefficients between the i th antenna of the T_x and the j th receive antenna of the R_l .

- The majority rule is applied. From the set of $i_l^{\max}$, the index of the majority transmit antenna i^{maj} is determined, i.e.,

$$i^{\text{maj}} = \text{Maj}(i_l^{\max}). \quad (2)$$

In Eq. 2, $\text{Maj}(\cdot)$ is the majority function which determines the i^{maj} .

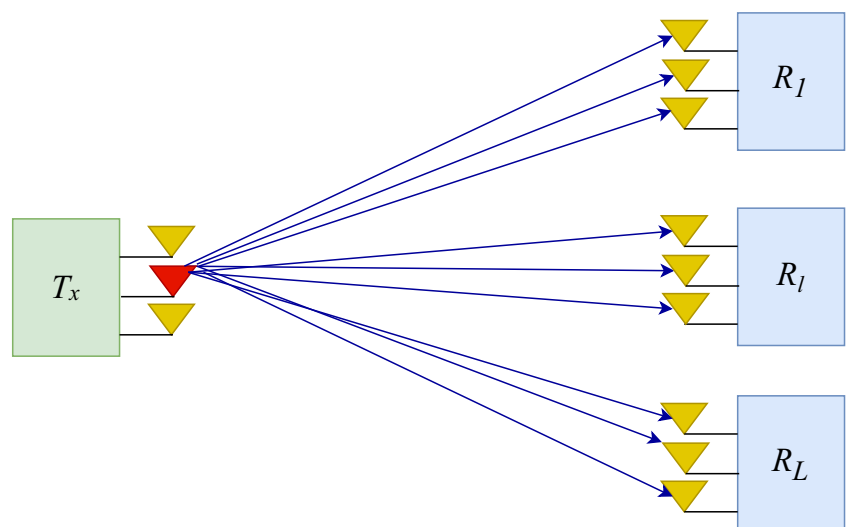
Regarding NOMA principle, the T_x utilizes the SC. So the superimposed signal of the transmitter can be written as

$$x = \sum_{l=1}^L \alpha_l s_l. \quad (3)$$

In Eq. 3, s_l is R_l 's signal, $\alpha_l = \sqrt{P a_l}$, P is T_x 's power, and a_l is R_l 's power factor. The received signal of the R_l can be written as

$$r_l = \mathbf{h}_l^{\text{maj}} x + \mathbf{w}_l$$

Fig. 1 System model



$$\begin{aligned}
&= \mathbf{h}_l^{maj} \sum_{l=1}^L \alpha_l s_l + \mathbf{w}_l \\
&= \mathbf{h}_l^{maj} \sum_{i=1}^{l-1} \alpha_i s_i + \mathbf{h}_l^{maj} \alpha_l s_l + \mathbf{h}_l^{maj} \sum_{i=l+1}^L \alpha_i s_i + \mathbf{w}_l. \quad (4)
\end{aligned}$$

In Eq. 4, $\mathbf{h}_l^{maj}$ denotes the fading channel coefficient of the selected majority transmit antenna between the T_x and R_l with $\Omega_l = E[\|\mathbf{h}_l^{maj}\|^2]$, and $\mathbf{w}_l$ is the noise with a variance of σ_l^2 . With the optimal receive weight vector $\mathbf{Q}_l = \frac{(\mathbf{h}_l^{maj})^H}{\|\mathbf{h}_l^{maj}\|}$, $\|\mathbf{Q}_l\| = 1$, the $\mathbf{r}_l$ are combined using MRC. Hence, the combined received signal at the R_l can be written as

