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Original research article

On the investigation of optical soliton solutions of cubic–quartic Fokas–Lenells and Schrödinger–Hirota equations

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ABSTRACT

Purpose: When it comes to third and higher-order dispersion, the Schrödinger–Hirota equation is one of the main models developed outside the classical NLSE management models for optical soliton transmission. The cubic–quartic Fokas–Lenells equation is also one of the recently developed equations, which has importance in the field of telecommunications regarding optical soliton transmission in the absence of chromatic dispersion. In this study, in order to examine the optical solitons, the Schrödinger–Hirota equation in the presence of the chromatic dispersion and the cubic–quartic Fokas–Lenells equation discarding the chromatic dispersion were investigated. For this intent, by obtaining certain soliton types using the unified Riccati equation expansion method (UREEM), optical soliton solutions were obtained for both models and graphical representations and comments were made.
Methodology: By developing appropriate computer algorithms and applying UREEM in the following ways, symbolic calculation software was made and analytical optical soliton solutions were obtained.
Findings: Through computer algebra software, we plotted the obtained results via 3D, 2D views and we also illustrated the investigation of wave behavior caused by parameter change on 2D graphics.
Originality: Different soliton behavior under the parameters effect of the Schrödinger–Hirota equation having chromatic dispersion and the cubic–quartic Fokas-Lenells equation is investigated and the obtained results are reported.

1. Introduction

The role of the nonlinear partial differential equations (NLPDEs) in explaining many events or actions in the universe has undisputed importance. Especially many studies have been done last 50 years; moreover, the studies have gained acceleration in the last 25 years. Progression in software technologies parallel with innovation in computer science and electronics caused these studies have been increased. In particular, obtaining effective results in computational mathematics and symbolic programming (with Matlab, Maple, Mathematica, etc.) and supporting these results via computer base also affected the increase. Many researchers can introduce new theories, areas and different solution techniques through computer-based processes. Nonlinear wave propagation and

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especially nonlinear optics have a unique research field in these areas. The invention of lasers and usage in many fields and parallel with advances in fiber technologies, communication and fiber usage in data transfer, reasoned that many researchers intend these fields.

After approaches such as Hirota bilinear form, Lax pair, and perturbation [1–5], many models have been made in this area. Schrödinger–Hirota [6], Fokas–Lenells [7], complex Ginzburg–Landau [8], Chen–Lee–Liu model [9], Manakov model [10], Radhakrishnan–Kundu–Lakshmanan [11], Kundu–Mukherjee–Naskar [12], Sasa–Satsuma [13], Gerdjikov–Ivanov [14], Lakshmanan–Porsezian–Daniel [15], Kaup–Newell [16], Biswas–Arshed [17], Biswas–Milovic [18] as examples of models developed only in the field of nonlinear optics. Although these models seem not to be too many, they have different dimensional forms (1+1, 2+1, 3+1, 4+1), different structures (dimensionless, perturbed, unperturbed, cubic, quartic, quintic, cubic–quartic, septic, sextic etc.), different forms of nonlinearity (Kerr, power, parabolic, dual-power, quadratic–cubic, log, anti-cubic, generalized anti-cubic, triple-power, polynomial, arbitrary refractive index [19–21]), different dispersion terms (group velocity dispersion (GVD), spatio-temporal dispersion (STD), polarization mode dispersion (PMD), the third order dispersion (TOD), the fourth order dispersion) (FOD), self phase modulation (SPM), cross phase modulation (XPM), inter modal dispersion (IMD) etc. [22–29] considering many other factors such as having or not, the range of work in this field includes thousands of problems. All these have also led to the development of many methods and different methods have been applied in many studies. For example, Lie symmetry analysis [30], modified Kudryashov's method [31], the improved extended tanh-equation method [32], the extended trial equation method [33], the dromion-like structures [34], new mapping scheme [35], stationary solitons [36], semi-inverse variational principle [37,38], the integration scheme [39], sine-Gordon equation approach [40], G'/G–expansion scheme [41], Laplace–Adomian decomposition [42], the improved Adomian decomposition scheme [43].

