
Work density analysis and thermoeconomic optimisation of modified Carnot cycle engine

Asim Sinan Karakurt* and Guven Gonca

Naval Architecture and Marine Engineering Department,

Yildiz Technical University,

34349 Besiktas/Istanbul, Turkey

Email: asinan@yildiz.edu.tr

Email: ggonca@yildiz.edu.tr

*Corresponding author

Bahri Sahin

Department of Aeronautical Engineering,

Istanbul Gelisim University,

34310 Avcilar/Istanbul, Turkey

Email: bsahin@gelisim.edu.tr

Abstract: The modified Carnot cycle, a novel thermodynamic cycle, is introduced, and the cycle's performance is analysed in order to more accurately depict the performance and design parameters of actual heat engines. Work output, thermal efficiency, and work density are all considered in conjunction during the performance analysis. In terms of maximum pressure, maximum volume, work density, thermal efficiency, and size, the results demonstrate that the modified cycle performs better than the classic Carnot cycle at simulating the real heat engine. In the performance analysis framework, a thermoeconomic optimisation has been carried out based on an objective function defined as the total cost per unit work output, which depends on an economic parameter, work density, and thermal efficiency. The findings might be used to select the modified cycle's ideal design and operation parameters, which can be applied to actual engines.

Keywords: Carnot cycle; modified cycle; work density; performance analysis; thermoeconomic optimisation.

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Biographical notes: Asim Sinan Karakurt is working as an Assistant Professor in the Naval Architecture and Marine Engineering Department at Yildiz Technical University, Istanbul, Turkey. He is currently working in the area of design, analysis, and optimisation of thermal systems.

Guven Gonca is working as a Professor in the Naval Architecture and Marine Engineering Department at Yildiz Technical University, Istanbul, Turkey. He is currently working in the area of design, analysis, and optimisation of thermal systems.

Bahri Sahin is an Emeritus Professor and former Rector of the Yildiz Technical University. He is currently working as a Professor and Rector at Istanbul Gelisim University. His research areas are mainly thermodynamics and energy systems.

1 Introduction

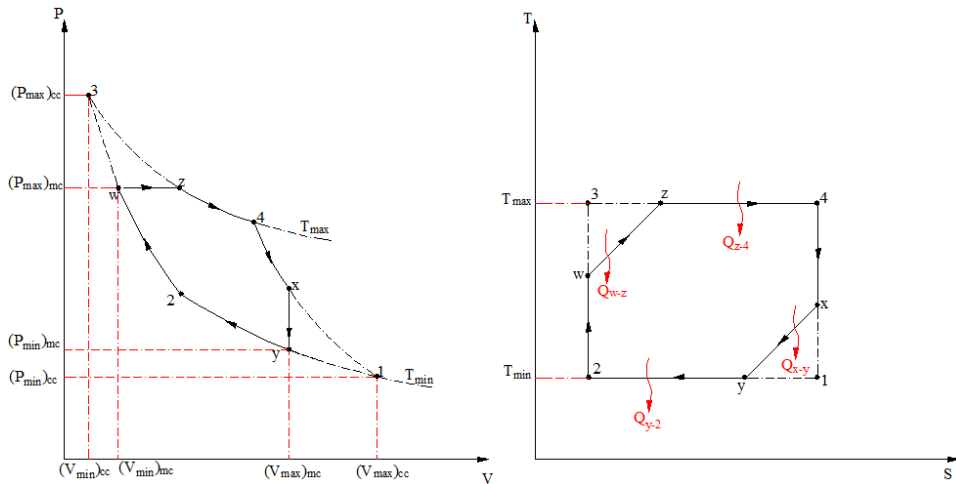
In recent years, numerous researches on endoreversible and irreversible heat engines have been conducted. The optimum design parameters were found in these optimisation experiments based on criteria such as power, power density, ecological coefficient of performance, exergy, exergy density, minimal entropy generation, and thermoeconomic goal. These studies were reviewed in detail by Durmayaz et al. (2004), Chen et al. (1999) and Bejan (1996). The Carnot heat engine model was used in a significant part of the performance optimisation study (Gutkowicz-Krusin et al., 1978; Salamon et al., 1980; Salamon and Nitzan, 1981; Rubin and Andresen, 1982; Wu, 1988a, 1988b; Bejan, 1988; Wu and Walker, 1989; Gordon, 1989; Angulo-Brown, 1991; Goktun et al., 1993; Ait-Ali, 1995; Chen et al., 1996; Goktun, 1996; Moukalled et al., 1996). According to the idea of internal and external reversibilities, the energy efficiency of the Carnot cycle represents the theoretical maximum for heat engines operating within the same temperature ranges ($T_{\max}$ and $T_{\min}$). Due to thermodynamic equilibrium, this limit can only be reached with infinitely slow processes. In this case, the cycle time will go to infinity. Therefore, it is impossible to generate a positive power ($\dot{W}$) with a finite heat transfer area (A). So, if $0 < A < \infty$ then $\dot{W} = 0$. To obtain positive power in a finite time, the heat transfer area must be infinite. So, if $A \rightarrow \infty$, $\dot{W} > 0$ (Durmayaz et al., 2004; Sahin et al., 2004). Although it has a maximum thermal efficiency, the reversible Carnot engine model cannot be used as a comparison benchmark for actual heat engines in terms of power and size. Chambadal (1957), Novikov (1958), and (Curzon and Ahlborn, 1975) expanded the reversible Carnot heat engine model to the endoreversible Carnot engine model, which accounts for the irreversibilities of heat transfer to achieve a certain amount of power in a finite size. Based on this heat engine model, an upper limit for power was determined by investigating the maximum power ($\dot{W}_{\max}$) conditions. It has also been demonstrated that the energy efficiency of the endoreversible heat engine under conditions of peak power (η_{mp}) is a lower limit for actual heat engines. Highly intense performance optimisation experiments were conducted in order to provide more realistic comparisons for actual heat engines, based on irreversible Carnot engine model and several performance criteria (Bejan, 1996; Chen et al., 1999; Durmayaz et al., 2004). For the optimal energy efficiency (η_{opt}) and optimal power ($\dot{W}_{opt}$) range of real engines, as a general result of these studies are given as following $\eta_{mp} \leq \eta_{opt} \leq \eta_{cc}$, $\dot{W}_{\max} \geq \dot{W}_{opt} \geq \dot{W}_{cc} = 0$. The reversible Carnot heat engine, on the other hand, while serving as a reasonable benchmark for comparison in terms of thermal efficiency, is an incorrect benchmark when it comes to maximum pressure ($P_{\max}$) and maximum volume ($V_{\max}$), which are crucial design factors for actual heat engines. These two factors are very important in reversible Carnot heat engines because they influence the size and investment costs of heat engines.

