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Finite Element Analysis for the Static Response of Functionally Graded Porous Sandwich Beams

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Abstract- In this paper, the finite element method is used to analyze the static response of the functionally graded porous (FGP) sandwich beams subjected to uniformly distributed loads along the beam span. The core of the beam is made up of functionally graded porous material while the top and bottom layers are made up of isotropic homogenous materials. Uniform distribution and symmetric distribution of pores are used as two different types of porous material. Shear deformation is considered in the analysis by utilizing BEAM189 in ANSYS which is a finite element package program. This element is based on the first-order shear deformation theory. The influence of porosity coefficient, boundary conditions, and type of the porous material on the static response of the considered structures is presented in detail. The results demonstrate that the porosity coefficient has an important impact on the static response of the FGP sandwich beams.

Keywords Static analysis, Finite element method, FGM, Porous materials, Sandwich beam

1. Introduction

Engineering designers are aimed to provide superior structure performance regards various aspects. The structure needs to withstand applied loads in addition to other requirements that need to be met. Users and designer requirements are continuously developed to the level that conventional materials cannot give acceptable performance. Damping, strength, stiffness, thermal isolation, lightweight, environmental needs, vibration characteristics, and many more properties that may not be met with separate materials. For this reason, functionally graded materials (FGMs) should be preferred. Although fabrication technology of FGMs is in its beginning, they provide various benefits such as high stiffness, lightweight, and thermal isolation.

Sandwich beams and plates that were found in the 1980s showed acceptable performance, durability, and multi-objective structures. These structures can be used in civil, mechanical, aerospace, manufacturing, automobiles, and many more fields. VARTM technology is applied to manufacture sandwich structural elements like sandwich beams or plates [1]. The sandwich beam is considered from cellular materials that its properties can be modified according to its usage [2].

The sandwich beams have a variety of characterizations. Strong, durable, thin load member, stiffness, and strength over weight ratios for sandwich beam are very high. The solid metal faces give an acceptable stiffness. Where the core may be used as graded porous in different patterns. The core materials can be bone, coral, ceramic, polymer, wood, and many more. The core porosity can be formed based on the required final performance. The increase of the porosity may reduce the overstrength, but it will provide superior characteristics in other aspects [3]. A review paper can provide more knowledge about functionally graded sandwich beams like in [4-6]. Higher-order shear deformation theory [7], Euler-Bernoulli beam theory [8], quasi-3D theory [9], and first-order shear deformation theory [10] can be applied in the case of structural elements analysis for various types of structures. In functionally graded beams, an analysis was implemented by [11] with the aid of the Ritz method on the basis of Timoshenko revealed that material selection and modulus of Elasticity have a major role in beam behavior related to stress distribution and displacement. The research was performed by [12] on some basic sandwich beam concepts for their fitness of application to uni-dimensional functionally graded sandwich beams. A perfect convention is found between finite element analysis and higher-order theory. A finite element model was developed by [13] to analyze stress distribution for functionally graded sandwich beams across the cross-section thickness. The model was validated using numerical methods results.

The quasi-3D theory was used by [14] to present the static behavior of functionally graded sandwich beams, which includes both thickness stretching and shear deformation effects. In [15], it is carried out that functionally graded sandwich beams with less length to height (L/h) ratio than five can be simulated, and solved using symmetric smoothed particle hydrodynamics method combined with applying of quasi-3D theory. Damping properties of sandwich cantilever beams were investigated by [16] where they mentioned that viscoelastic core thickness has a great effect in reducing the damping factor. Wang et al [17] utilized shock tube experiments to represent the dynamic behavior of sandwich panels. Model analysis for functionally graded beams was built by [18] depending on shear deformation from first order regarding static bending behavior under different boundary conditions. The model verification process of [18] was successfully determined for accuracy and efficiency by comparison of their results with those of the available literature. They considered the slenderness ratio, the thickness of the core, and metal faces, and power low index in their research. Through the application of the repeated shear deformation theory, the static analysis of functionally graded beams was performed by [19].