Chromatic dispersion (CD) and nonlinear refractive index (NRI) are two elements of the fundamentals of soliton propagation in optical fibers. As it is known, the CD is the phase and group velocity of light emitted in a transparent medium depends on the optical frequency. So, the CD is caused by changes in wavelength and propagation velocity. Although the effect of the CD in fibers can be reset, the CD is a natural process in producing glass fibers. The CD is compulsory for adjusting (minimizing) the effect of the nonlinear terms in wavelength-division multiplexing systems.

Nevertheless, in general, adjusting these balances is a big problem. The issue of the CD occurs in fibers as corrupted signal quality, requiring dispersion compensation. The fiber gratings approach is the most effective method for the CD issue [44–47]. Much work has been done on CD, especially on single-mode fibers [48–50]. In recent years, not only theoretical but also some experimental studies have been carried out regarding the role of CD in soliton conduction. The most notable among these studies is the structural experimental study, replacing the term CD with the fourth-order dispersion term [51]. With this study, the concept of pure-quartic soliton entered the literature and some studies were carried out and reported [52–55], some of which were experimental, by replacing the term CD and some perturbation terms with both TOD and FOD terms due to ignoring [56–60].

In the framework of the manuscript, the first problem which is Schrödinger–Hirota equation examined by taking into account the CD term. The effect of CD was investigated for different parameter values and the second model equation, the CQ-FL equation, which was formed by ignoring the CD term, was examined as the second step. The Schrödinger–Hirota equation with chromatic dispersion is given [61]:

$$iu_t + a_1 u_{xx} + b_1 |u|^2 u + i\left(c_1 u_{xxx} + d_1 |u|^2 u_x\right) = 0, \tag{1}$$

where a_1, b_1, c_1 and d_1 are the coefficients of GVD, Kerr-law nonlinearity, TOD and nonlinear dispersion terms, respectively. The cubic–quartic Fokas Lenell equation by discarding the chromatic dispersion is given [62]:

$$iu_t + ia_2 u_{xxx} + b_2 u_{xxxx} + |u|^2\left(c_2 u + id_2 u_x\right) - i\left[\lambda(|u|^2 u)_x + \mu(|u|^2)_x u\right] = 0, \tag{2}$$

where Eq. (2) $a_2, b_2, c_2, d_2, \lambda, \mu$ stand for TOD, FOD, Kerr-law nonlinearity, nonlinear dispersion, self-steepening, higher-order dispersion terms coefficients. Various modeling studies have been carried out regarding optical solitons. Among the main purposes of these studies is to maintain the shape and quality of the soliton in soliton transmission and to control it by nonlinear effects. In this context, terms such as Lie transform, Hamiltonian perturbation, and quasi-stationarity are among the main actors that come to the fore [63]. Other studies related to these models are given in [31,43,61,62,64–75].

The organization of the paper is as follows; in Section 2 we did a mathematical analysis of the models. Section 3 includes the application of the UREEM on both models. Then, Section 4 includes the plotting of the obtained results. Lastly, Section 5 includes the conclusion part of the study.

2. Mathematical analysis

2.1. Obtaining NODE form of the Schrödinger–Hirota equation

In this section, we obtained the nonlinear ordinary differential equation (NODE) form of Eq. (1). Consider the following wave transformation:

$$u(x,t) = U(\xi)e^{i\theta}, \quad \xi = x - \upsilon t, \quad \theta = -\alpha x + \beta t + \psi_0. \tag{3}$$

Where υ is the velocity, α is frequency, β wave number and ψ_0 is phase constant of soliton. Insert Eq. (3) into Eq. (1) and separate the real and imaginary parts as follows:

$$\left(\alpha d_1 + b_1\right) U^3 + \left(-\alpha^3 c_1 - a_1 \alpha^2 - \beta\right) U + \left(3\alpha c_1 + a_1\right) U'' = 0, \tag{4}$$

$$d_1U^2U' + \left(-3\alpha^2c_1 - 2a_1\alpha - v\right)U' + c_1U''' = 0, \tag{5}$$

where ($'$) sign denotes ordinary differential as $\frac{dU}{d\xi}$. If we integrate Eq. (5) with respect to the ξ and assume the integration constant as zero, we obtain;

$$d_1U^3 + \left(-9\alpha^2c_1 - 6a_1\alpha - 3v\right)U + 3c_1U'' = 0. \tag{6}$$