$$\begin{aligned}
y_l &= \mathbf{Q}_l \mathbf{r}_l \\
&= \mathbf{Q}_l \mathbf{h}_l^{maj} \sum_{i=1}^{l-1} \alpha_i s_i + \mathbf{Q}_l \mathbf{h}_l^{maj} \alpha_l s_l \\
&\quad + \mathbf{Q}_l \mathbf{h}_l^{maj} \sum_{i=l+1}^L \alpha_i s_i + \mathbf{Q}_l \mathbf{w}_l \\
&= \frac{(\mathbf{h}_l^{maj})^H}{\|\mathbf{h}_l^{maj}\|} \mathbf{h}_l^{maj} \sum_{i=1}^{l-1} \alpha_i s_i + \frac{(\mathbf{h}_l^{maj})^H}{\|\mathbf{h}_l^{maj}\|} \mathbf{h}_l^{maj} \alpha_l s_l \\
&\quad + \frac{(\mathbf{h}_l^{maj})^H}{\|\mathbf{h}_l^{maj}\|} \mathbf{h}_l^{maj} \sum_{i=l+1}^L \alpha_i s_i + \frac{(\mathbf{h}_l^{maj})^H}{\|\mathbf{h}_l^{maj}\|} \mathbf{w}_l \\
&= \|\mathbf{h}_l^{maj}\| \sum_{i=1}^{l-1} \alpha_i s_i + \|\mathbf{h}_l^{maj}\| \alpha_l s_l + \|\mathbf{h}_l^{maj}\| \sum_{i=l+1}^L \alpha_i s_i \\
&\quad + \frac{(\mathbf{h}_l^{maj})^H}{\|\mathbf{h}_l^{maj}\|} \mathbf{w}_l. \quad (5)
\end{aligned}$$

The R_l applies SIC and for perfect SIC, and the $\|\mathbf{h}_l^{maj}\| \sum_{i=1}^{l-1} \alpha_i s_i$ is eliminated. Therefore, the received signal at the R_l can be rewritten as

$$\mathbf{r}_l = \|\mathbf{h}_l^{maj}\| \phi_l + \mathbf{w}_l. \quad (6)$$

In Eq. 6, $\phi_l = \alpha_l s_l + \sum_{i=l+1}^L \alpha_i s_i$. Suppose that the information signal is modulated by using BPSK modulation, i.e.,

$$\phi_l^* = * \alpha_l + \sum_{i=l+1}^L \alpha_i s_i. \quad (7)$$

In Eq. 7, $* \in +, -$ once $s_l = +1$ or -1 , respectively. s_i may be $+1$ and/or -1

$$\phi_l^* = * \alpha_l + \sum_{i=l+1}^L \pm \alpha_i. \quad (8)$$

This means that we have multiple combinations of ϕ_l^* , where $\phi_{l,v}^*$ is the ϕ_l^* 's v th combination and $v = 1, \dots, \delta_l = 2^{L-l}$.

Now, the complexity for the optimal TAS/MRC (TAS-opt/MRC) algorithm is given as $O(N_t N_r^3)$ by hypothetically assuming that the optimal solution always exists. On the other hand, for the TAS-maj/MRC scheme (Algorithm 1), the complexity can be written as $O(3N_t N_r + 12N_t)$. Now, for the sake of simplicity, assume that $N_t = N_r = N$. Accordingly, the complexities of the TAS-opt/MRC and TAS-maj/MRC algorithms are given as $O(N^4)$ and $O(3N^2 + 4N)$ and $O(3N^2 + 12N)$, respectively. Note that the TAS-maj/MRC can remarkably reduce the computational complexity compared to the TAS-opt/MRC.

3 Channel model

The noise is modelled as $\mathbf{w}_l \sim CN(0, \mathbf{I}_{N_r} \sigma_l^2)$, and its probability density function (PDF) can be stated as

$$f_{\mathbf{w}_l}(x) = \frac{1}{\sqrt{2\pi} \sigma_l^2} e^{-\frac{x^2}{2\sigma_l^2}}. \quad (9)$$

In this paper, we assume that the fading channel is Nakagami- m and the cumulative distribution function (CDF) and PDF of its square can be written, respectively, as

$$F_R(x) = \frac{\Psi(m, \frac{mx}{\Omega})}{\Gamma(m)}, \quad (10)$$

and

$$f_R(x) = \frac{(m/\Omega)^m x^{m-1}}{\Gamma(m)} e^{-\frac{mx}{\Omega}}. \quad (11)$$