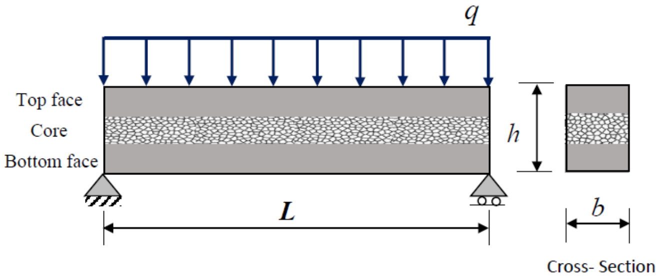
The displacement aspect established on higher-order shear deformation theory is applied by [20] to study the static behavior of functionally graded metal-ceramic functionally graded material beams under surrounded temperature. By the method of complementary functions, the bending of functionally graded beams is defined by [21]. Depending on the theory of Bernoulli, and Timoshenko the equations governing the bending response of sandwich beams were obtained by [22]. Furthermore, the method of complementary functions is applied in order to solve the obtained equations numerically. Also, they studied the effects of (L/h) ratios, different boundary conditions, layer ratios, and material difference coefficients on the bending response of the beams. In [23] it is carried out that functionally graded core sandwich beam which is loaded by the transverse load can be analyzed using elasticity solution. Core properties variation was not affected by normal stress which varies linearly with cross-section thickness. Results of [24] show that the sandwich beam behavior is dependent on the power-law index, porosity, and FG porous metal core thicknesses. Under different types of distributed loads, linear and nonlinear flexural analyzes of sandwich beams in which the core was functionally graded have been performed by [25].

The literature survey shows that the bending response of functionally graded porous sandwich beams has not been investigated by using the BEAM189 element via ANSYS [26] yet. This paper aims to examine the static response of FGP sandwich beams with the aid of the finite element method. The sandwich beams are assumed to have isotropic homogenous face sheets and an FGP core. Results are obtained and compared for various values of porosity coefficients and clamped - clamped, clamped - pinned, clamped - free, and pinned - pinned boundary conditions. The influence of the porosity coefficients on maximum vertical displacement and stress values is presented. The effect of shear deformation is taken into account in the analysis procedure. The uniform porosity and symmetric porosity are used as two different types of FGP materials.

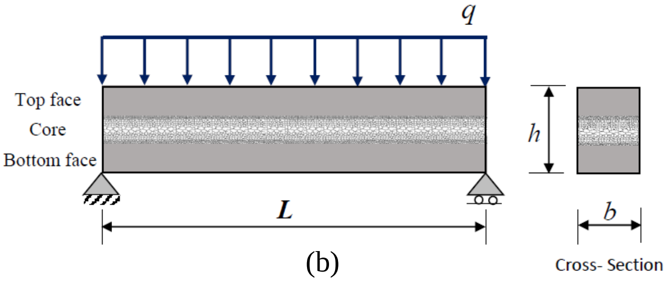
To present this research paper in a better way, it is organized as follows: Section 2 shows the functions of FGP materials and gives information about the finite element type used in the analysis. Section 3 presents the numerical results and discussion. and Section 4 gives the most important conclusion of this paper.

2. Materials and Method

The functionally graded porous (FGP) sandwich beam shown in figure 1 with length *L*, thickness *h*, and width *b* is considered. In this paper, it is assumed that the sandwich beam is consist of three layers with different thicknesses. The top and bottom layers of the beam are isotropic homogenous and the core layer is FGP. Two different types of porosity distributions are used.



(a) FGP sandwich beam with uniform porosity distribution



(b) FGP sandwich beam with symmetric porosity distribution

Fig. 1: Simply supported FGP sandwich beam with different porosity distributions

These types are symmetric and uniform porosity distributions. The Poisson's ratio is taken as constant. The symmetric FG porosity can be described by Eq. 1 and the uniform FG porosity can be expressed by Eq. 2 [27].

$$E\,(z) = E_1\left[1 - e_0 cos\left(\frac{\pi z}{h_c}\right)\right] \qquad (1)$$

$$E\,(z) = E_1[1 - e_0\varphi] \qquad (2)$$

where h_c is the core thickness and the value of φ can be calculated as follows:

$$\varphi = \frac{1}{e_o} - \frac{1}{e_o}\left(\frac{2}{\pi}\sqrt{1 - e_o} - \frac{2}{\pi} + 1\right)^2 \qquad (3)$$

