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RESEARCH PAPER

Multi-objective Solid Transportation Problem in Uncertain Environment

Hasan Dalman[1] · **Mustafa Sivri**[2]

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Abstract Although new information technologies decrease information and transaction costs, transportation problem of physical goods is still relevant. Globalization of trade increased the uncertainty that companies are facing, it also increased the importance of supply chain management in world economies drastically. Because transportation systems are crucial for operation management, the problem of finding efficient and sustainable solutions under such uncertain environments needs to be studied. Thus, this paper focuses on "multi-objective solid transportation problem (STP) in uncertain environment" and presents some approaches to find the compromised optimal solution of multi-objective STP. Then, compatible with uncertainty, fuzzy programming and some techniques of the fuzzy set theory are implemented to solve the problem by using Maple 17.02. A numerical example is executed and presented here to illustrate suggested procedures.

Keywords Multi-objective solid transportation · Interval numbers · Goal programming · Fuzzy programming · Global criteria

1 Introduction

The traditional transportation problem (TP) in real life deals with various issues such as selection of sources and delivery routes giving the destinations; handling and packing; financing and insurance; duty and taxes, and is related to other operations such as selection of production place and capacity, decision on outsourcing of production, hiring human capital, etc. Firms have to make related complex decisions under highly competitive and uncertain environments; this makes the transportation problem an important issue which affects a firm's competitive advantage and ability to satisfy customer expectations.

Traditional transportation programming uses deterministic coefficients which are determined by experts. However, in real life, experts decide under uncertainty and their knowledge is limited. That is why stochastic, interval and fuzzy methods are used to treat imprecise elements in real life decision tasks in operation research literature. While in real life the membership functions of decision makers are not well specified, in fuzzy programming, the membership functions of constraints and objectives are presumed to be known, and they are taken as fuzzy sets. In that case, uncertain interval numbers may be transformed into crisp numbers by using weight factors.

The solid transportation problem (STP) is an extension of the traditional TP in which three-dimensional properties are taken in the objective and constraint set instead of the supply and destination. The obligation of considering this private type of TP appears when heterogeneous conveyances are available for shipment of products. In many real life problems, a homogeneous product is obtained by its supply to a destination by way of distinct modes of transport called conveyances, such as trucks, cargo flights, goods trains, and ships. These conveyances are considered as the third dimension.

The TP was first developed by Hitchcock (1941). Shell (1955) first formulated the STP and then, Haley (1962) introduced a solution procedure of the STP.

Because of the complexity of the real life problems, we generally meet uncertain conditions in formulating the model

✉ Hasan Dalman
hsandalman@gmail.com

[1] Department of Computer Engineering, Gelisim University, Avcılar, 34310 Istanbul, Turkey

[2] Department of Mathematical Engineering, Yildiz Technical University, Esenler, 34210 Istanbul, Turkey

of real applications. For such a case, we commonly employ the uncertain variables (i.e., fuzzy, interval or stochastic) to the models. The concept of the fuzzy set theory is introduced by Bellman and Zadeh (1970). Zimmermann (1978) suggested the fuzzy programming approach for solving several objective functions. Up to now, the STP is studied and solved by many investigators in deterministic as well as uncertain environments. The STP based on the concept of fuzzy sets was suggested by Jimenez and Verdegay (1998, 1999). Bit et al. (1993) obtained an efficient solution of the STP using a fuzzy programming approach.

Nowadays, many types of multi-objective STP models with uncertain variables are investigated by Dalman et al. (2016), Kundu et al. (2014), Pramanik et al. (2013).

Model parameters in most mathematical programming problems need to be addressed as interval parameters due to weak data for a certain evaluation, but with familiar extreme limits of the parameters. The interval uncertainty theory was presented by Moore (1966). Ishibuchi and Tanaka (1990) used it for solving linear programming problems with interval objective functions by transforming those into the multi-objective programming problem. Chanas and Kuchta (1996) suggested α-level-cut of the intervals. Thus, they obtained the deterministic non-linear functions in the objective and constraints. In empirical science, some basic data analysis can be done with linear methods, but in general, these problems, also are non-linear, such as Ma and Lee (2009), Ma and Fan (2011), Mohyud-Din et al. (2009a, b, 2011a, b) and Noor et al. (2006). In interval uncertainty, Dalman et al. (2013) suggested a solution procedure for solving non-linear multi-objective TP.

In this paper, a multi-objective STP in uncertain environment is studied. The interval coefficients are replaced by equivalent deterministic numbers using weighted factor for any value between 0 and 1. Thus, a multi-objective STP in an uncertain environment is transformed into a deterministic one. Then, determining the upper and lower limits of each objective function subject to constraints, membership functions of each objective function are constructed using the upper and lower limits of the objective functions. Thus, two solution methods are used to obtain the optimal solution for the multi-objective STP in uncertain environment. Finally, a numerical example is presented to demonstrate the efficiency of the suggested procedures.

The rest of this paper is organized as follows: In Sect. 2, we recall some preliminary information about uncertain constraint programming and we formulate multi-objective uncertain STP with uncertain inequality constraint and also provide general information about the fuzzy goal programming method, global criterion method. Multi-objective STP with uncertain inequality constraints is introduced in Sect. 2. A numerical example is solved using Maple 17.02 in Sect. 3.

2 Problem Formulation

Definition 2.1 A interval number A is defined by known lower limit and upper limit but with unknown deterministic information for A. A interval number is denoted by $A = \left[\underset{\sim}{a}, \tilde{a}\right]$.

Definition 2.2 Let $A = \left[\underset{\sim}{a}, \tilde{a}\right]$ be a closed interval and θ be any constant number such that $0 \le \theta \le 1$, $(i = 1, 2, \ldots, n)$. A closed interval converted a deterministic number by θ-cut of $A = \left(\underset{\sim}{a}\theta + (1-\theta)\tilde{a}\right)$. This transform is called a weight factor.

2.1 The Multi-objective Solid Transportation Problem

In the solid transportation problem, the following concepts are satisfied:

- $m =$ number of sources of the TP.
- $n =$ number of destinations of the TP.
- $k =$ number of conveyances of different modes of transportation.
- $\left[\underset{\sim}{a}_i, \tilde{a}_i\right]$ uncertain amount of products available at ith origin.
- $\left[\underset{\sim}{b}_j, \tilde{b}_j\right]$ uncertain demand at jth destination.
- $\left[\underset{\sim}{e}_k, \tilde{e}_k\right]$ uncertain total capacity of conveyances of the TP.
- $X_{ijk} =$ the uncertain amount to be transported from ith origin to jth destination by means of kth conveyance of decision variables.
- $\left[\underset{\sim}{c}^r_{ijk}, \tilde{c}^r_{ijk}\right]$ per unit uncertain transportation cost from ith origin to jth destination by means of kth conveyance of rth, the objective function.