In Eqs. 10 and 11, $\Omega = E[|R|^2]$ and m are Nakagami's parameters. In this paper, we attend to avoid the majority function's complexity and assume the special case of three users and two transmit antennas. The CDF of the TAS-maj/MRC can be expressed as [18]

$$F_{\phi_l^2}(x) = \sum_{q=1}^6 \zeta_{(l,q)} (F_X(x))^q, \quad (12)$$

where

$$F_X(x) = \frac{\Psi(mN_r, \frac{mx}{\Omega})}{\Gamma(mN_r)}. \quad (13)$$

By taking the derivative of $F_{\varphi_l^2}(x)$ in Eq. 12, the corresponding PDF can be stated as

$$f_{\varphi_l^2}(x) = \sum_{q=1}^6 \zeta_{(l,q)} q f_X(x) (F_X(x))^{q-1}, \quad (14)$$

where

$$f_X(x) = \frac{(m/\Omega)^{mN_r} x^{mN_r-1}}{\Gamma(mN_r)} e^{-\frac{mx}{\Omega}}, \quad (15)$$

and the values of $\zeta_{(m,q)}$ can be labeled as

$$\zeta = \begin{bmatrix} 1.5 & 1.5 & -4.5 & 3.75 & -1.5 & 0.25 \\ 0 & 0 & 3 & -3/4 & -2.25 & 1 \\ 0 & 0 & 0 & 0 & 1.5 & -0.5 \end{bmatrix} \quad (16)$$

4 BER analysis

In this section, we derive theoretical expressions of the BER for the R_l under the context of the BPSK modulation in Nakagami- m fading channels. Then, we obtain the corresponding asymptotic BER expressions in the high SNR regime in order to provide deeper insights into the BER performance of the system. The expressions and derivations of the BER and asymptotic BER are given in Theorems 1 and 2, respectively.

Theorem 1 *The exact BER for the R_l under the context of the BPSK modulation in Nakagami- m fading channels can be expressed as*

$$P_l(E) = \sum_{v=1}^{\delta_l} \sum_{q=1}^6 \sum_{p=0}^{q-1} \sum_{k=0}^{p(mN_r-1)} \binom{q-1}{p} (-1)^p \zeta_{(l,q)} \times \frac{q \mu_k(p, mN_r) (2(\psi_{l,v}^*)^2 \gamma)^{-(mN_r+k)} \Gamma(2(mN_r+k)+1)}{\delta_l \Gamma(mN_r) (m/\Omega)^{-mN_r}} \times {}_2F_1\left(mN_r+k, mN_r+k+\frac{1}{2}, mN_r+k+1, \frac{2(p+1)}{(\psi_{l,v}^*)^2 \gamma \Omega}\right), \quad (17)$$

where ${}_2F_1(a, b, c, z)$ denotes regularized Gauss hypergeometric function.

Proof The conditional PDF of y_l given $(\varphi_l, \phi_{l,v}^*)$ can be written as

$$f(y_l/\varphi_l, \phi_{l,v}^*) = \frac{1}{\sqrt{2\pi\sigma_l^2}} e^{-\frac{(y_l-\varphi_l\phi_{l,v}^*)^2}{2\sigma_l^2}}. \quad (18)$$

In Eq. 18, $\varphi_l = \|\mathbf{h}_l^{maj}\|$. Integrating Eq. 18, the error probability can be stated as

$$P_r(E/\varphi_l, \phi_{l,v}^*) = Q\left(\frac{\varphi_l \phi_{l,v}^*}{\sqrt{\sigma_l^2}}\right), \quad (19)$$

where $Q(x)$ is the Q -function and can be expressed as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{x^2}{2}} dx. \quad (20)$$

Note that $\sigma_l^2 = \sigma^2$ is assumed and $\gamma = \frac{P}{\sigma^2}$ is the SNR and $\beta_{l,v}^* = \frac{\phi_{l,v}^*}{\sqrt{P}}$. Then, $P_r(E/\varphi_l, \phi_{l,v}^*)$ in Eq. 19 can be rewritten as

$$\begin{aligned} P_r(E/\varphi_l, \phi_{l,v}^*) &= Q\left(\sqrt{\frac{\varphi_l^2 (\phi_{l,v}^*)^2}{\sigma^2}}\right) \\ &= Q\left(\sqrt{\varphi_l^2 (\phi_{l,v}^*)^2 \frac{P}{P\sigma^2}}\right) \\ &= Q\left(\sqrt{(\beta_{l,v}^*)^2 \gamma \varphi_l^2}\right). \end{aligned} \quad (21)$$

