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M/M/c/K state-dependent models for controlling the pilgrims and the design of hajj facilities

Khalidur Rahman[1], Noraida Abdul Ghani[2], Anton Abdulbasah Kamil[3]

[1]Department of Statistics, School of Physical Sciences, Shahjalal University of Science and Technology, Sylhet-3114, Bangladesh
[2]School of Distance Education, Universiti Sains Malaysia, Penang 11800, Malaysia
[3]Faculty of Economics, Administrative and Social Sciences, Istanbul Gelisim University, Istanbul 34310, Turkey

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ABSTRACT

In Hajj overflow of pilgrims causes queuing delay and in turn, is controlled by the capacity of facilities. The concept of flow control can be used to avoid the building of extreme queues. Thus, in this paper, a new analytical tool for measuring the performances of pilgrim movements in Hajj has been developed. The application of M/M/c/K queuing models for controlling the pilgrims and the design of facilities in Hajj is discussed. The paper also used some data on the flow of pilgrims for implementing the use of M/M/c/K queuing models for the above mentioned controlling and design. The result show that, the average number of pilgrims waiting in queue almost zero, because maybe most of the pilgrims only spend least then 12 second in the system.

Corresponding Author:

Anton Abdulbasah Kamil,
Faculty of Economics,
Administrative and Social Sciences,
Istanbul Gelisim University, Istanbul 34310, Turkey
Email: kamil.antonabdulbasah@gmail.com
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1. INTRODUCTION

Congestion or high density is a common phenomenon that impedes the smooth movement of pedestrians in public walkways (e.g. sidewalks in Central Business Districts). It accounts for a great percent of variation in pedestrian speeds [1]. Congestion is not only observed in walking facilities for transportation including sidewalks, crosswalks, stairways, subway tunnels, and transportation terminals but is also observed in various types of public facilities including at factories, sporting events (stadia), pilgrims or graduate assembly, pop concerts, high rise buildings etc. In contrast to vehicular traffic, congestions in pedestrian traffic are not only annoying, but might become very dangerous [2]. In an emergency situation, congestion is the main reason which generates injury and even death toll. It might lead to stampedes when the number of pedestrians exceeds the capacity of pedestrian facilities as occurred during the ritual Ramy Al-Jamarāt (Stoning of the

Devil) on the last day of the Hajj in Mina in 12 January, 2006. During the event, at least 346 pilgrims died and more than 289 pilgrims were injured (Figure 1 Source: http://en.wikipedia.org/wiki/Incidents_during_the_Hajj, Retrieved on 3 July 2013). Thus, an understanding of the system dynamics contributing to congestion is a necessary prerequisite to developing appropriate congestion management and control techniques, as well as design standards for congested facilities [3].

Figure 1. Congestion at stoning of the devil in hajj

Experienced with persistent congestion, people from different disciplines unanimously say "congestion is awful", but they may provide different solutions to ease the condition, according to their subject's aspect. Solution in terms of urban planning would be considered by geographers. Economists would formulate the problem as a demand-supply gap and try to mitigate the gap by finding an equilibrium condition between the demand for flows and the supply of available facilities/capacities. Engineers would provide a suggestion to reconstruct the existing infrastructure or to construct a new facility for proper movement of pedestrians. Operational researchers would develop and use an appropriate analytical tool to assist the policy makers in optimizing the operational conditions and the design of the facilities in a cost effective manner. Efficient management and control of congestion is a determinant of the quality of the human experience in public walkways and facilities [3]. Research on pedestrian movements and flow characteristics are limited compared with those that focus on vehicle traffic flow [4]. Such research began in the 1960s, mostly in developed countries.

According to Al-Gadhi [3] and Ventre, Stahl, and Turner [5] pedestrian movement behaviour (B) may be characterised to variation in three factors: pedestrian attributes or flow characteristics (A), facility design and layout (F), and management practices (M). That is:

$$B = f(A, F, M) \tag{1}$$

If a facility manager or designer has a model which precisely identifies the relationship among these factors, then he/she could predict, logically strongly, pedestrian movement behaviour at a particular facility on the basis of anticipated pedestrian attributes, facility design conditions and management decisions. Thus, pedestrian studies should mainly focus on the identification of key factors and to systematically manage these factors with the appropriate design and managing approach to facilitate safe, comfortable and prompt walking provision.