Then, the mathematical model of multi-objective STP can be given as follows:

$$\min Z^r(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\left[\underset{\sim}{c}^r_{ijk}, \tilde{c}^r_{ijk}\right]X_{ijk}$$

$$s.t. \begin{cases} \sum_{i=1}^{m} x_{ijk} \le \left[\underset{\sim}{a}_i, \tilde{a}_i\right], & i = 1, 2, \ldots, m \\ \sum_{j=1}^{n} x_{ijk} \ge \left[\underset{\sim}{b}_j, \tilde{b}_j\right], & j = 1, 2, \ldots, n \\ \sum_{k=1}^{p} x_{ijk} \le \left[\underset{\sim}{e}_k, \tilde{e}_k\right], & k = 1, 2, \ldots, p \\ x_{ijk} \ge 0, \forall ijk \end{cases}, \quad (1)$$

Definition 2.3 The function $z :\to I \ (I \in R)$ is called a closed and bounded interval function on the R^n and defined as $Z^r(X) = \left[\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \underset{\sim}{c}^{r}_{ijk} X_{ijk}, \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \tilde{c}^{r}_{ijk} X_{ijk}\right]$ where $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \underset{\sim}{c}^{r}_{ijk} X_{ijk}$ and $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \tilde{c}^{r}_{ijk} X_{ijk}$ are the lower limit and the upper limit of interval, respectively. Then, we have $\forall x_{ijk} \in X$ (X is the feasible region of problem) $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \underset{\sim}{c}^{r}_{ijk} X_{ijk} \le \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \tilde{c}^{r}_{ijk} X_{ijk}$.

Definition 2.4 $x^0 \in X$ is a non-dominate solution of problem (1), if and only if there is no other $x \in X$ such that $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X_{ijk} \le \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X^0_{ijk}$ for all $r = 1, 2, \ldots, k$. and $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X_{ijk} < \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X^0_{ijk}$ for some $r = 1, 2, \ldots, k$.

Definition 2.5 $x^0 \in X$ is an efficient solution of problem (1), iff there is no other $x \in X$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X_{ijk} \le \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X^0_{ijk} \quad \text{for all } r = 1, 2, \ldots, k,$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X_{ijk} \ne \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] X^0_{ijk} \quad \text{for all } r = 1, 2, \ldots, k.$$

Then, each closed and bounded interval is converted into deterministic ones by means of the combination of their left and right limits using weighted factor (from Definition 2.2) as follows:

$$\begin{cases} \left[\underset{\sim}{c}^{r}_{ijk}, \tilde{c}^{r}_{ijk}\right] \cong \underset{\sim}{c}^{r}_{ijk}\, \theta^r_{ijk} + (1-\theta^r_{ijk})\tilde{c}^r_{ijk}, \quad (i = 1, 2, \ldots, m), \\ \qquad (j = 1, 2, \ldots, n), \ (k = 1, 2, \ldots, p), \ (r = 1, 2, \ldots, k) \\ \left[\underset{\sim}{a}_i, \tilde{a}_i\right] \cong \underset{\sim}{a}_i, \alpha_i + (1-\alpha_i)\tilde{a}_i, \quad (i = 1, 2, \ldots, m), \\ \left[\underset{\sim}{b}_j, \tilde{b}_j\right] \cong \underset{\sim}{b}_j, \beta_j + (1-\beta_i)\tilde{b}_i, \quad (j = 1, 2, \ldots, n), \\ \left[\underset{\sim}{e}_k, \tilde{e}_k\right] \cong \underset{\sim}{e}_k, \delta_k + (1-\delta_i)\tilde{e}_k, \quad (k = 1, 2, \ldots, p), \end{cases} \tag{2}$$

where $\theta^r_{ijk}, \alpha_i, \beta_i, \delta_k \in [0, 1]$.

Using the deterministic parameters (2), the multi-objective STP in uncertain environment is converted to the following deterministic multi-objective STP:

$$\min Z^r(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \underset{\sim}{c}^{r}_{ijk}\, \theta^r_{ijk} + (1-\theta^r_{ijk})\tilde{c}^r_{ijk}, \quad r = 1, 2, \ldots, k$$

s.t.

$$G = \begin{cases} \sum_{i=1}^{m} x_{ijk} \le \underset{\sim}{a}_i\, \alpha_i + (1-\alpha_i)\tilde{a}_i, \quad i = 1, 2, \ldots, m \\ \sum_{j=1}^{n} x_{ijk} \ge \underset{\sim}{b}_j, \beta_j + (1-\beta_j)\tilde{b}_j, \quad j = 1, 2, \ldots, n \\ \sum_{k=1}^{p} x_{ijk} \le \underset{\sim}{e}_k, \delta_k + (1-\delta_K)\tilde{e}_k, \quad k = 1, 2, \ldots, p \\ x_{ijk} \ge 0, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, p \\ \lambda, \alpha_i, \beta_j, \delta_k, \theta^r_{ijk} \in [0, 1] \end{cases} \tag{3}$$

2.2 An Interactive Fuzzy Goal Programming Approach for the Multi-objective STP

Bellman and Zadeh (1970) introduced the basic concepts of fuzzy goals for fuzzy decision. Zimmermann (1978) suggested a fuzzy programming approach to solve the multi-objective problem and he obtained efficient solutions of the multi-objective problem.

Here, we first introduce the fuzzy method for solving multi-objective STP. After determining the upper and lower limit values of each objective function (3), we can construct the membership functions of each objective as follows:

$$\mu(Z^r(x)) = \begin{cases} 1 & Z^r(x) \le (Z^r)^{\min} \\ \dfrac{(Z^r)^{\max} - Z^r}{(Z^r)^{\max} - (Z^r)^{\min}}, & (Z^r)^{\min} \le Z^r(x) \le (Z^r)^{\max} \\ 0 & Z^r \ge (Z^r)^{\max} \end{cases} \tag{4}$$

where $(Z^r)^{\min} = \min Z^r$, $(r = 1, 2, \ldots, k)$, is the lower limit of the objective function r and $(Z^r)^{\max} = \max Z^r$, $(r = 1, 2, \ldots, k)$ is the upper limit of each objective function r.