Now, with $P_r(\phi_{l,v}^*) = \frac{1}{\delta_l}$ and Eq. 21, $P_r(E/\varphi_l, \phi_l^*)$ can be expressed as

$$\begin{aligned} P_r(E/\varphi_l, \phi_l^*) &= \sum_{v=1}^{\delta_l} P_r(E/\varphi_l, \phi_{l,v}^*) P_r(\phi_{l,v}^*) \\ &= \frac{1}{\delta_l} \sum_{v=1}^{\delta_l} Q\left(\sqrt{(\beta_{l,v}^*)^2 \gamma \varphi_l^2}\right). \end{aligned} \quad (22)$$

Next, if $\psi_{l,v}^* = \sqrt{(\beta_{l,v}^*)^2}$ and $P_r(\phi_l^*) = 0.5$, $P_l(E/\varphi_l)$ can be stated as

$$\begin{aligned} P_l(E/\varphi_l) &= \sum_{*} P_r(E/\varphi_l, \phi_l^*) P_r(\phi_l^*) \\ &= \sum_{*} \frac{1}{\delta_l} \sum_{v=1}^{\delta_l} Q\left(\sqrt{(\psi_{l,v}^*)^2 \gamma \varphi_l^2}\right) \times 0.5 \\ &= \frac{1}{\delta_l} \sum_{v=1}^{\delta_l} Q\left(\sqrt{(\psi_{l,v}^*)^2 \gamma \varphi_l^2}\right). \end{aligned} \quad (23)$$

In general, the average bit error probability for R_l in fading channels is given by

$$\begin{aligned} P_l(E) &= \int_0^\infty P_r(E/\varphi_l) f_{\varphi_l}(x) dx \\ &= \int_0^\infty P_r(E/\varphi_l^2) f_{\varphi_l^2}(x) dx. \end{aligned} \quad (24)$$

Using Eqs. 23 and 14 in 24,

$$\begin{aligned}
 P_l(E) &= \sum_{v=1}^{\delta_l} \sum_{q=1}^6 \frac{q \zeta(l, q) (m/\Omega)^{mN_r}}{\delta_l \Gamma(mN_r)} \int_0^\infty Q(\sqrt{(\psi_{l,v}^*)^2 \gamma x}) x^{mN_r-1} \\
 &\quad \times e^{-\frac{mx}{\Omega}} \left(\frac{\Psi(mN_r, \frac{mx}{\Omega})}{\Gamma(mN_r)} \right)^{q-1} dx \\
 &= \sum_{v=1}^{\delta_l} \sum_{q=1}^6 \frac{q \zeta(l, q) (m/\Omega)^{mN_r}}{\delta_l \Gamma(mN_r)} \int_0^\infty Q(\sqrt{(\psi_{l,v}^*)^2 \gamma x}) x^{mN_r-1} \\
 &\quad \times e^{-\frac{mx}{\Omega}} \left(1 - e^{-\frac{mx}{\Omega}} \sum_{k=0}^{mN_r-1} \left(\frac{mx}{\Omega} \right)^k \frac{1}{k!} \right)^{q-1} dx. \quad (25)
 \end{aligned}$$

The binomial term in Eq. 25 can be expressed as

$$\begin{aligned}
 \left(1 - e^{-\frac{mx}{\Omega}} \sum_{k=0}^{mN_r-1} \left(\frac{mx}{\Omega} \right)^k \frac{1}{k!} \right)^{q-1} &= \sum_{p=0}^{q-1} \sum_{k=0}^{p(mN_r-1)} \\
 \binom{q-1}{p} (-1)^p \mu_k(p, mN_r) x^k e^{-\frac{mp}{\Omega} x}. \quad (26)
 \end{aligned}$$

In Eq. 26, $\mu_a(b, c_d)$ is the multinomial coefficient [[52], eq.(0.314)] and can be calculated as

$$\mu_a(b, c_d) = \sum_{u=1}^{a \geq 1} \frac{(u(b+1) - x) g_u \mu_{a-b}(b, c_d)}{a g_0}, \quad (27)$$

where $g_u = \frac{(c_d/\Omega_c)^u}{u!}$, $\mu_0(b, c_d) = 1$, and $\mu_a(b, c_d) = 0$ if $u > c_d - 1$.