Khan [6] investigated how Agent-Based Modelling and Simulation (ABMS) and Queuing Theory (QT) techniques help manage mass gathering crowds in Hajj, Saudi Arabia. He found that QT allows Hajj authorities to better predict pilgrim arrival, departure, and waiting times along with pilgrim queue length for different combination of pilgrim arrival and service times at Hajj sites. But no suggestion was provided on the designing of the facilities in Hajj. What this study intends to do is to use an appropriate analytical model to the study of circulation system of pilgrims and design of facilities in Hajj. The study also recommends the required data on the flow of pilgrims that are necessary to collect for implementing the use of the analytical model for managing and design of facilities for pilgrims.

2. METHOD

2.1 Congestion Model and Required Data for the Present Study

The recognized Bureau of Public Roads (BPR) [7] model has been generally utilized by traffic researchers and policy makers for estimating travel times on road networks over the last five decades. This model has also been accepted, in the following form, in estimating pedestrian travel time on different walking facilities:

$$t(k) = t_0 + A.(k/k_j)^s \tag{2}$$

where t(k) = travel time (sec) at flow level or density k;
t0 = free-flow travel time (sec);
k = pedestrian flow (peds/m/sec) or density (peds/m2);
A, s = constants to be estimated in the model fitting procedure;
k_j = the capacity of the pedestrian facility (peds/m/sec or peds/m2) and
k/k_j = pedestrian demand to capacity ratio.

The queuing time as well as the time required for a pedestrian to travel along the facility is usually considered as the measurement of travel time [8]. The formula in Eq.(2) is only applicable when the value of k is less than the value of kj. Although the waiting time or delay experienced by a pedestrian on a walking facility was considered in empirical measurement in previous studies based on Eq.(2), however it was not explicitly reflected in the formulation of the model for travel time. Thus, Rahman, Ghani, Kamil, Mustafa, and Chowdhury [9] have formulated a pedestrian travel time / speed model based on queuing theory. Their model is specially formulated for sidewalks and can be used to assess the influence of relevant determinants and functional factors: free flow speed, maximum density, effective width of the facility, level of service (LOS), and lateral spacing required for a pedestrian to move on the facility. They have formulated the effective pedestrian speed as:

$$v = \frac{v_f}{\left[1 + \dfrac{(k/k_j)^s}{s(1-k/sk_j)(k/k_j)^s + s(s!)(1-k/sk_j)^2 \displaystyle\sum_{n=0}^{s-1} \frac{(k/k_j)^n}{n!}}\right]} \tag{3}$$

where v = average walking speed (m/sec);
v_f = average free flow speed of a pedestrian (m/sec);
k *and* k_j = prevailing density (peds/m^2) and maximum/jam density (peds/m^2), respectively;
W = the width of the facility (m);
L = the length of the facility (m);
s = $\frac{W-1.07}{b}$ = number of pedestrian lanes on the facility, where 1.07 m of width is reduced to calculate the effective width of the facility [10] and
b = lateral spacing required for a pedestrian to move on the facility (m) = 0.8m [11].

Here, we have adopted the above speed-density relationship for the study of circulation system of pilgrims in Hajj, which can be expressed as the following as a function of the number of pilgrims on a particular facility.

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$$v_m = \frac{v_f}{\left[1 + \dfrac{\left(\dfrac{m}{c}\right)^s}{s\left(1-\dfrac{m}{s*c}\right)\left(\dfrac{m}{c}\right)^s + s(s!)\left(1-\dfrac{m}{s*c}\right)^2 \sum_{n=0}^{s-1}\left(\dfrac{\left(\dfrac{m}{c}\right)^n}{n!}\right)}\right]} \tag{4}$$

where v_m = average walking speed of *m* pilgrims on the facility (m/sec);
v_f = average free flow speed of a pilgrim (m/sec);
c = $k_j * W * L$ = the normal capacity of a facility (pil);
m = number of pilgrims on the facility, $m = 1, 2, \ldots, c, \ldots, K(=\phi\, c)$;
W, L, s, b = as defined above.