Using fuzzy decision of Bellman and Zadeh (1970), multi-objective STP (3) can be constructed as a single objective programming problem as follows:

$$\max \lambda$$
$$\text{s.t.}\begin{cases} \lambda \leq \mu(Z^r) \\ \sum_{i=1}^{m} x_{ijk} \leq \underset{\sim}{a}_i, \alpha_i + (1-\alpha_i)\tilde{a}_i, \quad i = 1,2,\ldots,m \\ \sum_{j=1}^{n} x_{ijk} \geq \underset{\sim}{b}_j, \beta_j + (1-\beta_i)\tilde{b}_i, \quad j = 1,2,\ldots,n \\ \sum_{k=1}^{p} x_{ijk} \leq \underset{\sim}{e}_k, \delta_k + (1-\delta_i)\tilde{e}_k, \quad k = 1,2,\ldots,p \\ \lambda, \alpha_i, \beta_j, \delta_k, \theta_{ijk}^r \in [0,1], \quad (i = 1,2,\ldots,m), \\ \quad (j = 1,2,\ldots,n), \quad (k = 1,2,\ldots,p) \\ x_{ijk} \geq 0, \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n, \quad k = 1,2,\ldots,p \end{cases} \tag{5}$$

where $\lambda = \min\{\mu(Z^r(x)\}, (r = 1,2,\ldots k)$

Now, we will formulate a fuzzy goal programming model. Tiwari et al. (1987) formulated a fuzzy goal programming using the weighted sum method. Then, the fuzzy goal programming approach to multi-objective programming problems was introduced by Mohamed (1997). There are many fuzzy goal programming approaches for many mathematical programming problems such as El-Wahed and Lee (2006) and Sakawa (1993).

Here, we will present a fuzzy goal programming method for solving multi-objective STP (3). To construct the problem (5) as a fuzzy goal programming problem, we consider the negative deviational variables $n^r \leq 0$ and the positive deviational variables $p^r \geq 0$ and goals of each objective function in (3) can be written as follows:

$$Z^r - p^r + n^r \leq (Z^r)^{\min} \tag{6}$$

where $(Z^r)^{\min}$ is the aspiration level of the objective function r Then an interactive fuzzy goal programming model can be formulated as follows:

$$\max \lambda$$
$$\text{s.t.}$$
$$\begin{cases} \lambda \leq \mu(Z^r) \\ Z^r - p_{ijk}^{+} + n_{ijk}^{-} \leq (Z^r)^{\min} \\ \sum_{i=1}^{m} x_{ijk} \leq \underset{\sim}{a}_i, \alpha_i + (1-\alpha_i)\tilde{a}_i, \quad i = 1,2,\ldots,m \\ \sum_{j=1}^{n} x_{ijk} \geq \underset{\sim}{b}_j, \beta_j + (1-\beta_i)\tilde{b}_i, \quad j = 1,2,\ldots,n \\ \sum_{k=1}^{p} x_{ijk} \leq \underset{\sim}{e}_k, \delta_k + (1-\delta_i)\tilde{e}_k, \quad k = 1,2,\ldots,p \\ \lambda, \alpha_i, \beta_j, \delta_k, \theta_{ijk}^r \in [0,1], \quad (i = 1,2,\ldots,m), \\ \quad (j = 1,2,\ldots,n), \quad (k = 1,2,\ldots,p) \\ x_{ijk} \geq 0, \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n, \quad k = 1,2,\ldots,p \end{cases} \tag{7}$$

where $\lambda = \min\{\mu(Z^r(x)\}, (r = 1,2,\ldots k)$

This problem can be easily solved as a single objective programming problem. Then, the optimal solutions of multi-objective STP are obtained.

Moreover, if the decision maker is satisfied by the current solution, then, the solution procedure terminates. Otherwise, in (6), the aspiration level is changed with the obtained solutions of problem (7).

2.3 Global Criteria Method for the Multi-objective STP

The global criteria method gives a compromised optimal solution of multi-objective programming problem. This method is a way of achieving compromise in minimizing the sum in derivations of the best solutions (ideal solutions) from the relative objective functions. The solution procedure for the multi-objective STP can be summarized as follows:

- Solve the multi-objective STP (3) for each objective function with constraints of $Z^r(x), (r = 1,2,\ldots,k)$ separately.
- Generate the ideal objective vector $(Z^r)^{\min}$ and $(Z^r)^{\max}$ $(r = 1,2,\ldots,k)$
- Then, the multi-objective STP (3) can be converted to a single objective problem using global criteria method as follows:

$$\min \; Z^r(x), (r = 1,2,\ldots,k)$$
$$\text{s.t.}$$
$$G = \begin{cases} \sum_{i=1}^{m} x_{ijk} \leq \underset{\sim}{a}_i, \alpha_i + (1-\alpha_i)\tilde{a}_i, \quad i = 1,2,\ldots,m \\ \sum_{j=1}^{n} x_{ijk} \geq \underset{\sim}{b}_j, \beta_j + (1-\beta_i)\tilde{b}_i, \quad j = 1,2,\ldots,n \\ \sum_{k=1}^{p} x_{ijk} \leq \underset{\sim}{e}_k, \delta_k + (1-\delta_i)\tilde{e}_k, \quad k = 1,2,\ldots,p \\ \lambda, \alpha_i, \beta_j, \delta_k, \theta_{ijk}^r \in [0,1], \\ x_{ijk} \leq 0, \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n, \\ \quad k = 1,2,\ldots,p \end{cases} \tag{8}$$

where $Z^r(x) = \min\left\{\sum_{r=1}^{k}\left(\frac{Z_r(x)-(Z^r)^{\min}}{(Z^r)^{\min}}\right)^q\right\}^{1/q}$ or $Z^r(x) = \min\left\{\sum_{r=1}^{k}\left(\frac{Z_r(x)-(Z^r)^{\min}}{(Z^r)^{\max}-(Z^r)^{\min}}\right)^q\right\}^{1/q}$ where $1 \leq q \leq \infty$. In this method, generally taken as $q = 2$ in L_2 norms.

3 A Numerical Example

To illustrate the proposed approach, let us consider the data in Tables 1 and 2.

Transportation cost for 1st objective $\left[\underset{\sim}{c}^{1}_{ijk}, \tilde{c}^{1}_{ijk}\right]$

$$\underset{\sim}{c}^{1}_{ijk} = \begin{bmatrix} \underset{\sim}{c}^{(1)}_{111} & \underset{\sim}{c}^{(1)}_{112} & \underset{\sim}{c}^{(1)}_{121} \\ \underset{\sim}{c}^{(1)}_{122} & \underset{\sim}{c}^{(1)}_{131} & \underset{\sim}{c}^{(1)}_{132} \\ \underset{\sim}{c}^{(1)}_{211} & \underset{\sim}{c}^{(1)}_{212} & \underset{\sim}{c}^{(1)}_{221} \\ \underset{\sim}{c}^{(1)}_{222} & \underset{\sim}{c}^{(1)}_{231} & \underset{\sim}{c}^{(1)}_{232} \end{bmatrix} = \begin{bmatrix} 13/2 & 10 & 5 \\ 7 & 11 & 8 \\ 9 & 21/2 & 13/2 \\ 7 & 12 & 15 \end{bmatrix},$$

$$\tilde{c}^{(1)}_{ijk} = \begin{bmatrix} \tilde{c}^{(1)}_{111} & \tilde{c}^{(1)}_{112} & \tilde{c}^{(1)}_{121} \\ \tilde{c}^{(1)}_{122} & \tilde{c}^{(1)}_{131} & \tilde{c}^{(1)}_{132} \\ \tilde{c}^{(1)}_{211} & \tilde{c}^{(1)}_{212} & \tilde{c}^{(1)}_{221} \\ \tilde{c}^{(1)}_{222} & \tilde{c}^{(1)}_{231} & \tilde{c}^{(1)}_{232} \end{bmatrix} = \begin{bmatrix} 10 & 14 & 10 \\ 11 & 15 & 13 \\ 14 & 14 & 17/2 \\ 11 & 33/2 & 17 \end{bmatrix},$$