By substituting Eqs. 26 into 25,

$$\begin{aligned}
 P_l(E) &= \sum_{v=1}^{\delta_l} \sum_{q=1}^6 \sum_{p=0}^{q-1} \sum_{k=0}^{p(mN_r-1)} \binom{q-1}{p} \frac{(-1)^p \zeta(l, q) \mu_k(p, mN_r)}{\delta_l \Gamma(mN_r) (m/\Omega)^{-mN_r}} \\
 &\quad \times q \underbrace{\int_0^\infty Q(\sqrt{(\psi_{l,v}^*)^2 \gamma x}) x^{mN_r+k-1} e^{-\frac{m(p+1)}{\Omega} x} dx}_{\Lambda}. \quad (28)
 \end{aligned}$$

Using [[53], eq.(15.2.2)], the integral in Eq. 27 can be calculated as

$$\begin{aligned}
 \Lambda &= (2\psi_{l,i}^* \gamma)^{-(mN_r+k)} \Gamma(2(mN_r+k) + 1) \\
 &\quad \times {}_2F_1\left(mN_r+k, mN_r+k+\frac{1}{2}, mN_r+k+1, \frac{2(p+1)}{\psi_{l,i}^* \gamma \Omega}\right). \quad (29)
 \end{aligned}$$

With some substitution, the $P_l(E)$ is obtained in Theorem 1. $\square$

From Theorem 1, it is difficult to get an insight from the BER expression. Therefore, in the following theorem, we examine the BER behavior in the high SNR region.³

Theorem 2 The asymptotic BER expression for R_l in the high SNR regime can be expressed as

$$\begin{aligned}
 P_l^{as}(E) &= \sum_{v=1}^{\delta_l} \sum_{q=1}^6 \frac{\zeta(l, q) q \left(\frac{m}{\Omega}\right)^{mqN_r-1} 2^{mqN_r-\frac{1}{2}}}{\delta_l \Gamma(mN_r) \Gamma(mN_r+1) q^{q-1} \sqrt{2\pi} (mqN_r)} \\
 &\quad \frac{\Gamma(mqN_r + \frac{1}{2})}{(\psi_{l,v}^*)^{2mqN_r}} \gamma^{-mqN_r}. \quad (30)
 \end{aligned}$$

Proof Exploiting the properties of incomplete Gamma function from [54] and [[55], eq. (45.9.1)], the $\Psi(mN_r, \frac{mx}{\Omega})$ in the high SNR regime can be expressed as

$$\Psi(mN_r, \frac{mx}{\Omega}) \approx \frac{(\frac{mx}{\Omega})^{mN_r}}{mN_r}. \quad (31)$$

Therefore, the corresponding CDF and PDF can be written, respectively, as

$$F_X^\infty(x) = \frac{(\frac{m}{\Omega})^{mN_r} x^{mN_r}}{\Gamma(mN_r+1)}, \quad (32)$$

and

$$f_X^\infty(x) = \frac{(\frac{m}{\Omega})^{mN_r} x^{mN_r-1}}{\Gamma(mN_r)}. \quad (33)$$

Now, the PDF of the TAS-maj/MRC in the high SNR region can be expressed as

$$\begin{aligned}
 f_{\varphi_l^2}^\infty(x) &= \sum_{q=1}^6 \zeta(l, q) q f_X^\infty(x) (F_X^\infty(x))^{q-1} \\
 &= \sum_{q=1}^6 \zeta(l, q) q \frac{(\frac{m}{\Omega})^{mN_r} x^{mN_r-1}}{\Gamma(mN_r)} \left(\frac{(\frac{m}{\Omega})^{mN_r} x^{mN_r}}{\Gamma(mN_r+1)} \right)^{q-1} \\
 &= \sum_{q=1}^6 \frac{\zeta(l, q) q \left(\frac{m}{\Omega}\right)^{mqN_r-1}}{\Gamma(mN_r) \Gamma(mN_r+1) q^{q-1}} x^{mqN_r-1}. \quad (34)
 \end{aligned}$$