In these equations e_o is the porosity coefficient, *h* is the thickness of the cross-section, E_1 is the maximum value for the modulus of elasticity. The distribution of the elasticity material along the cross-section of the beam is given as follows:

$$\begin{array}{ll} E = E_1 & -\frac{h}{2} \le z \le -\frac{7h}{18} \\ E = E(z) & -\frac{7h}{18} \le z \le +\frac{7h}{18} \\ E = E_1 & +\frac{7h}{18} \le z \le +\frac{h}{2} \end{array} \qquad (4)$$

In the finite element solution of the problem in the hand with ANSYS, the BEAM189 element is used. This element is a quadratic element that has three nodes. There are six degrees of freedom at each node of this element. The shear deformation is considered in this element based on the first-order shear deformation theory. For more detailed information about the assumptions and restrictions of this element see [28].

To define the functionally graded materials for the FGP sandwich beam the modulus of elasticity is calculated and entered into the ANSYS model. The section of the beam is divided into 36 layers (Figure 2) along the thickness direction, similar to [29]. To obtain detailed values of stress along the length of the beam, it is divided into 100 finite elements in the longitudinal direction. The boundary conditions are given in Table 1. The load is implemented in the z-direction.

Table 1: Boundary conditions

Type of the support	Boundary conditions	
	i	j
Fixed – Fixed (F-F)	Rotz = Roty = Rotx= Uz = Uy = Ux = 0	Rotz = Roty = Rotx= Uz = Uy = Ux = 0
Fixed – Pinned (F-P)	Rotz = Roty = Rotx= Uz = Uy = Ux = 0	Uz = Ux = Uy = 0
Pinned – Pinned (P-P)	Uz = Ux = Uy = 0	Uz = Ux = Uy = 0
Fixed – Free (F-FR)	Rotz = Roty = Rotx= Uz = Uy = Ux = 0	-----

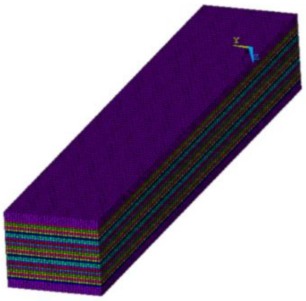


Fig. 2: The finite element model of an FGP sandwich beam

3. Numerical results and discussion

The analyzed beam dimensions are (3 × 0.5 × 0.5) m as length, width, and height respectively. The beam is a functionally graded sandwich beam by thickness direction with a varied elasticity value. The implemented load is 10 N/m as a uniformly distributed load. It is worth to be mentioned that there is no specific reason for choosing these loads. It is considered only to investigate the effect of porosity on the static response of the FGP beams.The outer faces are made up of steel material, and the core is FGP material. Two main groups used are symmetric material constitutive relationships (SMCR) FG beam and uniform porosity distribution FG beam. For each one of these two FGP material groups, forty-four different models were generated and analyzed using the finite element procedure.

To outline the effect of porosity on the static response of the considered structure, results are obtained for several values of porosity coefficients and boundary conditions. The material properties are assumed to be E_1= 210 GPa and ν = 0.3. The maximum displacement values are obtained and presented in Tables (2 -3).

Table 2: Maximum transverse displacement values for FGP sandwich beam with symmetric distribution (m)

e_o	F-F	F-FR	P-F	P-P
0.10	2.81E-09	1.02E-07	5.14E-09	1.10E-08
0.20	2.90E-09	1.04E-07	5.28E-09	1.13E-08
0.25	2.96E-09	1.04E-07	5.35E-09	1.14E-08
0.30	3.01E-09	1.05E-07	5.44E-09	1.15E-08
0.40	3.14E-09	1.07E-07	5.61E-09	1.18E-08
0.50	3.30E-09	1.10E-07	5.83E-09	1.21E-08
0.60	3.50E-09	1.12E-07	6.09E-09	1.24E-08
0.70	3.78E-09	1.15E-07	6.45E-09	1.28E-08
0.75	3.97E-09	1.16E-07	6.69E-09	1.31E-08
0.80	4.23E-09	1.18E-07	7.01E-09	1.35E-08
0.90	5.23E-09	1.24E-07	8.19E-09	1.46E-08