Sahin et al. (1996) and Kodal et al. (2000) have conducted performance optimisation studies for reversible, endoreversible and irreversible engine models based on the power density ($\dot{W}_d = \dot{W}/V_{\max}$), which takes into account both power and dimensions and is described as the ratio of power to peak cycle volume. It was demonstrated that the design parameters that optimise power density allowed the development of smaller heat engines for a given power, as well as a better energy efficiency than that at maximum power (η_{mp}).

Also, some studies (De Vos, 1995; Sahin and Kodal, 2001; Kodal and Sahin, 2003) have conducted based on a thermoeconomic optimisation method for an endoreversible and irreversible Carnot engine model. In these studies, the maximum heat transfer from the hot reservoir to the cold reservoir (De Vos, 1995) and the area of heat transfer as a measure of investment cost and heat consumption as a measure of fuel cost were considered (Kodal and Sahin, 2003; Sahin and Kodal, 2001).

In this study, performance optimisation and investigations of the modified Carnot cycle as in Figure 1 have been performed based on the work density and thermoeconomic criteria. By contrasting the modified cycle's performance results with those of the conventional Carnot cycle in terms of work density, net work, thermal efficiency, maximum pressure, and maximum volume, it has been demonstrated that the modified cycle has a more useful comparison specification for practical engines.

Figure 1 P-V and T-S diagram of Carnot and Modified Carnot Cycles (see online version for colours)



2 Theoretical model

The P-V and T-S diagrams of the modified Carnot cycle (mc) (y-2-w-z-4-y) model are shown in Figure 1. It is thought that practical heat engines are represented more realistically by improving the work density performance. The modified cycle model consists of two constant temperature processes (y-2, z-4), two isentropic processes (2-w, 4-x), one constant pressure process (w-z) and one constant volume process (x-y). In

Figure 1, the classical Carnot cycle (*cc*) (1-2-3-4-1) is given for comparison with the modified cycle performance.