Here, kj is the maximum density at which pilgrims usually move in a particular facility in Hajj. This quantity as well as vf , average free flow speed of a pilgrim, have to be measured from the practical movement of pilgrims. By fitting the Eq. (3) to the empirical data, we can easily estimate the 'usual jam' density, , that generally occurs in a particular facility in Hajj. In addition, by dividing the by the observed kj, we can also estimate the buffering size of the facility . Thus, we would be able to use the above Eq. (4) to establish a proper design and managing approach of pilgrim facilities in Hajj based on M/M/c/K queuing models.

2.2 The M/M/C/K State-Dependent Model for The Pilgrims

By knowing the prevailing density or the number of pilgrims in a particular facility, using Eq. (3) or Eq. (4), we can estimate the average speed of the pilgrims in that facility. When the number of pilgrims and speed are fixed and interrelated, we can manage/control the flow of pilgrims to response to changes in the pilgrim traffic.

For a facility of length L, average travel time required for a pilgrim (when there are n pilgrims in the facility) can be defined as:

$$t_m = \frac{L}{v_m} \tag{5}$$

As inverse of travel time can be considered as the service rate at which the pilgrims are being served in the particular facility, we can describe the overall service rate as a function of number of pilgrims in the facility. That is, the service rate is state-dependent and can be defined as:

$$\mu_m = m\frac{v_m}{L} \tag{6}$$

To express the service rate in terms of expected service time, , required for a pedestrian in free flow condition, we can rewrite Eq. (6) as the following:

$$\mu_m = m\frac{v_f}{L}.\frac{v_m}{v_f} = m.\frac{f(m)}{E(S)} \tag{7}$$

where, f(m) is the ratio of average speed of m pilgrims to free flow speed on the facility.

Now, considering that in the M/M/c/K queuing system the arrival rate is constant and can be managed/controlled, and the service rate is state-dependent as defined in Eq. (6) or (7), the steady-state probabilities pm (m = 1, 2,…..,c,…..K) of m pilgrims on the facility can be computed as:

$$p_m = \begin{cases} \dfrac{\lambda^m}{\prod_{i=1}^{m} \mu_i} p_0 & for\ 1 \le m \le c \\[2ex] \dfrac{\lambda^m}{(\mu_c)^{m-c} \prod_{i=1}^{m} \mu_i} p_0 & for\ c \le m \le K \end{cases} \tag{8}$$

and

$$p_0 = \left(1 + \sum_{m=1}^{c-1} \frac{\lambda^m}{\prod_{i=1}^{m} \mu_i} + \sum_{m=c}^{K} \frac{\lambda^m}{(\mu_c)^{m-c} \prod_{i=1}^{m} \mu_i} \right)^{-1} \tag{9}$$

By replacing the expression for , from Eq. (7) into Eq. (8) and Eq. (9), we obtain the steady-state probabilities as:

$$p_m = \begin{cases} \dfrac{[\lambda E(S)]^m}{m! \prod_{i=1}^{m} f(i)} p_0 & for\ 1 \le m \le c \\[2ex] \dfrac{[\lambda E(S)]^m}{c!(c)^{m-c} \prod_{i=1}^{m} f(i)} p_0 & for\ c \le m \le K \end{cases} \tag{10}$$

and

$$p_0 = \left(1 + \sum_{m=1}^{c-1} \frac{[\lambda E(S)]^m}{m! \prod_{i=1}^{m} f(i)} + \sum_{m=c}^{K} \frac{[\lambda E(S)]^m}{c!(c)^{m-c} \prod_{i=1}^{m} f(i)} \right)^{-1} \tag{11}$$

The use of M/M/c/K queuing models for analysing a single facility allows us to compute certain performance measures in equilibrium condition. The most relevant performance measures include:

i. The probability of balking (p_{Balk}) is equal to p_K,

ii. $E(Q) = \sum_{m=c+1}^{K} (m-c) p_m$ is the average number of pilgrims waiting in the queue,

iii. $E(T) = \dfrac{\sum_{m=1}^{K} m p_m}{\theta}$ is the expected amount of time a pilgrim spends on the facility,

iv. $\theta = \lambda(1 - p_{Balk})$ is the throughput through the facility.

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3. RESULTS AND DISCUSSIONS

3.1 Experiment with the Model

We will consider three number of servers (= C): the first jamrah (al-jamrah al-ula), the middle jamrah (al-jamrah al-wusta), the largest jamrah (jamrah of aqaba), then calculate K = queue capacity (pilgrim/m2), λ = arrival rates (in minutes), μm = service rates (pilgrim/minutes). Service rate here, refers to the number of pilgrim finish throwing (in minutes).