Transportation cost for 2nd objective $\left[\underset{\sim}{c}^{2}_{ijk}, \tilde{c}^{2}_{ijk}\right]$

$$\underset{\sim}{c}^{2}_{ijk} = \begin{bmatrix} \underset{\sim}{c}^{(2)}_{111} & \underset{\sim}{c}^{(2)}_{112} & \underset{\sim}{c}^{(2)}_{121} \\ \underset{\sim}{c}^{(2)}_{122} & \underset{\sim}{c}^{(2)}_{131} & \underset{\sim}{c}^{(2)}_{132} \\ \underset{\sim}{c}^{(2)}_{211} & \underset{\sim}{c}^{(2)}_{212} & \underset{\sim}{c}^{(2)}_{221} \\ \underset{\sim}{c}^{(2)}_{222} & \underset{\sim}{c}^{(2)}_{231} & \underset{\sim}{c}^{(2)}_{232} \end{bmatrix} = \begin{bmatrix} 19/2 & 12 & 13/2 \\ 13/2 & 21/2 & 27/2 \\ 12 & 15 & 8 \\ 10 & 13 & 27/2 \end{bmatrix},$$

$$\tilde{c}^{(2)}_{ijk} = \begin{bmatrix} \tilde{c}^{(2)}_{111} & \tilde{c}^{(2)}_{112} & \tilde{c}^{(2)}_{121} \\ \tilde{c}^{(2)}_{122} & \tilde{c}^{(2)}_{131} & \tilde{c}^{(2)}_{132} \\ \tilde{c}^{(2)}_{211} & \tilde{c}^{(2)}_{212} & \tilde{c}^{(2)}_{221} \\ \tilde{c}^{(2)}_{222} & \tilde{c}^{(2)}_{231} & \tilde{c}^{(2)}_{232} \end{bmatrix} = \begin{bmatrix} 25/2 & 29/2 & 11 \\ 10 & 12 & 12 \\ 13 & 19 & 13 \\ 27/2 & 17 & 31/2 \end{bmatrix},$$

Here, insert data for origins and destination, two different conveyances, respectively, as follows:

Table 1 Interval cost functions for first objective function Z_1

I	J			J		
	1	2	3	1	2	3
1	[13/2,10]	[5, 10]	[11, 15]	[10, 14]	[7, 11]	[8,13]
2	[9,14]	[13/2,17/2]	[12,33/2]	[21/2,14]	[7, 11]	[15,17]
K	1			2		

$$a_1 = \left[\frac{45}{2}, 27\right], \quad a_2 = [30, 36],$$

$$b_1 = \left[15, \frac{41}{2}\right], \quad b_2 = \left[\frac{37}{2}, \frac{47}{2}\right], \quad b_3 = \left[\frac{27}{2}, \frac{39}{2}\right],$$

$$e_1 = \left[\frac{95}{2}, 52\right], \quad e_2 = \left[52, \frac{115}{2}\right].$$

Then, using these data tables, the multi-objective STP in uncertain environment can be formulated as follows:

$$\min Z^1(X) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underset{\sim}{c}^{1}_{ijk}, \tilde{c}^{1}_{ijk}\right]X_{ijk}$$

$$\min Z^2(X) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underset{\sim}{c}^{2}_{ijk}, \tilde{c}^{2}_{ijk}\right]X_{ijk}$$

subject to

$$\begin{cases} \sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} \leq\ = a_1 = \left[\frac{45}{2}, 27\right], \sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} \\ \quad \leq\ a_2 = [30, 36]; \ (j = 1,2,3), \ (k = 1,2), \\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i1k} \geq b_1 = \left[15, \frac{41}{2}\right], \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i2k} \ \geq \\ \quad b_2 = \left[\frac{37}{2}, \frac{47}{2}\right] \ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i3k} \geq b_3 = \left[\frac{27}{2}, \frac{39}{2}\right]; \\ \quad (i = 1,2), \ (k = 1,2). \\ \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij1} \leq\ e_1 = \left[\frac{95}{2}, 52\right] \ , \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij2} \\ \quad \leq\ e_2 = \left[52, \frac{115}{2}\right]; \ (i = 1,2), \ (j = 1,2,3), \\ x_{ijk} \geq 0, \ i = 1,2, \ j = 1,2,3, \ k = 1,2. \end{cases}$$

$\Rightarrow$

$$\min Z^1 = \left[\frac{13}{2}, 10\right]x_{111} + [5, 10]x_{121} + [11, 15]x_{131} + [9, 14]x_{211} + \left[\frac{13}{2}, \frac{17}{2}\right]x_{221} + \left[12, \frac{33}{2}\right]x_{231} + [10, 14]x_{112} + [7, 11]x_{122} + [8, 13]x_{132}$$

Table 2 Interval cost functions for second objective function Z_2

I	J			J		
	1	2	3	1	2	3
1	[19/2,25/2]	[13/2.11]	[21/2,12]	[12,29/2]	[13/2,10]	[9,12]
2	[12,13]	[8,13]	[13,17]	[15,19]	[10,27/2]	[27/2,31/2]
K	1			2		

$$+\left[\frac{21}{2},4\right]x_{212}+[7,11]x_{222}+[15,17]x_{232}$$

$$\min Z^2=\left[\frac{19}{2},13\right]x_{111}+\left[\frac{13}{2},11\right]x_{121}+\left[\frac{21}{2},12\right]x_{131}+[12,13]x_{211}+[8,13]x_{221}+[13,17]x_{231}+\left[12,\frac{29}{2}\right]x_{112}+\left[\frac{13}{2},10\right]x_{122}+[9,12]x_{132}+[15,19]x_{212}+\left[10,\frac{27}{2}\right]x_{222}+\left[\frac{27}{2},\frac{31}{2}\right]x_{232}$$

$$s.t\begin{cases}X_{111}+X_{121}+X_{131}+X_{112}+X_{122}+X_{132}\le a_1=\left[\frac{45}{2},27\right]\\ X_{211}+X_{221}+X_{231}+X_{212}+X_{222}+X_{232}\le a_2=[30,36]\\ X_{111}+X_{211}+X_{112}+X_{212}\ge b_1=\left[15,\frac{41}{2}\right]\\ X_{121}+X_{221}+X_{222}+X_{122}\ge b_1=\left[15,\frac{41}{2}\right]\\ X_{131}+X_{231}+X_{132}+X_{232}\ge b_3=\left[\frac{27}{2},\frac{39}{2}\right]\\ X_{111}+X_{121}+X_{131}+X_{221}+X_{231}\le e_1=\left[\frac{95}{2},52\right]\\ X_{112}+X_{122}+X_{132}+X_{212}+X_{232}\le e_2=\left[52,\frac{115}{2}\right]\\ X_{ijk}\ge 0,\ i=1,\ 2,\ j=1,2,3,\ k=1,2.\end{cases}\tag{9}$$