Essential design parameters characterising the performance of the modified cycle: Cycle temperature ratio is α , isothermal compression ratio is $\varepsilon_{T,mc}$, maximum pressure parameter defined as ratio of maximum pressure of the modified cycle ($P_z = P_w = P_{\max,mc}$) to maximum pressure ($P_3 = P_{\max,cc}$) of the Carnot cycle is ψ , and maximum volume parameter described as the ratio of the maximum volume ($V_x = V_y = V_{\max,mc}$) of the modified cycle to the maximum volume ($V_1 = V_{\max,cc}$) of the Carnot cycle is ϕ . They are stated as follows:

$$\alpha = \frac{T_{\max}}{T_{\min}} \quad (1)$$

$$\psi = \frac{P_{\max,mc}}{P_{\max,cc}} = \frac{P_z}{P_3} \quad (2)$$

Valid range for ψ parameter;

$$P_4 < P_z < P_3 \quad \text{and} \quad \frac{P_4}{P_3} < \frac{P_z}{P_3} < 1 \quad (3)$$

The isothermal expansion ratio of the Carnot cycle (this ratio is equal to the isothermal compression ratio);

$$\varepsilon_{T,cc} = \frac{V_4}{V_3} = \frac{V_1}{V_2} = \frac{P_3}{P_4} = \frac{P_2}{P_1} \quad (4)$$

By using equation (3) and equation (4), valid range for the ψ is given as below:

$$\frac{1}{\varepsilon_{T,cc}} < \psi < 1 \quad (5)$$

$$\phi = \frac{V_{\max,mc}}{V_{\max,cc}} = \frac{V_x}{V_1} \quad (6)$$

Valid range for ϕ parameter;

$$V_2 < V_x < V_1 \quad \text{and} \quad \frac{V_2}{V_1} < \frac{V_x}{V_1} < 1 \quad (7)$$

Depending on equation (7), valid range for the ϕ is given as below:

$$\frac{1}{\varepsilon_{T,cc}} < \phi < 1 \quad (8)$$

Isothermal compression ratio of the modified cycle:

$$\varepsilon_{T,mc} = \frac{V_y}{V_2} = \frac{V_x}{V_2} \quad (9)$$

By substituting equation (4) and equation (6)

$$\varepsilon_{T,mc} = \phi \varepsilon_{T,cc} \quad (10)$$

In the following expression, ε_T symbol is used instead of the compression ratio of the Carnot cycle ($\varepsilon_{T,cc}$).

Total compression ratio ($r_v = V_{\max}/V_{\min}$) and extreme pressure ratio ($r_p = P_{\max}/P_{\min}$) of the modified cycle and Carnot cycle depending on design parameters such as α , ε_T , ψ and ϕ are stated as below:

$$r_{v,mc} = \left(\frac{V_{\max}}{V_{\min}} \right)_{mc} = \varepsilon_T \phi \psi^{\frac{1}{k}} \alpha^{\frac{1}{k-1}} \quad (11)$$

$$r_{p,mc} = \left(\frac{P_{\max}}{P_{\min}} \right)_{mc} = \varepsilon_T \phi \psi \alpha^{\frac{k}{k-1}} \quad (12)$$

In the equations above, k is the ratio of the constant-pressure specific heat to the constant-volume specific heat ($k = C_p/C_v$).

If $\phi = 1$ and $\psi = 1$ are accepted in the equation (11) and equation (12), total compression ratio and extreme pressure ratio of the Carnot cycle are obtained as below:

$$r_{v,cc} = \left(\frac{V_{\max}}{V_{\min}} \right)_{cc} = \varepsilon_T \alpha^{\frac{1}{k-1}} \quad (13)$$

$$r_{p,cc} = \left(\frac{P_{\max}}{P_{\min}} \right)_{cc} = \varepsilon_T \alpha^{\frac{k}{k-1}} \quad (14)$$

When the equations (11), (12), (13) and (14) are investigated, total compression ratio ($r_{v,mc}$) and extreme pressure ratio ($r_{p,mc}$) of the modified cycle are lower compared to those of the Carnot cycle.

Net work, thermal efficiency and work density analyses for the modified cycle demonstrated in Figure 1 are given in the following equations.

The heat inputs to the modified cycle are Q_{w-z} (at constant pressure process) and Q_{z-4} (at isothermal process), and the heat outputs are Q_{x-y} (at constant volume process) and Q_{y-2} (at isothermal process). By assuming that the working fluid is an ideal gas which has constant specific heat, they can be written as follows:

$$(Q_{w-z})_{mc} = m C_p (T_z - T_w) \quad (15)$$

$$(Q_{z-4})_{mc} = m R T_z \ln \frac{P_z}{P_4} \quad (16)$$

$$(Q_{x-y})_{mc} = m C_v (T_x - T_y) \quad (17)$$

$$(Q_{y-2})_{mc} = m R T_y \ln \frac{V_x}{V_z} \quad (18)$$

where m , C_p , C_v and R ($R = C_p - C_v$) are the mass of the working fluid, the constant-pressure specific heat at, the constant-volume specific heat, and the gas constant. The modified cycle's heat input and heat output are calculated as follows:

$$Q_{in,mc} = m \left[C_p (T_Z - T_W) + RT_Z \ln \frac{P_Z}{P_4} \right] \quad (19)$$

$$Q_{out,mc} = m \left[C_v (T_X - T_Y) + RT_Y \ln \frac{V_X}{V_2} \right] \quad (20)$$

Depending on α , ε_T , ψ and ϕ defined in equations (1), (2), (4) and (6), heat input and heat output can be derived as follows:

$$Q_{in,mc} = mRT_{\min} \alpha \left[\frac{k}{k-1} \left(1 - \psi^{\frac{k-1}{k}} \right) + \ln(\psi \varepsilon_T) \right] \quad (21)$$

$$Q_{out,mc} = mRT_{\min} \left[\frac{1}{k-1} \left(\frac{1}{\phi^{k-1}} - 1 \right) + \ln(\phi \varepsilon_T) \right] \quad (22)$$

where k is specific heat ratio.