If the homogeneous balance principle is applied between Eqs. (4) and (6), the following formula is gained:

$$\frac{\left(\alpha d_1 + b_1\right)}{d_1} = \frac{\left(-\alpha^3c_1 - a_1\alpha^2 - \beta\right)}{\left(-9\alpha^2c_1 - 6a_1\alpha - 3v\right)} = \frac{\left(3\alpha c_1 + a_1\right)}{3c_1}. \tag{7}$$

From Eq. (7), we get the following constraint conditions;

$$d_1 = \frac{3b_1c_1}{a_1},\quad \beta = \frac{8\alpha^3c_1^2 + 8a_1\alpha^2c_1 + 2a_1^2\alpha + 3\alpha c_1v + a_1v}{c_1}. \tag{8}$$

Under these constraints, we can consider Eq. (4) (or Eq. (6)) as a NODE representation of Eq. (1) in the following form:

$$\left(\alpha d_1 + b_1\right)U^3 + \left(-\alpha^3c_1 - a_1\alpha^2 - \beta\right)U + \left(3\alpha c_1 + a_1\right)U'' = 0. \tag{9}$$

Applying the balancing principle between U^3 and U'', the balance number is computed as $N = 1$.

2.2. Obtaining NODE form of CQ-FL equation

In this section, we obtained the NODE form of the CQ-FL equation. Let us assume that the following wave transformation:

$$u(x,t) = U(\xi)e^{i\theta},\quad \xi = x - vt,\quad \theta = -kx + \omega t + \psi_0, \tag{10}$$

where v is the velocity, k is frequency, ω wave number and ψ_0 is phase constant. Substituting Eq. (10) into Eq. (2), we derive the following equations;

$$\left(\left(-\lambda + d_2\right)k + c_2\right)U^3 + \left(b_2k^4 - a_2k^3 - \omega\right)U + \left(-6b_2k^2 + 3a_2k\right)U'' + b_2U^{(4)} = 0, \tag{11}$$

$$-3\left(\lambda - \frac{1}{3}d_2 + \frac{2}{3}\mu\right)U^2U' + \left(4b_2k^3 - 3a_2k^2 - v\right)U' + \left(-4b_2k + a_2\right)U''' = 0. \tag{12}$$

From Eq. (12), we obtain the following constraint conditions;

$$\lambda = \frac{1}{3}d_2 - \frac{2}{3}\mu,\quad v = 4b_2k^3 - 3a_2k^2,\quad a_2 = 4b_2k. \tag{13}$$

Substituting these constraints into Eq. (11), the NODE form of Eq. (2) is derived:

$$\left(\left(\frac{2}{3}d_2 + \frac{2}{3}\mu\right)k + c_2\right)U^3 + \left(-3b_2k^4 - \omega\right)U + 6b_2k^2U'' + b_2U^{(4)} = 0. \tag{14}$$

Referencing the balancing rule between the terms U^3 and $U^{(4)}$, balancing constant is calculated as $N = 2$.

3. Application

3.1. Implementation of UREEM on the Schrödinger–Hirota equation

Considering the balance constant $N = 1$, we assume that the NODE in Eq. (9) has a solution as following form;

$$U(\xi) = \sigma_0 + \sigma_1\phi(\xi), \tag{15}$$

where σ_0 and σ_1 are constants with $\sigma_1 \neq 0$, also $\phi(\xi)$ feeds the following formula;

$$\frac{d\phi}{d\xi} = \kappa_0 + \kappa_1\phi(\xi) + \kappa_2\phi(\xi)^2. \tag{16}$$

Eq. (16) has the following conditional solutions.

- If $\Delta > 0$, $r_2 \neq r_1$ and $r_1^2 + r_2^2 \neq 0$:

$$\phi_1(\xi) = -\frac{\kappa_1}{2\kappa_2} - \frac{\sqrt{\Delta}\left(r_1\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right) + r_2\right)}{2\kappa_2\left(r_1 + r_2\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right)}. \tag{17}$$

- If $\Delta<0$ and $r_3^2+r_4^2\neq 0$:

$$\phi_2(\xi)=-\frac{\kappa_1}{2\kappa_2}-\frac{\sqrt{-\Delta}\left(r_3\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)-r_4\right)}{2\kappa_2\left(r_3+r_4\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right)}. \tag{18}$$