Table 1. Performance of pilgrims in ramy Al-Jamarat for different arrival rates

C	K (number of pilgrims/ m^2)	λ (minutes)	μ_m (number of pilgrims/ minutes)	L	E(Q)	W	E(T)	ρ	Probabilities		
									Pm	Time in System	Time in Queue
3	3.5	30	244.00	2.9933	0	0.0998	0.0957	0.041	0.8843	0	0.9997
3	3.9	27	215.75	2.9931	0	0.1109	0.1062	0.0417	0.8821	0	0.9997
3	4.7	29	327.87	2.9964	0	0.1033	0.1003	0.0295	0.9153	0	0.9999
3	4.5	25	208.23	2.9936	0	0.1197	0.1149	0.04	0.8869	0	0.9997
3	3.6	36	170.27	2.982	0.0001	0.0829	0.077	0.0705	0.8094	0	0.9986
3	4.4	31	205.45	2.9902	0	0.0965	0.0916	0.0503	0.8599	0	0.9995
3	4.6	33	210.18	2.9895	0	0.0906	0.0858	0.0523	0.8547	0	0.9994
3	6.5	30	175.82	2.9878	0	0.0996	0.0939	0.0569	0.8431	0	0.9993
3	4.0	39	151.85	2.9747	0.0002	0.0763	0.0697	0.0856	0.7735	0	0.9976
3	4.0	30	182.17	2.9885	0	0.0996	0.0941	0.0549	0.8482	0	0.9993
3	7.6	33	256.37	2.9927	0	0.0907	0.0868	0.0429	0.8792	0	0.9997
3	6.6	27	215.58	2.9931	0	0.1109	0.1062	0.0417	0.8823	0	0.9997
3	6.0	45	141.56	2.9638	0.0005	0.0659	0.0588	0.106	0.7276	0	0.9956
3	4.7	33	107.3	2.9657	0.0004	0.0899	0.0806	0.1025	0.7352	0	0.996
3	8.0	39	196.19	2.9839	0.0001	0.0765	0.0714	0.0663	0.8197	0	0.9989
3	7.0	30	83.28	2.9556	0.0008	0.0985	0.0865	0.1201	0.6974	0	0.9938

Where:
L = Average number of pilgrims in the system.
E(Q) = pilgrims in queue (Expected amount of pilgrims waiting to be served).
W = Average time waiting in line (Average time spend by a pilgrim from arrival until fully served).
E(T) = Average time waiting in line (Average time it takes a pilgrim to start being served).
P = Server utilization (percentage of time a server is being utilized by a pilgrim).

For three servers with the number of pilgrims/m2 varying from 3-8 pilgrims, the arrival rate is between 25-45 minutes, then we can calculate the number of service rates, that the number of pilgrims/minute is around 84-328 pilgrims and this refers to the number of throws completed by pilgrims (in minute). the average number of pilgrims in this system is about 3 pilgrims, but those who are queuing or waiting to be served are almost 0. The average waiting time in line or the average time spent by pilgrims from arrival to being fully served is less than 12 seconds and the average time it takes a pilgrim to start being served is also less than 12 seconds. The percentage of server time used by pilgrims is about 12 second. Steady-state probability for the queuing system if the arrival rate is constant above 70%. Probability of time in system is 0, and probability of time in queue is above 99%.

Most of the pilgrims deciding to join the queue, then pBalk is close to zero. This make Θ ≈ λ. From the analysis, we can see that the average number of pilgrims waiting in queue almost zero, because maybe most of the pilgrims only spend least then 12 second in the system.

4. CONCLUSION

In this study a new analytical tool for measuring the performances of pilgrim movements in Hajj has been developed. Further effort is necessary to measure the observed maximum density and free flow speed of pilgrims that will enable us to calculate the performance measures of pilgrim flow as mentioned in the previous section. In addition, the influences of the factors: pilgrim arrival flow rate, , length of the facility L, width of the facility W and pilgrim personal capacity vf on the performances can be determined. Thus, the current study is helpful to manage the pilgrim flow to meet performance measures certain magnitudes that are convenient for pilgrim movements. The study is also helpful to determine the cost-effective length and width of the facility, which in turn helps in the design of the facilities in Hajj.

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