By using equations (21) and (22), dimensionless net work is found as below:

$$\bar{W}_{mc} = \frac{W_{mc}}{mRT_{\min}} = \frac{Q_{in,mc} - Q_{out,mc}}{mRT_{\min}} \quad (23)$$

$$\bar{W}_{mc} = \alpha \left[\frac{k}{k-1} \left(1 - \psi^{\frac{k-1}{k}} \right) + \ln(\psi \varepsilon_T) \right] - \left[\frac{1}{k-1} \left(\frac{1}{\phi^{k-1}} - 1 \right) + \ln(\phi \varepsilon_T) \right] \quad (24)$$

When $\phi = 1$ and $\psi = 1$, the modified cycle happens Carnot cycle and dimensionless net work is defined as follows:

$$\bar{W}_{cc} = (\alpha - 1) \ln(\varepsilon_T) \quad (25)$$

The work density, which is defined as the ratio of net work to maximum cycle volume, may be calculated as follows.

$$W_{d,mc} = \frac{W_{mc}}{V_{\max,mc}} = \frac{W_{mc}}{V_x} = \frac{Q_{in,mc} - Q_{out,mc}}{V_x} \quad (26)$$

By substituting equations (6), (21) and (22) into equation (26), dimensionless work density of the modified cycle is found as below:

$$\bar{W}_{d,mc} = \frac{W_{d,mc}}{RT_{\min} / V_1} \quad (27)$$

$$\bar{W}_{d,mc} = \alpha \left[\frac{1}{\phi} \ln(\varepsilon_T \psi) + \frac{k}{k-1} \left(\frac{1 - \psi^{\frac{k-1}{k}}}{\phi} \right) \right] - \left[\frac{1}{\phi} \ln(\varepsilon_T \phi) + \frac{1}{k-1} \left(\phi^{-k} - \frac{1}{\phi} \right) \right] \quad (28)$$

The energy efficiency of the modified cycle is calculated using the equation given below.

$$\eta_{th,mc} = \frac{W_{mc}}{Q_{in,mc}} = \frac{Q_{in,mc} - Q_{out,mc}}{Q_{in,mc}} = 1 - \frac{Q_{out,mc}}{Q_{in,mc}} \quad (29)$$

By substituting equations (21) and (22), into equation (29), the thermal efficiency is obtained with respect to the design parameters as follows:

$$\eta_{th,mc} = 1 - \frac{\frac{1}{k-1} \left(\frac{1}{\phi^{k-1}} - 1 \right) + \ln(\varepsilon_T \phi)}{\alpha \left[\frac{k}{k-1} \left(1 - \psi^{\frac{k-1}{k}} \right) + \ln(\varepsilon_T \psi) \right]} \quad (30)$$

When $\phi = 1$ and $\psi = 1$, the modified cycle will be Carnot cycle and thermal efficiency becomes $\eta_{cc} = 1 - 1/\alpha$.

Work density is an indicator of engine size and therefore, investment cost, and thermal efficiency is an indicator of fuel cost for a heat engine. Therefore, optimum values of the maximum volume parameter (ϕ), total compression ratio ($r_{v,mc}$), extrem pressure ratio ($r_{p,mc}$) of the modified cycle can be determined by defining an objective function that takes into account work density, thermal efficiency and net work altogether. The ratio of the overall cost (the sum of the investment cost and the fuel consumption cost) to the net work will be a good objective function for optimisation. The objective function is described as follows,

$$F_{mc} = \frac{C_i + C_f}{W_{mc}} \quad (31)$$

In equation (31), C_i and C_f can be considered as annual or per cycle costs. In this case, net work should also be per year or per cycle.

The total cost is calculated as the sum of the investment cost (capital expenditure) and the fuel cost, with the assumption that the investment cost is commensurate with the maximum cycle volume, an adequate measure of size, and the fuel consumption cost is related to the heat input to the heat engine.

$$C_i + C_f = aV_{\max,mc} + bQ_{in,mc} \quad (32)$$

where a and b are investment cost parameter and fuel cost parameter, respectively.