Table 3: Maximum transverse displacement values for FGP sandwich beam with uniform distribution (m)

e_o	F-F	F-FR	P-F	P-P
0.10	2.83E-09	1.03E-07	5.20E-09	1.12E-08
0.20	2.95E-09	1.07E-07	5.40E-09	1.16E-08
0.25	3.02E-09	1.09E-07	5.51E-09	1.18E-08
0.30	3.08E-09	1.11E-07	5.62E-09	1.20E-08
0.40	3.24E-09	1.15E-07	5.88E-09	1.25E-08
0.50	3.42E-09	1.20E-07	6.17E-09	1.31E-08
0.60	3.64E-09	1.25E-07	6.52E-09	1.37E-08
0.70	3.92E-09	1.32E-07	6.96E-09	1.45E-08
0.75	4.10E-09	1.35E-07	7.23E-09	1.49E-08
0.80	4.31E-09	1.40E-07	7.55E-09	1.55E-08
0.90	4.94E-09	1.51E-07	8.46E-09	1.69E-08

It can be seen in Table (2-3) that the material porosity coefficient has a significant influence on the maximum displacement of FGP sandwich beams. The minimum displacement occurs in fixed-fixed support with a porosity coefficient of 0.1 while the maximum displacement occurs in the cantilever beam with a porosity coefficient of 0.9.

To better interpret the given results in Tables (2-3) the graphical form of the results is illustrated in Figures (3- 10).

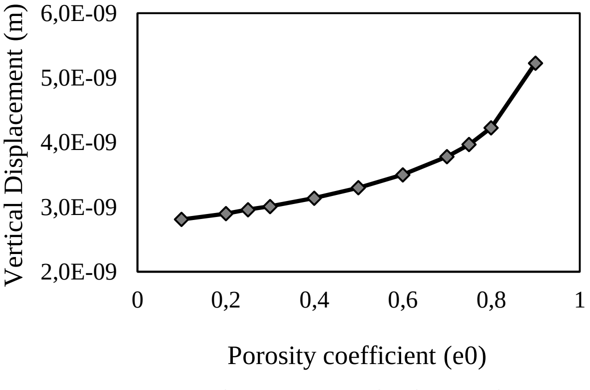


Fig. 3: Porosity – deflection curve for fixed – fixed FGP sandwich beam with symmetric distribution

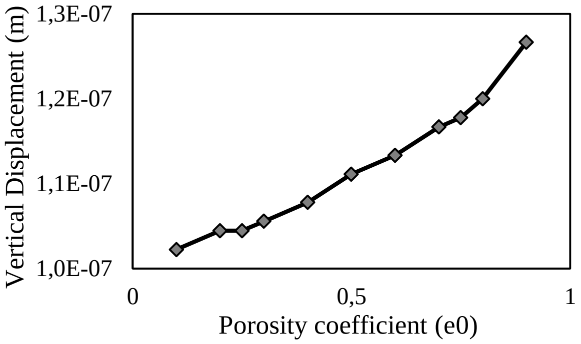


Fig. 4: Porosity – deflection curve for fixed – free FGP sandwich beam with symmetric distribution

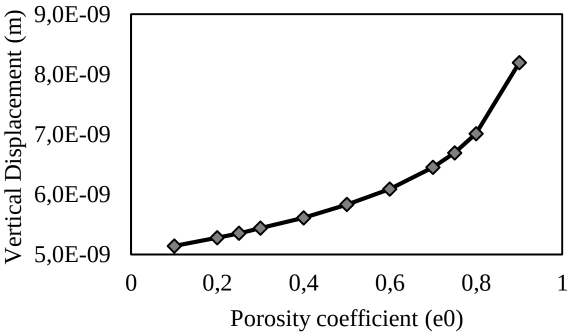


Fig. 5: Porosity – deflection curve for fixed – pinned FGP sandwich beam with symmetric distribution

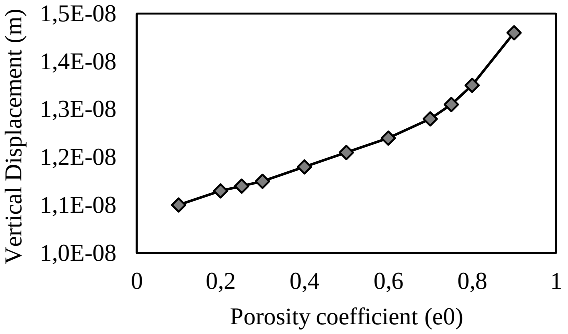


Fig. 6: Porosity – deflection curve for pinned – pinned FGP sandwich beam with symmetric distribution