3.1 Method 1

Using Definition 2.2 and Eq. (2), problem (9) can be converted into deterministic programming problem as follows:

$$\min Z^1=\left(-\frac{7}{2}\theta_{111}^{(1)}+10\right)x_{111}+(-5\theta_{121}^{(1)}+10)x_{121}+(-4\theta_{131}^{(1)}+15)x_{131}+(-5\theta_{211}^{(1)}+14)x_{211}+\left(-2\theta_{221}^{(1)}+\frac{17}{2}\right)x_{221}+\left(-\frac{9}{2}\theta_{231}^{(1)}+\frac{33}{2}\right)x_{231}+(-4\theta_{112}^{(1)}+14)x_{112}+(-4\theta_{122}^{(1)}+11)x_{122}+(-5\theta_{132}^{(1)}+13)x_{132}+\left(-\frac{7}{2}\theta_{212}^{(1)}+14\right)x_{212}+(-4\theta_{222}^{(1)}+11)x_{222}+(-2\theta_{232}^{(1)}+17)x_{232}$$

$$\min Z^2=\left(-3\theta_{111}^{(2)}+\frac{25}{2}\right)x_{111}+\left(-\frac{9}{2}\theta_{121}^{(2)}+11\right)x_{121}+\left(-\frac{3}{2}\theta_{131}^{(2)}+12\right)x_{131}+(-\theta_{211}^{(2)}+13)x_{211}+(-5\theta_{221}^{(2)}+13)x_{221}+(-4\theta_{231}^{(2)}+17)x_{231}+\left(-\frac{5}{2}\theta_{112}^{(2)}+\frac{29}{2}\right)x_{112}+\left(-\frac{7}{2}\theta_{122}^{(2)}+10\right)x_{122}+(-3\theta_{132}^{(2)}+12)x_{132}+(-4\theta_{212}^{(2)}+19)x_{212}+\left(-\frac{7}{2}\theta_{222}^{(2)}+\frac{27}{2}\right)x_{222}+\left(-2\theta_{232}^{(2)}+\frac{31}{2}\right)x_{232}$$

$$\text{s.t.}\begin{cases}X_{111}+X_{121}+X_{131}+X_{112}+X_{122}+X_{132}\\ \quad\le -4.5\alpha_1+27,\\ X_{211}+X_{221}+X_{231}+X_{212}+X_{222}+X_{232}\\ \quad\le -6.5\alpha_2+36,\\ -6.5\beta_1+20.5\le X_{111}+X_{211}+X_{112}+X_{212},\\ -5\beta_2+23.5\le X_{121}+X_{221}+X_{222}+X_{122},\\ -6\beta_3+19.5\le X_{131}+X_{231}+X_{132}+X_{232},\\ X_{111}+X_{121}+X_{131}+X_{221}+X_{231}\le 4.5\delta_1+52,\\ X_{112}+X_{122}+X_{132}+X_{212}+X_{232}\\ \quad\le -6.5\delta_2+57.5,\\ \alpha_i,\beta_j,\delta_k,\theta_{ijk}^{(1)},\theta_{ijk}^{(2)}\in[0,1],\ i=1,2,\\ =1,2,3,\ k=1,2.\\ X_{ijk}\ge 0,\ i=1,2,\ j=1,2,3,\ k=1,2.\end{cases}\tag{10}$$

The best lower limit and worst upper limits of each objective function are calculated as follows:

$$(Z^1)^{\min}=329.5,(Z^1)^{\max}=915 \text{ and } (Z^2)^{\min}=415.75,(Z^2)^{\max}=983.75$$

The upper and lower limits of each objective function can be arranged as follows:

$329.5\le Z_1\le 915$ and $415.75\le Z_2\le 983.75$

The membership functions of each objective function in Eq. (4) are determined as follows:

$$\mu(Z^1(x))=\begin{cases}1 & Z^1(x)\le 329.5\\ \dfrac{329.5-Z^1(x)}{915-329.5}, & 329.5\le Z^r(x)\le 915,\\ 0 & Z^r\ge 915\end{cases}\tag{11}$$

$$\mu(Z^2(x))=\begin{cases}1 & Z^2(x)\le 415.75\\ \dfrac{415.75-Z^2(x)}{983.75-415.75}, & 415.75\le Z^2(x)\le 983.75\\ 0 & Z^2\ge 983.75\end{cases}\tag{12}$$

$\Rightarrow$

$$\begin{aligned}\mu(z^1) = 1.56 &- 0.00171(-3.50\theta_{111}^{(1)} + 10.)x_{111} \\ &- 0.00171(-5._{121}^{(1)} + 10.)x_{121} - 0.00171(-4._{131}^{(1)} + 15.)x_{131} \\ &- 0.00171(-5.\theta_{211}^{(1)} + 14.)x_{211} - 0.00171(-2\theta_{221}^{(1)} + 850)x_{221} \\ &- 0.00171(-4.50\theta_{231}^{(1)} + 16.5)x_{231} \\ &- 0.00171(-4.\theta_{112}^{(1)} + 14.)x_{112} - 0.00171(-4\theta_{122}^{(1)} + 11)x_{122} \\ &- 0.00171(-5.\theta_{132}^{(1)} + 13.)x_{132} \\ &- 0.00171(-3.50\theta_{212}^{(1)} + 14.)x_{212} - 0.00171(-4.\theta_{222}^{(1)} + 11.)x_{222} \\ &- 0.00171(-2.\theta_{232}^{(1)} + 17.)x_{232}\end{aligned} \tag{13}$$

$$\begin{aligned}\mu(z^2) = 1.73 &- 0.00176(-3.\theta_{111}^{(2)} + 12.5)x_{111} \\ &- 0.00176(-4.50\theta_{121}^{(2)} + 11.)x_{121} \\ &- 0.00176(-1.50\theta_{131}^{(2)} + 12.)x_{131} \\ &- 0.00176(-1.\theta_{211}^{(2)} + 13.)x_{211} \\ &- 0.00176(-5\theta_{221}^{(2)} + 13.)x_{221} \\ &- 0.00176(-4.\theta_{231}^{(2)} + 17.)x_{231} \\ &- 0.00176(-2.50\theta_{112}^{(2)} + 14.5)x_{112} \\ &- 0.00176(-3.50\theta_{122}^{(2)} + 10.)x_{122} \\ &- 0.00176(-3.\theta_{132}^{(2)} + 12.)x_{132} \\ &- 0.00176(-4.\theta_{212}^{(2)} + 19.)x_{212} \\ &- 0.00176(-3.50\theta_{222}^{(2)} + 13.5)x_{222} \\ &- 0.00176(-2.\theta_{232}^{(2)} + 15.5)x_{232}\end{aligned} \tag{14}$$