Substituting equation (32) into equation (31) the objective function yields:

$$F_{mc} = \frac{aV_{\max,mc} + bQ_{in,mc}}{W_{mc}} \quad (33)$$

From equation (26) and equation (29), depending on the work density and thermal efficiency, the objective function can be expressed as

$$F_{mc} = \frac{a}{W_{d,mc}} + \frac{b}{\eta_{mc}} \quad (34)$$

The dimensionless objective function is defined as follows:

$$\bar{F}_{mc} = \frac{F_{mc}}{b} = \frac{\lambda}{W_{d,mc}} + \frac{1}{\eta_{mc}} \quad (35)$$

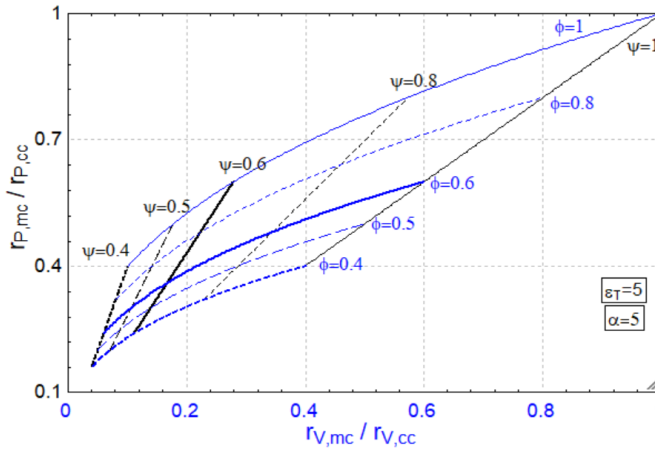
Where the ratio of the investment cost parameter to the fuel cost parameter ($\lambda = a/b$) is an economic parameter that shows the weight of investment and fuel cost in total cost.

Substituting equation (28) and equation (30) into equation (35), the objective function can be expressed depending on the important design parameters (ϕ , ψ , ε_T , α) and economical parameter (λ). In this study, optimum design parameters that minimise the objective function and optimum performance for different economical parameter (λ) values are numerically determined.

3 Results and discussion

The ratio of the total compression and extreme pressure ratios of the modified cycle to those of the Carnot cycle ($r_{V,mc}/r_{V,cc}$ and $r_{P,mc}/r_{P,cc}$) and the effects of maximum pressure and maximum volume parameters (ϕ and ψ) defined in equations (2) and (6) on these ratios can be observed in Figure 2. The total compression ratio and the extreme pressure ratio of the modified cycle are considerably lower compared to those of Carnot cycle.

Figure 2 The effects of maximum pressure and maximum volume parameters on the total compression and extreme pressure ratios (see online version for colours)



For example; If $\phi = \psi = 0.6$, the total compression ratio and extreme pressure ratio of the modified cycle are 16.7% and 36% of the Carnot cycle ($\phi = \psi = 1$). This ratio decreases as ϕ and ψ decrease.

This result suggests that the Carnot heat engine's dimensions are far higher than those of the modified cycle's heat engine, while those of actual engines are much smaller. The Carnot engine, on the other hand, outperforms all other heat engines operating within the same temperature range in terms of energy efficiency and net work, as can be shown in Figure 3.

When Figures 2 and 3 are examined together, the performance analysis to be made according to the work density criterion, which takes into account the net work and the dimensions of heat engine together, is important and meaningful. The changes in the work density of the modified cycle given in equation (28) and energy efficiency presented in equation (30) with respect to the parameters such as maximum volume ratio (ϕ), maximum pressure ratio (ψ) and extreme temperature ratio (α) are shown in Figure 4.

Figure 3 The variation of the thermal efficiency with respect to the dimensionless net work for various ψ and ϕ parameters (see online version for colours)