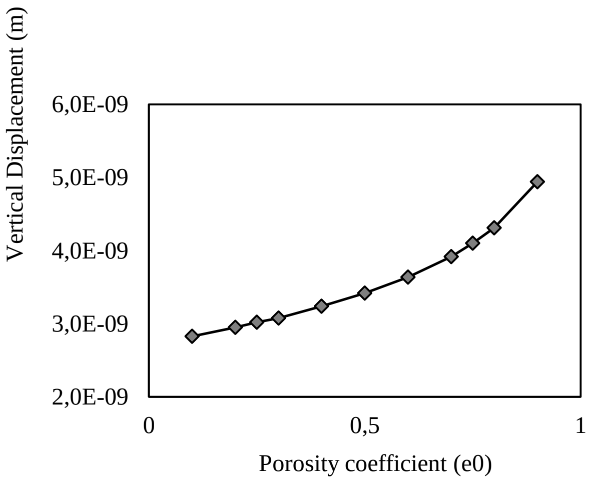


Fig. 7: Porosity – deflection curve for fixed – fixed FGP sandwich beam with uniform distribution

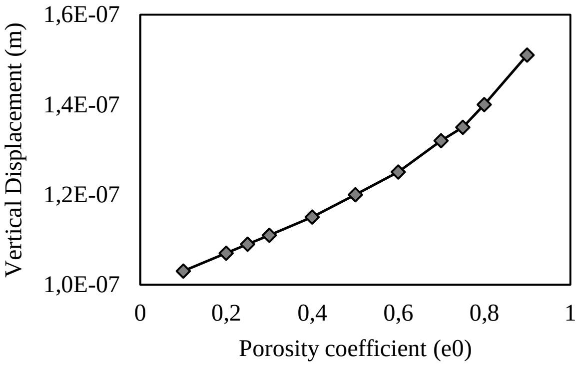


Fig. 8: Porosity – deflection curve for fixed – free FGP sandwich beam with uniform distribution

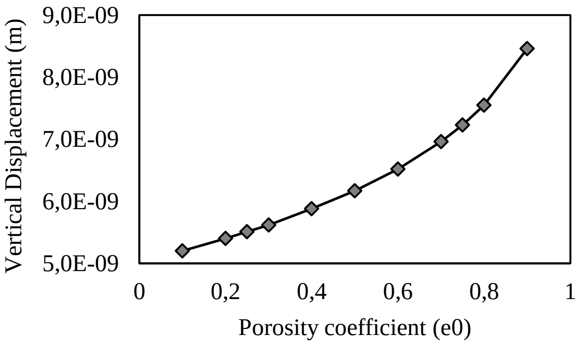


Fig. 9: Porosity – deflection curve for fixed – pinned FGP sandwich beam with uniform distribution

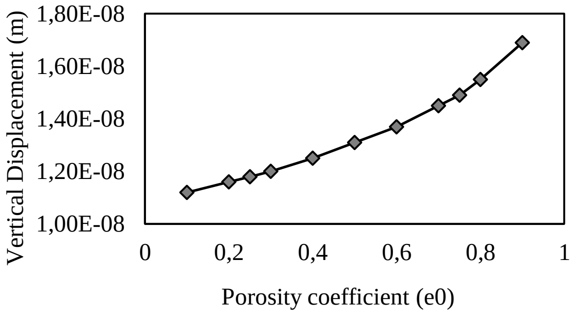


Fig. 10: Porosity – deflection curve for pinned – pinned FGP sandwich beam with uniform distribution

As can be clearly observed in Figures (3-10) the results reveal a positive correlation between porosity and deflection. In other words when the porosity coefficient increase higher deflection will be recorded. From the results gained we can arrange the beam from lower to higher deflection based on the beam supports comparison: fixed – fixed, fixed – pinned, pinned – pinned, and lastly fixed – free.

The diagrams for the vertical displacement and rotations along the axis of the beam are obtained for all cases discussed above. But only the vertical displacement (Uz) is illustrated in this section for fixed–fixed FGP sandwich beam with symmetry distribution (Figure 11). The material porosity coefficient is considered to be 0.5. The dashed line shows the undeformed shape.