Where $\Delta=r_1^2-4r_0r_2$. Substitute Eq. (15) with considering Eq. (16) into Eq. (9) and collecting each coefficients of the $\phi(\xi)^i, i=(0\dots 3)$ then equate every coefficient to the zero and get the following algebraic system:

$$\begin{aligned}
&\phi(\xi)^0: a_1c_1\sigma_1\kappa_1\kappa_0-2\sigma_0\left(a_1^2\alpha+\left(\frac{3}{2}c_1\alpha^2+\frac{1}{2}v\right)a_1-\frac{1}{2}\sigma_0^2b_1c_1\right)=0,\\
&\phi(\xi)^1: -2\sigma_1\left(a_1^2\alpha+\left(\left(\frac{3}{2}\alpha^2-\kappa_0\kappa_2-\frac{1}{2}\kappa_1^2\right)c_1+\frac{1}{2}v\right)a_1-\frac{3}{2}\sigma_0^2b_1c_1\right)=0,\\
&\phi(\xi)^2: 3\sigma_1c_1\left(a\kappa_1\kappa_2+b_1\sigma_0\sigma_1\right)=0,\\
&\phi(\xi)^3: 2a_1c_1\kappa_2^2\sigma_1+b_1c_1\sigma_1^3=0.
\end{aligned} \tag{19}$$

By using computer algebra software, we solve the system in Eq. (19) and get the following solution sets;

$$\mathbf{SET}_1:\left\{\begin{array}{c}\sigma_0=\frac{1}{2}\frac{\kappa_1\sigma_1}{\kappa_2}, b_1=\frac{1}{2}\frac{\left(6\alpha^2c_1-4c_1\kappa_0\kappa_2+c_1\kappa_1^2+2v\right)\kappa_2^2}{\alpha\sigma_1^2},\\ \sigma_1=\sigma_1, a_1=-\frac{1}{4}\frac{6\alpha^2c_1-4c_1\kappa_0\kappa_2+c_1\kappa_1^2+2v}{\alpha}\end{array}\right\} \tag{20}$$

$$\mathbf{SET}_2:\left\{\sigma_0=\frac{1}{2}\frac{\kappa_1\sigma_1}{\kappa_2}, b_1=-\frac{2a_1\kappa_2^2}{\sigma_1^2}, \sigma_1=\sigma_1, \kappa_0=\frac{1}{4}\frac{6\alpha^2c_1+c_1\kappa_1^2+4a_1\alpha+2v}{c_1\kappa_2}\right\}. \tag{21}$$

Substituting the obtained sets in Eqs. (20) and (21) into Eq. (15) with considering the solutions in Eqs. (17) and (18) together the constraint conditions in Eq. (8), then, applying the wave transformation in Eq. (3), we obtain the following solutions of Eq. (1);

$$u_{1,1}(x,t)=\left(\frac{\kappa_1\sigma_1}{2\kappa_2}+\sigma_1\left(-\frac{\kappa_1}{2\kappa_2}-\frac{\sqrt{\Delta_1}\left(r_1\tanh\left(\frac{\sqrt{\Delta_1}(-vt+x)}{2}\right)-r_2\right)}{2\kappa_2\left(r_1+r_2\tanh\left(\frac{\sqrt{\Delta_1}(-vt+x)}{2}\right)\right)}\right)\right)e^{i(-\alpha x+\beta t+\psi_0)}, \tag{22}$$

$$u_{1,2}(x,t)=\left(\frac{\kappa_1\sigma_1}{2\kappa_2}+\sigma_1\left(-\frac{\kappa_1}{2\kappa_2}-\frac{\sqrt{\Delta_2}\left(r_3\tan\left(\frac{\sqrt{\Delta_2}(-vt+x)}{2}\right)-r_4\right)}{2\kappa_2\left(r_3+r_4\tan\left(\frac{\sqrt{\Delta_2}(-vt+x)}{2}\right)\right)}\right)\right)e^{i(-\alpha x+\beta t+\psi_0)}, \tag{23}$$