³ In our paper, the term “high SNR region” refers to a regime where the signal power significantly exceeds the noise power. Note that power constraints and interference are common factors limiting the achievable SNR in practical communication systems. However, the “high SNR region” assumption is made for analytical convenience and to provide theoretical insights.

Next, the asymptotic BER for R_l can be calculated as

$$\begin{aligned} P_l^{as}(E) &= \int_0^\infty P_r(E/\varphi_l^2) f_{\varphi_l^2}^\infty(x) dx \\ &= \sum_{v=1}^{\delta_l} \sum_{q=1}^6 \frac{\zeta(l,q) q(\frac{m}{\Omega})^{mqN_r-1}}{\delta_l \Gamma(mN_r) \Gamma(mN_r+1)^{q-1}} \int_0^\infty \\ &\quad Q(\sqrt{(\psi_{l,v}^*)^2 \gamma x}) x^{mqN_r-1} dx. \end{aligned} \quad (35)$$

By using [54], the $P_l^{as}(E)$ is obtained in Theorem 2. $\square$

From Theorem 2, it can be noticed that the asymptotic BER depends on the SNR. Thus, the system achieves diversity gain in the high SNR region. Clearly, Theorem 2 shows that both the number of receive antenna (N_r) and channel condition (m) affect the system diversity order, as either N_r or m increases, the higher the diversity order, the better the achieved performance.

5 Results

In this section, we present numerical results to evaluate the performance of the MIMO-NOMA network using the TAS-maj/MRC scheme in Nakagami- m fading channels. Monte Carlo simulations are used to validate the analytical expressions provided in Section 4. The simulation assumes parameter values of $\{a_1, a_2, a_3\} = \{0.75, 0.2, 0.05\}$ and $\Omega = 1$. Here, 100,000,000 iterations have been generated for each value of the SNR to calculate the BER. Furthermore, the BPSK modulation scheme is utilized.

Figure 2 illustrates the impact of the number of receive antennas (N_r) on the BER performance in Rayleigh fading channels with $m = 1$. Meanwhile, Fig. 3 demonstrates the