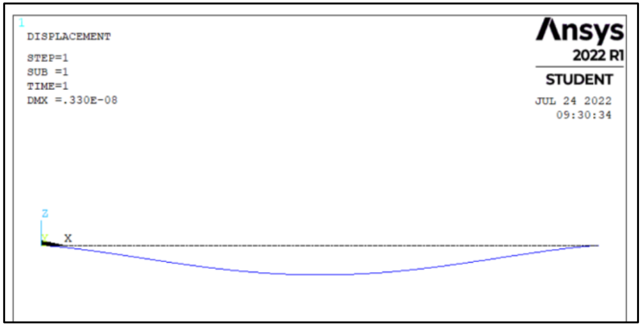


Fig. 11: Deformed shape of fixed – fixed FGP sandwich beam with symmetric distribution (e0=0.5)

The von Mises stress values are obtained for several cases and presented in Figures (12-16).

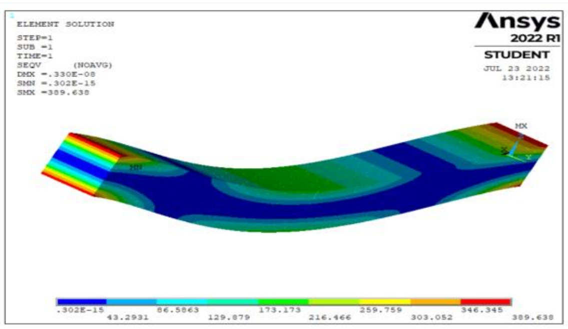


Fig. 12: The von Mises stress distribution for fixed – fixed FGP sandwich beam with symmetric distribution (e0=0.5)

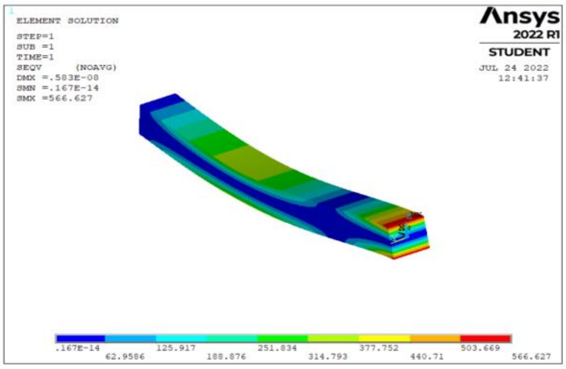


Fig. 13: The von Mises stress distribution for fixed – pinned FGP sandwich beam with symmetric distribution (e0=0.5)

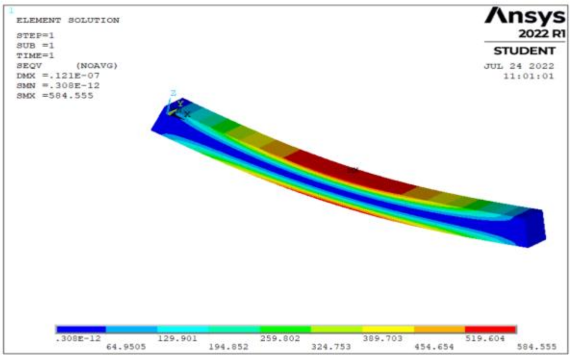


Fig. 14: The von Mises stress distribution for pinned-pinned FGP sandwich beam with symmetric distribution (e0=0.5)

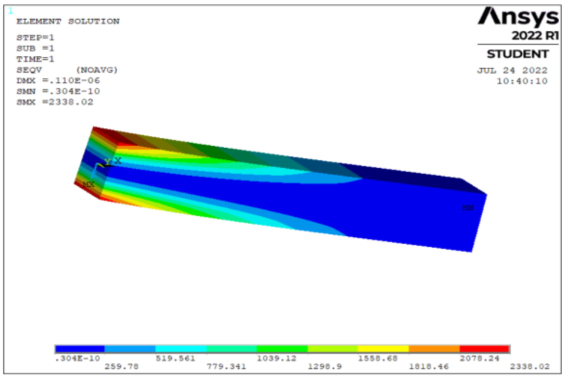


Fig. 15: The von Mises stress distribution for fixed – free FGP sandwich beam with symmetric distribution (e0=0.5)