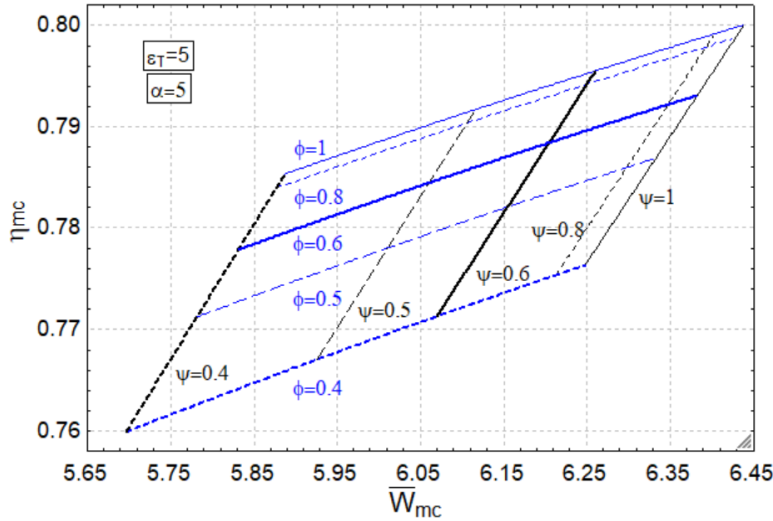
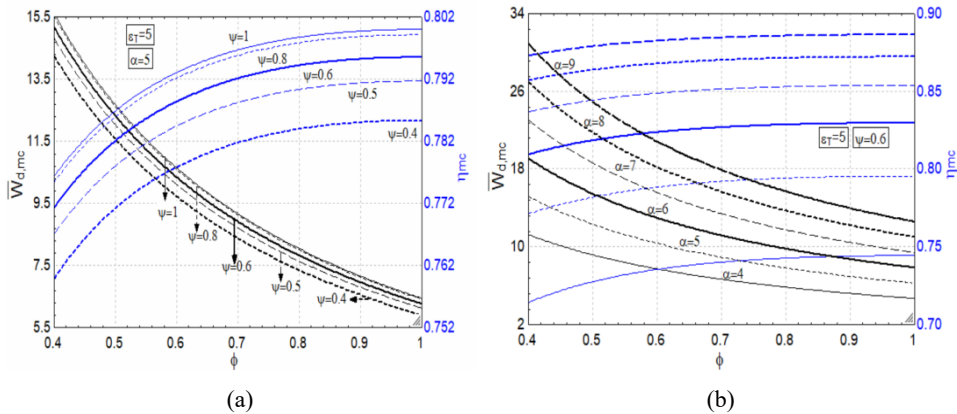


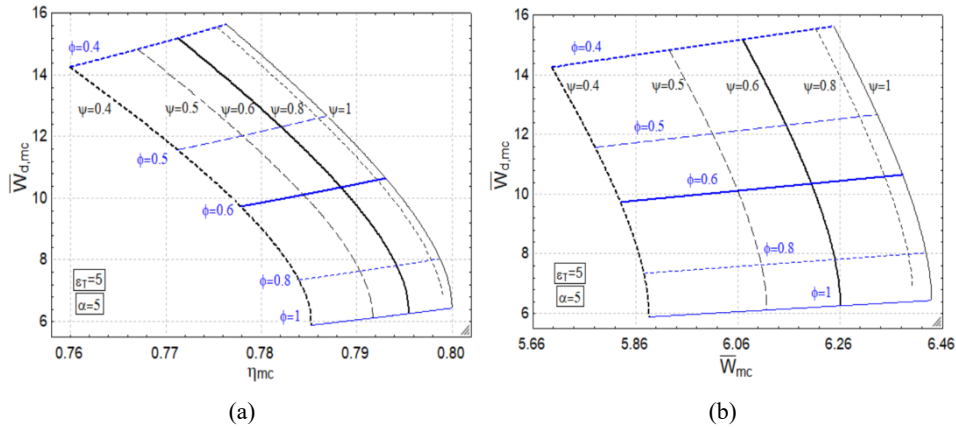
Figure 4 The changes in the dimensionless work density and thermal efficiency with respect to the maximum volume parameter (ϕ) (a) for varies maximum pressure parameter (ψ) and (b) for different extreme temperature ratio (α) (see online version for colours)



When Figure 4(a) is examined, although the thermal efficiency decreases as the ϕ decreases, the work density increases. However, the effect of ϕ on work density is more higher than its effect on thermal efficiency. For example, for $\psi = 0.6$, when ϕ changes from 1 to 0.4, the dimensionless work density increases from 6.3 to 15.2, and the thermal efficiency decreases from 0.795 to 0.772. In other words, while the work density increases by 2.4 times, the thermal efficiency decreases by only 3%. On the other hand, as ψ decreases, both work density and thermal efficiency decrease. For example, for $\phi = 0.6$, when ψ changes from 1 to 0.4, dimensionless work density decreases from 10.6 to 9.7 (8% reduction), and thermal efficiency decreases from 0.793 to 0.777 (2% reduction).

These results show that the effect of ϕ parameter on work density and thermal efficiency is higher than the effect of ψ parameter. The influence of the extreme temperature ratio (α) on the work density and energy efficiency of the modified cycle is shown in Figure 4(b). As the α increases for a given ϕ and ψ value, both work density and thermal efficiency increase significantly, as expected. For example, for $\phi = \psi = 0.6$, when α changes from 5 to 6, the dimensionless work density increases from 10.4 to 13, and the thermal efficiency increases from 0.788 to 0.825. In other words, work density increases by 30% and energy efficiency by 5%. It is clear that the effect of α on work density is higher than its effect on energy efficiency.