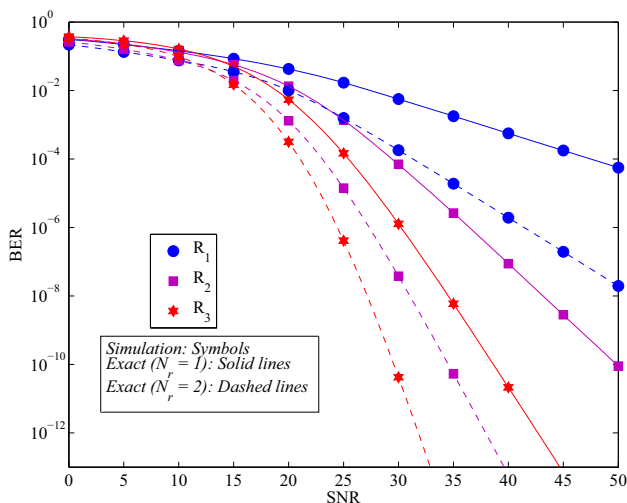


Fig. 2 Effect of N_r on BER performance of NOMA network

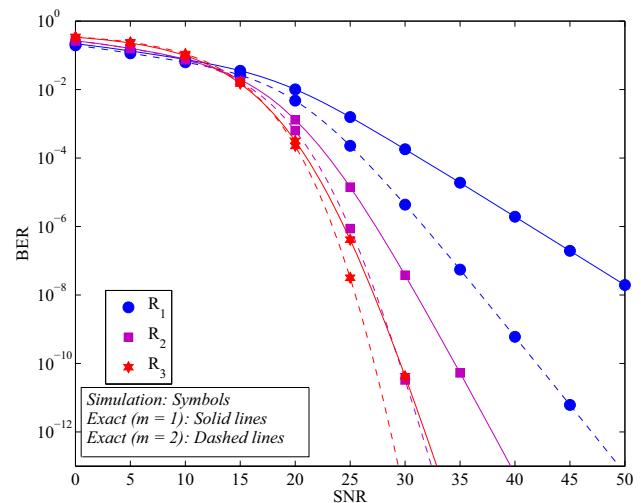


Fig. 3 Effect of m on BER performance of NOMA network

effect of the fading parameter m on the BER performance when each receiver is equipped with a single receive antenna ($N_r = 1$). These results show that the analytical expressions align well with the simulation results, validating the theoretical BER performance for all receivers. Furthermore, it is observed that the BER performance improves as the number of receive antennas increases and the channel condition becomes better. For ease of exposition, Tables 1 and 2 are given in order to provide, respectively, the values of the SNR that are required to achieve $OP = 10^{-3}$ and the SNR gain the advantage of different parameters of (m, N_r) for each receiver. Specifically, under the condition of $m = 1$ (Fig. 2), for a BER of 10^{-4} , there are SNR gain advantages of 16 dB, 7 dB, and 3 dB for the R_1 , R_2 , and R_3 , respectively, when $N_r = 2$ compared to $N_r = 1$. Similarly, under the condition of $N_r = 2$ (Fig. 3), for different channel parameters, to achieve a BER of 10^{-4} , there are approximately SNR gain advantages of 5 dB, 1 dB, and 1 dB for the R_1 , R_2 , and R_3 , respectively, when $m = 2$ compared to $m = 1$. Hence, it can be concluded that the BER performance of NOMA with the TAS-maj/MRC scheme is improved more by increasing the number of receive antennas than by enhancing the channel condition.

In Fig. 4, we validate the achieved analytical and asymptotic results by comparing them with simulation results. It is

Table 1 Values of SNR (in dB) that required to achieve $BER = 10^{-4}$ for each receiver with different parameters of (m, N_r)

(m, N_r)	R_1	R_2	R_3
(1, 1)	48	30	25
(1, 2)	32	23	22
(2, 2)	27	22	21

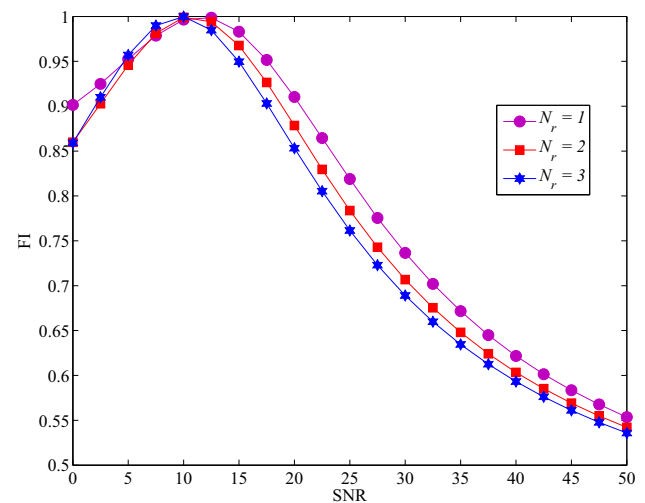
Table 2 SNR gain advantage of different parameters of (m, N_r) for each receiver

SNR gain advantage of (in dB)	R ₁	R ₂	R ₃
(1, 2) over (1, 1)	16	7	3
(2, 2) over (2, 1)	5	1	1

evident from this figure that the analytical results match the simulation ones. In addition, it can be seen that the asymptotic results converge to the analytical and simulation results at very high values of SNR. The figure clearly shows that the number of receive antenna and channel conditions affect the system diversity order, coinciding with the results achieved in Section 4. Clearly, as either N_r or m increases, the higher the diversity order, the better the achieved performance.