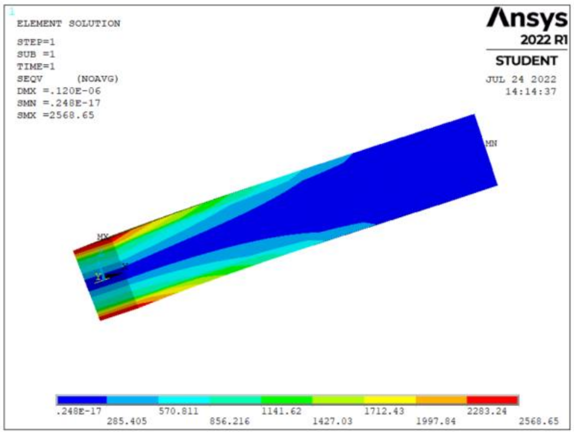


Fig. 16: The von Mises stress distribution for fixed – free FGP sandwich beam with uniform distribution (e0=0.5)

In Figures (12-15) it can be clearly seen that type of the support has a significant influence on the von Mises stress values of FGP sandwich beams. The maximum von Mises stress values for fixed–fixed, fixed–pinned, pinned–pinned, and fixed–free FGP sandwich beams with symmetric distribution (e0 = 0.5) are 389.63 Pa, 566.63 Pa, 584.56 Pa, and 2338.02 Pa. The same stress value for the cantilever FGP sandwich beam with uniform distribution (e0 = 0.5) is obtained as 2568.65 Pa (Figure 16). This indicates that the fixed–free beam has the highest von Mises stress while the fixed–fixed beam has the lowest. When symmetric and uniform porosity distributions are compared it can be carried out that for e0 = 0.5 porosity coefficients the stress values are greater in the uniform porosity distribution of FGP sandwich beams.

4. Conclusions

In this study, the impacts of porosity coefficient on the static response of FGP sandwich beams are investigated by the method of finite element. The results are obtained for several boundary conditions and material indices. Two types of porosity distribution, symmetric and uniform, are considered in the research. The analysis is done based on the Timoshenko Beam Theory.

Porosity is to decrease the weight of sandwich beams, but it also decreases the strength. The results showed that the values of the maximum displacement are directly proportional to porosity coefficients. Increasing the value of the porosity coefficient increases the values of displacements. The comparison between the symmetric and uniform distribution of the FGP materials indicates that the transverse displacement is greater when the uniform distribution is used for the values of e_0 less than 0.9. But for 0.9 it is vice versa. As one of the main results of this paper, it can be concluded that variations in von-Mises stresses depend on the boundary conditions and on the types of porous distribution and porosity coefficient at the core of the sandwich beams.

5. References

[1] M.A. Dweib, B.Hu, A.O'Donnell, H.W.Shenton and R.P.Woo, "All natural composite sandwich beams for structural applications". *Composite structures, 63*(2), pp. 147-157, 2004.

[2] L. J. Gibson and M.F. Ashby. *Cellular solids: structure and properties*. United Kingdom: Cambridge Univrsity Press, 1999.

[3] T. George, V. S. Deshpande and H. N.G. Wadley, "Mechanical response of carbon fiber composite sandwich panels with pyramidal truss cores." *Composites Part A: Applied Science and Manufacturing* 47, pp. 31-40, 2013.

[4] A. S. Sayyad and Y. M. Ghugal,"Modeling and analysis of functionally graded sandwich beams: A review." *Mechanics of Advanced Materials and Structures* 26.2, pp. 1776-1795, 2019.

[5] E. K. Njim, M. Al-Waily and S. H. Bakhy,"A review of the recent research on the experimental tests of functionally graded sandwich panels." *Journal of Mechanical Engineering Research and Developments* 44.3, pp. 420-441, 2021.

[6] H. Wu, J. Yang and S. Kitipornchai, "Mechanical analysis of functionally graded porous structures: A review." *International Journal of Structural Stability and Dynamics* 20.13 2020.

[7] T.Nguyen, T. T. Nguyen, T. P. Vo and H. Thai, "Vibration and buckling analysis of functionally graded sandwich beams by a new higher-order shear deformation theory." *Compos. Part B Eng.* 76, pp. 273–285, 2015.

[8] B.V. Sankar, "An elasticity solution for functionally graded beams." *Compos. Sci.Technol.* 61, pp. 689–696, 2001.

[9] T. P. Vo, H. Thai, T. Nguyen, F. Inam and J. Lee, "Static behaviour of functionally graded sandwich beams using a quasi-3D theory,." *Compos. Part B Eng.* 68, pp. 59–74, 2015.