Figure 5 The variation of the respect to (a) the dimensionless work density and the thermal efficiency and (b) the dimensionless work density and the dimensionless net work for various ψ and ϕ parameters (see online version for colours)

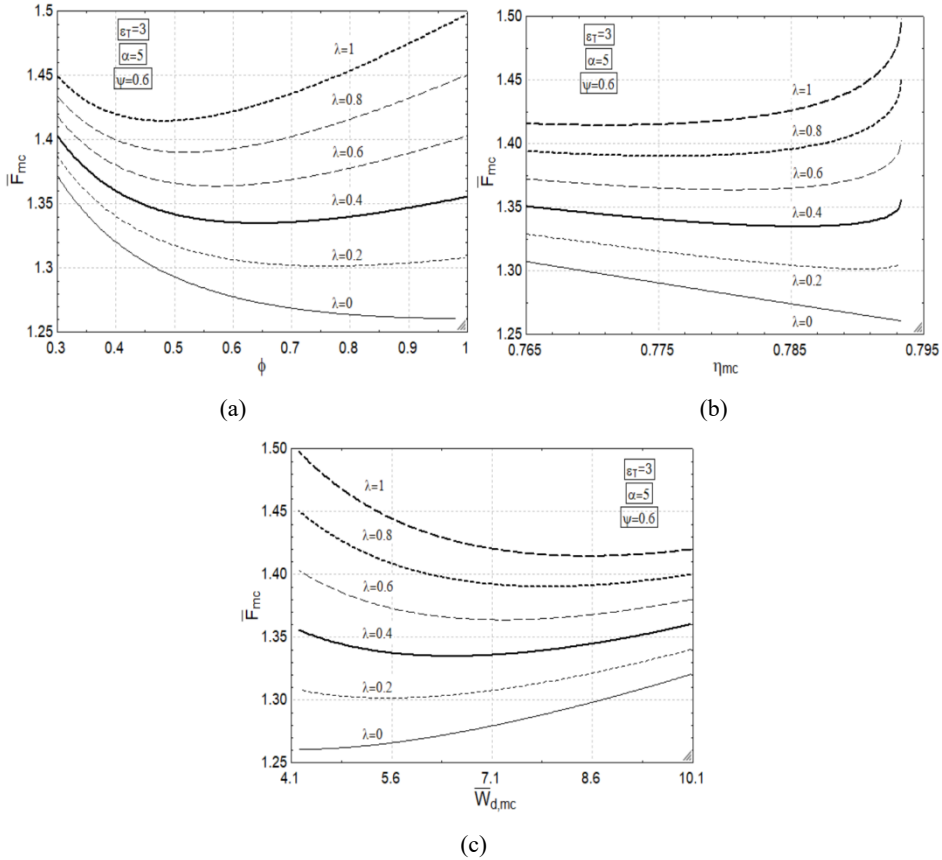


The alteration of the dimensionless work density of the modified cycle with the energy efficiency for different ψ and ϕ values is given in Figure 5(a), and the variation of it with net work is given in Figure 5(b). When these figures are examined, although the Carnot cycle ($\phi = \psi = 1$) has the highest performance in terms of thermal efficiency and net work, it shows a very low performance in terms of work density. Therefore, the performance of the modified cycle instead of the classic Carnot cycle, which is analysed according to the work density criterion, which takes into account the engine sizes, will be a good guide for practical engines.

Optimisation results of the modified cycle based on objective function introduced in equations (31)–(36) are presented in Figures 6–7. In Figure 6(a), the objective function can be plotted with respect to the maximum volume parameter (ϕ) for various economical parameter (λ). As it can be observed from Figure 6(a), there exist an optimal ϕ point that minimises the objective function for a given λ and the optimum ϕ value decreases as λ increases. This results means that the optimum ϕ value will lead to the design of smaller sized heat engines when investment cost dominant with respect to fuel cost. Figures 6(b) and 6(c) illustrate the alteration of the goal function with regard to energy efficiency and non-dimensional work density for varying λ . It is monitored from these figures that the λ enhances although the optimum thermal efficiency decreases while optimum work density increases. For a given λ , the optimum value of thermal efficiency and work density is obtained by substituting the optimum ϕ value in equations (28)–(30) for a given