Then, corresponding fuzzy programming model of problem (10) is given as follows:

$$\max \lambda$$

$$\text{s.t.}\begin{cases}\lambda \leq 1.56 - 0.00171(-3.50\theta_{111}^{(1)} + 10.)x_{111} \\ -0.00171(-5._{121}^{(1)} + 10.)x_{121} - 0.00171(-4._{131}^{(1)} + 15.)x_{131} \\ -0.00171(-5.\theta_{211}^{(1)} + 14.)x_{211} - 0.00171(-2\theta_{221}^{(1)} + 850)x_{221} \\ -0.00171(-4.50\theta_{231}^{(1)} + 16.5)x_{231} \\ -0.00171(-4.\theta_{112}^{(1)} + 14.)x_{112} - 0.00171(-4\theta_{122}^{(1)} + 11)x_{122} \\ -0.00171(-5.\theta_{132}^{(1)} + 13.)x_{132} \\ -0.00171(-3.50\theta_{212}^{(1)} + 14.)x_{212} - 0.00171(-4.\theta_{222}^{(1)} + 11.)x_{222} \\ -0.00171(-2.\theta_{232}^{(1)} + 17.)x_{232} \\ \lambda \leq 1.73 - 0.00176(-3.\theta_{111}^{(2)} + 12.5)x_{111} - 0.00176(-4.50\theta_{121}^{(2)} \\ +11.)x_{121} - 0.00176(-1.50\theta_{131}^{(2)} + 12.)x_{131} \\ -0.00176(-1.\theta_{211}^{(2)} + 13.)x_{211} - 0.00176(-5\theta_{221}^{(2)} + 13.)x_{221} \\ -0.00176(-4.\theta_{231}^{(2)} + 17.)x_{231} \\ -0.00176(-2.50\theta_{112}^{(2)} + 14.5)x_{112} \\ -0.00176(-3.50\theta_{122}^{(2)} + 10.)x_{122} - 0.00176(-3.\theta_{132}^{(2)} + 12.)x_{132} \\ -0.00176(-4.\theta_{212}^{(2)} + 19.)x_{212} \\ -0.00176(-3.50\theta_{222}^{(2)} + 13.5)x_{222} \\ -0.00176(-2.\theta_{232}^{(2)} + 15.5)x_{232} \\ X_{111} + X_{121} + X_{131} + X_{112} + X_{122} + X_{132} \leq -4.5\alpha_1 + 27, \\ X_{211} + X_{221} + X_{231} + X_{212} + X_{222} + X_{232} \leq -6.5\alpha_2 + 36, \\ -6.5\beta_1 + 20.5 \leq X_{111} + X_{211} + X_{112} + X_{212}, \\ -5\beta_2 + 23.5 \leq X_{121} + X_{221} + X_{222} + X_{122}, \\ -6\beta_3 + 19.5 \leq X_{131} + X_{231} + X_{132} + X_{232}, \\ X_{111} + X_{121} + X_{131} + X_{221} + X_{231} \leq 4.5\delta_1 + 52, \\ X_{112} + X_{122} + X_{132} + X_{212} + X_{232} \leq -6.5\delta_2 + 57.5, \\ X_{ijk} \geq 0, \alpha_i, \beta_j, \delta_k, \theta_{ijk}^{(1)}, \theta_{ijk}^{(2)} \in [0,1],\ i = 1,2,\ j = 1,2,3,\ k = 1,2\end{cases}$$

The corresponding optimal solutions of problem (10) are:

$$X_{ijk} = \begin{bmatrix} X_{111} & X_{112} & X_{113} \\ X_{122} & X_{131} & X_{132} \\ X_{211} & X_{212} & X_{221} \\ X_{222} & X_{231} & X_{232} \end{bmatrix} = \begin{bmatrix} 13.5 & 0 & 0 \\ 0 & 0 & 13.5 \\ 1.5 & 0 & 18/5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\theta_{ijk}^{(1)} \begin{bmatrix} \theta_{111}^{(1)} & \theta_{112}^{(1)} & \theta_{121}^{(1)} \\ \theta_{122}^{(1)} & \theta_{131}^{(1)} & \theta_{132}^{(1)} \\ \theta_{211}^{(1)} & \theta_{212}^{(1)} & \theta_{221}^{(1)} \\ \theta_{222}^{(1)} & \theta_{231}^{(1)} & \theta_{232}^{(1)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

$$\theta_{ijk}^{(2)} \begin{bmatrix} \theta_{111}^{(2)} & \theta_{112}^{(2)} & \theta_{121}^{(2)} \\ \theta_{122}^{(2)} & \theta_{131}^{(2)} & \theta_{132}^{(2)} \\ \theta_{211}^{(2)} & \theta_{212}^{(2)} & \theta_{221}^{(2)} \\ \theta_{222}^{(2)} & \theta_{231}^{(2)} & \theta_{232}^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\alpha_1 = 0, \alpha_2 = 1, \beta_1 = 1, \beta_2 = 1, \beta_3 = 1, \delta_1 = 1, \delta_2 = 1 \text{ and } \lambda = 1$$

The degree of membership functions (13) and (14) are $\mu(z^1) = 1$ and $\mu(z^2) = 1$, respectively.

Remark The feasible sets of problems (9) and (10) are identical to each other. Assume that problems (9) and (10) use the same solution procedures. Then (9) and (10) have the same optimal solutions.

Therefore, the corresponding optimal values of objective functions for problem (9) are $Z^1(x) = [329.5, 488.75]$, $Z^2(x) = [415.75, 590.75]$

Now, to construct a fuzzy goal programming, we consider the negative deviational variables $n^1, n^2 \geq 0$ and the positive deviational variables $p^1, p^2 \geq 0$ and goals of each objective function in (10) can be written as follows:

$$\begin{cases} z^1 = \left(-\frac{7}{2}\theta_{111}^{(1)} + 10\right)x_{111} + (-5\theta_{121}^{(1)} + 10)x_{121} \\ \quad + (-4\theta_{131}^{(1)} + 15)x_{131} + (-5\theta_{211}^{(1)} + 14)x_{211} + \left(-2\theta_{221}^{(1)} + \frac{17}{2}\right)x_{221} \\ \quad + \left(-\frac{9}{2}\theta_{231}^{(1)} + \frac{33}{2}\right)x_{231} + (-4\theta_{112}^{(1)} + 14)x_{112} + (-4\theta_{122}^{(1)} + 11)x_{122} \\ \quad + (-5\theta_{132}^{(1)} + 13)x_{132} + \left(-\frac{7}{2}\theta_{212}^{(1)} + 14\right)x_{212} \\ \quad + (-4\theta_{222}^{(1)} + 11)x_{222} + (-2\theta_{232}^{(1)} + 17)x_{232} - p^1 + n^1 = 915 \\ Z^2 = \left(-3\theta_{111}^{(2)} + \frac{25}{2}\right)x_{111} + \left(-\frac{9}{2}\theta_{121}^{(2)} + 11\right)x_{121} \\ \quad + \left(-\frac{3}{2}\theta_{131}^{(2)} + 12\right)x_{131} + (-\theta_{211}^{(2)} + 13)x_{211} + (-5\theta_{221}^{(2)} + 13)x_{221} \\ \quad + (-4\theta_{231}^{(2)} + 17)x_{231} + \left(-\frac{5}{2}\theta_{112}^{(2)} + \frac{29}{2}\right)x_{112} \\ \quad + \left(-\frac{7}{2}\theta_{122}^{(2)} + 10\right)x_{122} + (-3\theta_{132}^{(2)} + 12)x_{132} + (-4\theta_{212}^{(2)} + 19)x_{212} \\ \quad + \left(-\frac{7}{2}\theta_{222}^{(2)} + \frac{27}{2}\right)x_{222} + \left(-2\theta_{232}^{(2)} + \frac{31}{2}\right)x_{232} - p^2 + n^2 = 983.75 \end{cases}$$

Then the equivalent interactive fuzzy programming problem is constructed as follows:

max λ

s.t.

$$\begin{cases} z^1 = \left(-\frac{7}{2}\theta_{111}^{(1)} + 10\right)x_{111} + (-5\theta_{121}^{(1)} + 10)x_{121} \\ \quad + (-4\theta_{131}^{(1)} + 15)x_{131} + (-5\theta_{211}^{(1)} + 14)x_{211} \\ \quad + \left(-2\theta_{221}^{(1)} + \frac{17}{2}\right)x_{221} \\ \quad + \left(-\frac{9}{2}\theta_{231}^{(1)} + \frac{33}{2}\right)x_{231} + (-4\theta_{112}^{(1)} + 14)x_{112} \\ \quad + (-4\theta_{122}^{(1)} + 11)x_{122} + (-5\theta_{132}^{(1)} + 13)x_{132} + \left(-\frac{7}{2}\theta_{212}^{(1)} + 14\right)x_{212} \\ \quad + (-4\theta_{222}^{(1)} + 11)x_{222} + (-2\theta_{232}^{(1)} + 17)x_{232} - p^1 + n^1 = 915 \\ Z^2 = \left(-3\theta_{111}^{(2)} + \frac{25}{2}\right)x_{111} + \left(-\frac{9}{2}\theta_{121}^{(2)} + 11\right)x_{121} \\ \quad + \left(-\frac{3}{2}\theta_{131}^{(2)} + 12\right)x_{131} + (-\theta_{211}^{(2)} + 13)x_{211} + (-5\theta_{221}^{(2)} + 13)x_{221} \\ \quad + (-4\theta_{231}^{(2)} + 17)x_{231} + \left(-\frac{5}{2}\theta_{112}^{(2)} + \frac{29}{2}\right)x_{112} \\ \quad + \left(-\frac{7}{2}\theta_{122}^{(2)} + 10\right)x_{122} + (-3\theta_{132}^{(2)} + 12)x_{132} + (-4\theta_{212}^{(2)} + 19)x_{212} \\ \quad + \left(-\frac{7}{2}\theta_{222}^{(2)} + \frac{27}{2}\right)x_{222} + \left(-2\theta_{232}^{(2)} + \frac{31}{2}\right)x_{232} \\ \quad - p^2 + n^2 = 983.75 \end{cases}$$

$$\begin{cases} \lambda \leq 1.56 - 0.00171(-3.50\theta_{111}^{(1)} + 10.)x_{111} \\ \quad - 0.00171(-5.{}_{121}^{(1)} + 10.)x_{121} - 0.00171(-4.{}_{131}^{(1)} + 15.)x_{131} \\ \quad - 0.00171(-5.\theta_{211}^{(1)} + 14.)x_{211} - 0.00171(-2\theta_{221}^{(1)} + 850)x_{221} \\ \quad - 0.00171(-4.50\theta_{231}^{(1)} + 16.5)x_{231} \\ \quad - 0.00171(-4.\theta_{112}^{(1)} + 14.)x_{112} - 0.00171(-4\theta_{122}^{(1)} + 11)x_{122} \\ \quad - 0.00171(-5.\theta_{132}^{(1)} + 13.)x_{132} \\ \quad - 0.00171(-3.50\theta_{212}^{(1)} + 14.)x_{212} \\ \quad - 0.00171(-4.\theta_{222}^{(1)} + 11.)x_{222} \\ \quad - 0.00171(-2.\theta_{232}^{(1)} + 17.)x_{232} \\ \lambda \leq 1.73 - 0.00176(-3.\theta_{111}^{(2)} + 12.5)x_{111} \\ \quad - 0.00176(-4.50\theta_{121}^{(2)} + 11.)x_{121} \\ \quad - 0.00176(-1.50\theta_{131}^{(2)} + 12.)x_{131} \\ \quad - 0.00176(-1.\theta_{211}^{(2)} + 13.)x_{211} - 0.00176(-5\theta_{221}^{(2)} + 13.)x_{221} \\ \quad - 0.00176(-4.\theta_{231}^{(2)} + 17.)x_{231} \\ \quad - 0.00176(-2.50\theta_{112}^{(2)} + 14.5)x_{112} \\ \quad - 0.00176(-3.50\theta_{122}^{(2)} + 10.)x_{122} \\ \quad - 0.00176(-3.\theta_{132}^{(2)} + 12.)x_{132} \\ \quad - 0.00176(-4.\theta_{212}^{(2)} + 19.)x_{212} \\ \quad - 0.00176(-3.50\theta_{222}^{(2)} + 13.5)x_{222} \\ \quad - 0.00176(-2.\theta_{232}^{(2)} + 15.5)x_{232} \end{cases}$$

$$\begin{cases} X_{111}+X_{121}+X_{131}+X_{112}+X_{122}+X_{132}\leq -4.5\alpha_1+27, \\ X_{211}+X_{221}+X_{231}+X_{212}+X_{222}+X_{232}\leq -6.5\alpha_2+36, \\ -6.5\beta_1+20.5\leq X_{111}+X_{211}+X_{112}+X_{212}, \\ -5\beta_2+23.5\leq X_{121}+X_{221}+X_{222}+X_{122}, \\ -6\beta_3+19.5\leq X_{131}+X_{231}+X_{132}+X_{232}, \\ X_{111}+X_{121}+X_{131}+X_{221}+X_{231}\leq 4.5\delta_1+52, \\ X_{112}+X_{122}+X_{132}+X_{212}+X_{232}\leq -6.5\delta_2+57.5, \\ X_{ijk}\geq 0, p^1, p^2, n^1, n^2\geq 0, (\alpha_i, \beta_j, \delta_k, \theta^{(1)}_{ijk}, \theta^{(2)}_{ijk}) \\ \quad \in [0,1],\ i=1,2,\ j=1,2,3,\ k=1,2 \end{cases}$$