[10] H. Thai and D. Choi, "A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates." *Compos. Struct.* 66.101, pp. 332–340, 2013.

[11] M. Şimşek, "Static analysis of a functionally graded beam under a uniformly distributed load by Ritz method." *International Journal of Engineering and Applied Sciences (IJEAS)* 1.3, pp. 1-11, 2009.

[12] N. A. Apetre, B. V. Sankar and D. R. Ambur, "Analytical Modeling of Sandwich Beams with Functionally Graded Core." *Journal of Sandwich Structures and Materials* 10.1, pp. 53-74, 2008.

[13] S. Das and S. K. Sarangi, "Static analysis of functionally graded composite beams." *IOP Conference Series: Materials Science and Engineering*. Bangalore, 2016.

[14] T. P. Vo, H. Thai, T. Nguyen, F. Inam, J. Lee. "Static behaviour of functionally graded sandwich beams using a quasi-3D theory,." *Compos. Part B Eng.* 68, pp. 59–74, 2015.

[15] A. Karamanlı, "Bending behaviour of two directional functionally graded sandwich beams by using a quasi-3d shear deformation theory." *Composite Structures* 174, pp. 70-86, 2017.

[16] J. H. Yim, S. Y. Cho,Y. J. Seo, B. Z. Jang, "A study on material damping of 0° laminated composite sandwich cantilever beams with a viscoelastic layer." *Composite Structures* 60.4, pp. 367–374, 2003.

[17] E. Wang, N. Gardner and A. Shukla, "The blast resistance of sandwich composites with stepwise graded cores." *International Journal of Solids and Structures* 46, pp. 3492–3502, 2009.

[18] P. V. Van, "Static bending analysis of functionally graded sandwich beams using a novel mixed beam element based on first-order shear deformation theory." *Forces in Mechanics* 4 2021.

[19] T. P. Vo, H. Thai, T. Nguyen, F. Inam and J. Lee, "Static behaviour of functionally graded sandwich beams using a quasi-3D theory,." *Compos. Part B Eng.* 68, pp. 59–74, 2015.

[20] R. Kadoli, K. Akhtar and N. Ganesan, "Static analysis of functionally graded beams using higher order shear deformation theory." *Applied mathematical modelling* 32.12, pp. 2509-2525, 2008.

[21] A.R. Noori, T. A. Aslan and B. Temel, "Static analysis of FG beams via complementary functions method." *European Mechanical Science* 4.1, pp. 1-6, 2020.

[22] A.R. Noori, H. Rasooli, T. A. Aslan and B. Temel, "Fonksiyonel Derecelenmiş Sandviç Kirişlerin Tamamlayıcı Fonksiyonlar Yöntemi ile Statik Analizi." *Çukurova Üniversitesi Mühendislik-Mimarlık Fakültesi* 35.4, pp. 1091-1102, 2020.

[23] B. Srikarun, W. Songsuwan and N. Wattanasakulpong, "Analysis of sandwich beams with functionally graded core." *19th AIAA Applied Aerodynamics Conference*. Anaheim,Canada,U.S.A, 2001

[24] E. K. Njim, M. Al-Waily and S. H. Bakhy, "Experimental and Numerical Flexural Properties of Sandwich Structure with Functionally Graded Porous Materials." *Engineering and Technology Journal* 40.1, pp. 137-147, 2022.

[25] B. Srikarun, W. Songsuwan and N. Wattanasakulpong, "Linear and nonlinear static bending of sandwich beams with functionally graded porous core under different distributed loads." *Composite Structures* 276, 114538, 2021

[26] ANSYS, Inc Release 15.0, Canonsburg, PA, 2013.

[27] N. Wattanasakulpong and S Eiadtrong, “Transient responses of sandwich plates with a functionally graded porous core: Jacobi-Ritz method.” *International Journal of Structural Stability and Dynamics*,2022, https://doi.org/10.1142/S0219455423500396.

[28] Mechanical APDL Element Reference, 2013, Inc., 275 Technology Drive, Canonsburg, PA 15317

[29] A.R. Noori, T. A. Aslan and B. Temel, "Dynamic analysis of functionally graded porous beams using complementary functions method in the Laplace domain." *Composite Structures* 256, 113094, 2021.