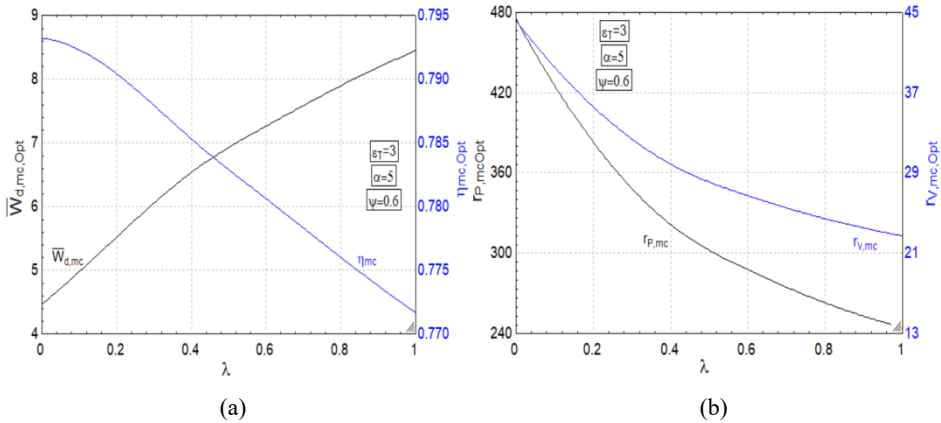
economical parameter, λ . The variations optimal thermal efficiency and optimal dimensionless work density with respect to λ are demonstrated in Figure 7(a). As λ increases, the share of investment cost in the total cost increases, optimum thermal efficiency decreases while optimum work density increases. The variation of the optimum values of compression ratio and extreme pressure ratio given equations (11) and (12) with respect to λ is depicted in Figure 7(b). When the figure is examined, both the optimal compression ratio and the optimal extreme pressure ratio decreases with the increase of λ .

Figure 6 The variation of the dimensionless thermoeconomic objective function with respect to (a) the maximum volume parameter, (b) the thermal efficiency and (c) the dimensionless work for various economical parameter (λ)



When the above optimisation results are examined, as the economic parameter λ increases, the modified cycle moves away from Carnot cycle to the real engine cycles under optimal design conditions, that is, while the thermal efficiency decreases, the work density increases and accordingly the size decreases [Figure 7(a)]. In addition, as λ enhances the optimum values of compression ratio and extreme pressure ratio decrease, and the modified cycle approaches the real heat engine cycles.

Figure 7 Variations of (a) the optimum dimensionless work density and optimum thermal efficiency and (b) the optimum total compression and optimum extreme pressure ratios with respect to economical parameter (see online version for colours)



4 Conclusions

A performance analysis of a new thermodynamic cycle was performed, which named modified Carnot cycle, based on the work density criterion. It has been illustrated that the work density of the modified cycle is much greater than that of the classical Carnot cycle. Therefore, a design based on the modified Carnot cycle can significantly reduce the size of heat engines due to high work density without a significant reduction in thermal efficiency. The effects of design parameters such as maximum volume parameter (ϕ), maximum pressure parameter (ψ) and extreme temperature ratio (α) on work density and thermal efficiency were examined and discussed in detail in the previous section. As a consequence of the work density study, the modified cycle may be a more relevant norm for practical heat engines than the Carnot cycle due to its significantly greater work density, lower maximum pressure, and lower maximum volume. In addition, the redesigned cycle provides an optimal compromise between work density and thermal efficiency. For this reason, a thermoeconomic optimisation is introduced by considering the work density as a measure of the size, that is the investment cost, and the energy efficiency as a measure of the fuel consumption, that is, the fuel cost. The objective function, which is defined as the sum of the investment and fuel cost per unit of work output, is expressed in point of work density, energy efficiency, and an economic parameter, and a foundation for determining the optimal design parameters and optimal performance characteristic based on the objective function is presented. The results could be evaluated so as to optimise the performance of actual heat engines with additional studies.

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Nomenclature

Indices

C	the specific heat (kJ/(kg.K))
C	per cycle cost (\$)
F	objective function (\$/kJ)
$\bar{F}$	dimensionless objective function
k	specific heat ratio
m	the mass of the working fluid (kg)
P	pressure (kPa)
Q	heat transfer (kJ)
R	ideal gas constant (kJ/(kg.K))
r_P	extreme pressure ratio
r_v	total compression ratio
t	temperature (K)
W	net work (kJ)
$\bar{W}$	dimensionless net work

W_d	the work density (kJ/m ³)
$\overline{W_d}$	dimensionless work density.

Sub-indices

1-4, x, w, y, z	cycle points
a	investment cost parameter
i	fuel cost parameter
cc	classical Carnot cycle
f	fuel
i	investment
in	inlet
max	maximum
mc	modified Carnot cycle
min	minimum
out	outlet
P	pressure
th	thermal
v	volume.

Greek symbols

α	cycle temperature ratio
ε_T	isothermal compression ratio
η	efficiency
λ	economical parameter
ϕ	maximum volume parameter
ψ	maximum pressure parameter.