The problem is solved as a single objective programming problem by using Maple 17.02 and the corresponding optimal results of problem (10) were:

$$X_{ijk} = \begin{bmatrix} X_{111} & X_{112} & X_{113} \\ X_{122} & X_{131} & X_{132} \\ X_{211} & X_{212} & X_{221} \\ X_{222} & X_{231} & X_{232} \end{bmatrix} = \begin{bmatrix} 13.5 & 0 & 0 \\ 0 & 0 & 13.5 \\ 1.5 & 0 & 18/5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\theta^{(1)}_{ijk} \begin{bmatrix} \theta^{(1)}_{111} & \theta^{(1)}_{112} & \theta^{(1)}_{121} \\ \theta^{(1)}_{122} & \theta^{(1)}_{131} & \theta^{(1)}_{132} \\ \theta^{(1)}_{211} & \theta^{(1)}_{212} & \theta^{(1)}_{221} \\ \theta^{(1)}_{222} & \theta^{(1)}_{231} & \theta^{(1)}_{232} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

$$\theta^{(2)}_{ijk} \begin{bmatrix} \theta^{(2)}_{111} & \theta^{(2)}_{112} & \theta^{(2)}_{121} \\ \theta^{(2)}_{122} & \theta^{(2)}_{131} & \theta^{(2)}_{132} \\ \theta^{(2)}_{211} & \theta^{(2)}_{212} & \theta^{(2)}_{221} \\ \theta^{(2)}_{222} & \theta^{(2)}_{231} & \theta^{(2)}_{232} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\alpha_1=0, \alpha_2=1, \beta_1=1, \beta_2=1, \beta_3=1, \delta_1=1, \delta_2=1, p^1 = 3.726, p^2=5.422, n^1=3.726, n^2=2.390 \text{ and } \lambda=1$$

The degree of membership functions (13) and (14) for the decision maker are $\mu(z^1)=1$ $\mu(z^1)=1$ and $\alpha_1=0, \alpha_2=1, \beta_1=1, \beta_2=1, \beta_3=1, \delta_1=1, \delta_2=1$, respectively. The corresponding optimal values of objective functions for problem (9) are $Z^1(x)=[329.5, 488.75]$, $Z^2(x)=[415.75, 590.75]$

3.2 Method 2

Now, we consider the above global criteria method for solving problem (10). According to global criteria algorithm, we generate ideal objective vector of problem (10) for each objective function.

Then ideal vectors are calculated from problem (10) as follows:

$$\left((Z^1)^{\min}=329.5,\quad (Z^2)^{\min}=415.75\right)$$

$$\text{and } \left((Z^1)^{\max}=915,\quad (Z^2)^{\max}=983.75\right)$$

and then, using (8), the global criteria method for the multi-objective STP (10) in L_2 norms can be written as follows:

$$\min\left\{\left(\frac{Z^1(x)-329.5}{329.5}\right)^2+\left(\frac{Z^2(x)-415.75}{415.75}\right)^2\right\}^{1/2}$$

$$s.t \begin{cases} X_{111}+X_{121}+X_{131}+X_{112}+X_{122}+X_{132}\leq -4.5\alpha_1+27, \\ X_{211}+X_{221}+X_{231}+X_{212}+X_{222}+X_{232}\leq -6.5\alpha_2+36, \\ -6.5\beta_1+20.5\leq X_{111}+X_{211}+X_{112}+X_{212}, \\ -5\beta_2+23.5\leq X_{121}+X_{221}+X_{222}+X_{122}, \\ -6\beta_3+19.5\leq X_{131}+X_{231}+X_{132}+X_{232}, \\ X_{111}+X_{121}+X_{131}+X_{221}+X_{231}\leq 4.5\delta_1+52, \\ X_{112}+X_{122}+X_{132}+X_{212}+X_{232}\leq -6.5\delta_2+57.5, \\ \alpha_i, \beta_j, \delta_k, \theta^{(1)}_{ijk}, \theta^{(2)}_{ijk}, \in [0,1], i=1,2, j=1,2,3, k=1,2. \\ X_{ijk}\geq 0,\ i=1,2,\ j=1,2,3,\ k=1,2. \end{cases}$$

If this problem is solved as a single objective by using Maple 17.02, the same results are obtained with previous methods as follows:

$$X_{ijk} = \begin{bmatrix} X_{111} & X_{112} & X_{113} \\ X_{122} & X_{131} & X_{132} \\ X_{211} & X_{212} & X_{221} \\ X_{222} & X_{231} & X_{232} \end{bmatrix} = \begin{bmatrix} 13.5 & 0 & 0 \\ 0 & 0 & 13.5 \\ 1.5 & 0 & 18/5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\theta^{(1)}_{ijk} \begin{bmatrix} \theta^{(1)}_{111} & \theta^{(1)}_{112} & \theta^{(1)}_{121} \\ \theta^{(1)}_{122} & \theta^{(1)}_{131} & \theta^{(1)}_{132} \\ \theta^{(1)}_{211} & \theta^{(1)}_{212} & \theta^{(1)}_{221} \\ \theta^{(1)}_{222} & \theta^{(1)}_{231} & \theta^{(1)}_{232} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

$$\theta^{(2)}_{ijk} \begin{bmatrix} \theta^{(2)}_{111} & \theta^{(2)}_{112} & \theta^{(2)}_{121} \\ \theta^{(2)}_{122} & \theta^{(2)}_{131} & \theta^{(2)}_{132} \\ \theta^{(2)}_{211} & \theta^{(2)}_{212} & \theta^{(2)}_{221} \\ \theta^{(2)}_{222} & \theta^{(2)}_{231} & \theta^{(2)}_{232} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\alpha_1=0, \alpha_2=1, \beta_1=1, \beta_2=1, \beta_3=1, \delta_1=1, \delta_2=1.$ The corresponding optimal values of objective functions for problem (9) are $Z^1(x)=[329.5, 488.75]$, $Z^2(x)=[415.78, 590.75]$

4 Conclusion

The multi-objective solid transportation problem in uncertain environment has been investigated in this paper. Some soft computing methods have been applied to obtain the optimal solutions of the models. First, an uncertain solid transportation problem has been converted to a deterministic solid transportation problem, and so the deterministic solid transportation problem has been solved by using the fuzzy programming, fuzzy goal programming and global criteria methods. Consequently, application of the model is discussed with a numerical model and the same results were obtained by three different methods. At each stage of the solution procedures, Maple 17.02 optimization toolbox was used. Furthermore, the suggested interactive fuzzy goal programming method can be applied to other complex transportation problem models (with multi-item, safety factor) under uncertain environments.

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