The fairness index (FI) in NOMA plays a crucial role in ensuring equitable resource allocation and user satisfaction within a communication system. There are several fairness indices, but one common metric used is Jain's fairness index. Jain's fairness index is a normalized metric ranging from 0 to 1, where 0 represents the least fair allocation, and 1 indicates a perfectly fair allocation. Figure 5 presents FI versus SNR for different numbers of receive antennas. Clearly, the proposed system with a high number of receive antennas needs less SNR to achieve maximum FI.

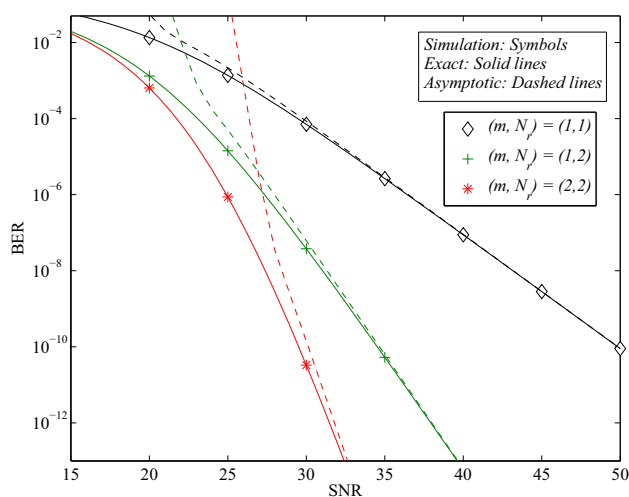
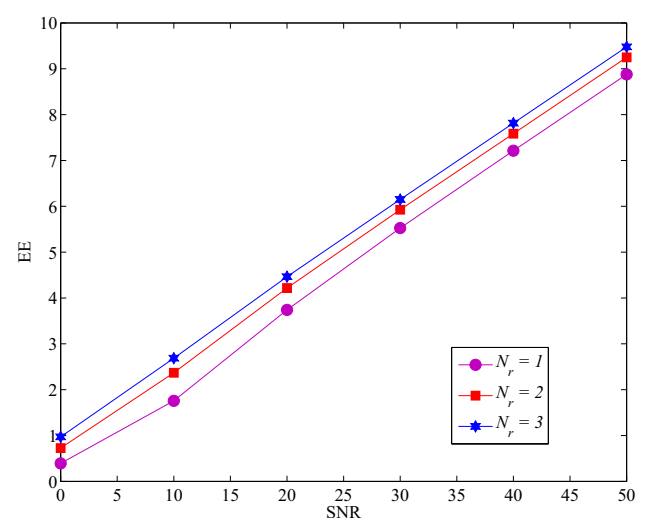
Energy efficiency (EE) is a critical measure in energy-constrained next-generation communication systems, defined by the division of total throughput (sum throughput) by the overall energy consumption [22, 23]. This metric holds paramount importance for optimizing the performance of such systems while considering energy limitations. Figure 6 plots the EE versus SNR. We notice that as SNR increases,

**Fig. 5** Effect of N_r on the FI performance of NOMA network

the throughput of users increases, which in turn increases the sum throughput, and hence, the EE increases. It is also observed that EE increases with an increasing number of receive antennas.

6 Conclusion

In this paper, we have investigated the BER performance for the MIMO-NOMA network, in which the TAS-maj/MRC scheme is employed. We have derived a closed-form expression for the exact BER for BPSK modulation in Nakagami- m fading channels. Next, we have obtained asymptotic expressions for BER in the high SNR region in order to gain further insights into the BER behavior of the system. Finally, the

**Fig. 4** Asymptotic BER with different parameters of (m, N_r) for each receiver**Fig. 6** Effect of N_r on the EE performance of NOMA network

simulation results validate the accuracy of the analytical results. The results demonstrate that diversity and code gains are achieved. Moreover, the BER performance is observed to significantly improve with an increase in the number of receive antennas or enhancement of channel conditions. In our future research endeavors, we plan to assess the performance of MIMO-NOMA networks in practical conditions such as imperfect CSI, imperfect SIC, and hardware impairments. Furthermore, we will derive the generalized BER expressions for m-array PSK.

Declarations

Conflict of interest The authors declare no competing interests.

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