$$u_{1,3}(x,t)=\left(\frac{\kappa_1\sigma_1}{2\kappa_2}+\sigma_1\left(-\frac{\kappa_1}{2\kappa_2}-\frac{\sqrt{\Delta_3}\left(r_3\tanh\left(\frac{\sqrt{\Delta_3}(-vt+x)}{2}\right)-r_4\right)}{2\kappa_2\left(r_3+r_4\tan\left(\frac{\sqrt{\Delta_3}(-vt+x)}{2}\right)\right)}\right)\right)e^{i(-\alpha x+\beta t+\psi_0)}, \tag{24}$$

$$u_{1,4}(x,t)=\left(\frac{\kappa_1\sigma_1}{2\kappa_2}+\sigma_1\left(-\frac{\kappa_1}{2\kappa_2}-\frac{\sqrt{\Delta_4}\left(r_3\tan\left(\frac{\sqrt{\Delta_4}(-vt+x)}{2}\right)-r_4\right)}{2\kappa_2\left(r_3+r_4\tan\left(\frac{\sqrt{\Delta_4}(-vt+x)}{2}\right)\right)}\right)\right)e^{i(-\alpha x+\beta t+\psi_0)}, \tag{25}$$

where $\Delta_1=-4\kappa_0\kappa_2+\kappa_1^2>0$, $\Delta_2=-\Delta_1$, $\Delta_3=-4\left(\frac{1}{4}\frac{6\alpha^2c_1+c_1\kappa_1^2+4a_1\alpha+2v}{c_1\kappa_2}\right)\kappa_2+\kappa_1^2>0$, $\Delta_4=-\Delta_3$, $\beta=\frac{\left(8\alpha^3c_1^2+8a_1\alpha^2c_1+2a_1^2\alpha+3\alpha c_1v+a_1v\right)}{c_1}$.

3.2. Implementation of UREEM on the CQ-FL equation

Considering the balance constant $N=2$, it is accepted that Eq. (26) is the solution of Eq. (14);

$$U(\xi)=\sigma_0+\sigma_1\phi(\xi)+\sigma_2\phi(\xi)^2, \tag{26}$$

where σ_0,σ_1 and σ_2 are constants with $\sigma_2\neq 0$, also $\phi(\xi)$ admits the following formula;

$$\frac{d\phi}{d\xi}=\kappa_0+\kappa_1\phi(\xi)+\kappa_2\phi(\xi)^2. \tag{27}$$

According to the UREEM, the Riccati equation in Eq. (27) has the same conditional solutions in Eqs. (17) and (18). Substitute Eq. (26) into Eq. (14) by considering Eq. (27) and separate each coefficients of the $\phi(\xi)^i, i=(0\dots 6)$ then equate every coefficient to the zero and get the following algebraic system:

$$\begin{aligned}
\phi(\xi)^0 &: 12\left(k^2+\frac{4}{3}\kappa_2\kappa_0+\frac{7}{6}\kappa_1^2\right)\kappa_0^2 b_2\sigma_2+\frac{1}{3}\left(18\kappa_1\left(k^2+\frac{4}{3}\kappa_2\kappa_0+\frac{1}{6}\kappa_1^2\right)\kappa_0\sigma_1-9k^4\sigma_0\right)b_2\\
&+\frac{2}{3}\sigma_0\left(\sigma_0^2\left(\mu+d_2\right)k+\frac{3}{2}\sigma_0^2 c_2-\frac{3}{2}\omega\right)=0,\\
\phi(\xi)^1 &: 36\kappa_1\left(k^2+\frac{10}{3}\kappa_2\kappa_0+\frac{5}{6}\kappa_1^2\right)\kappa_0 b_2\sigma_2-3\left(\left(k^4+\left(-4\kappa_0\kappa_2-2\kappa_1^2\right)k^2-\frac{16}{3}\kappa_2^2\kappa_0^2-\frac{22}{3}\kappa_2\kappa_0\kappa_1^2\right.\right.\\
&\left.\left.-\frac{1}{3}\kappa_1^4\right)b_2-\frac{2}{3}\sigma_0^2\left(\mu+d_2\right)k-\sigma_0^2 c_2+\frac{1}{3}\omega\right)\sigma_1=0,\\
\phi(\xi)^2 &: \frac{1}{3}\left(\left(-9k^4+\left(144\kappa_0\kappa_2+72\kappa_1^2\right)k^2+408\kappa_2^2\kappa_0^2+696\kappa_2\kappa_0\kappa_1^2+48\kappa_1^4\right)b_2+6\sigma_0^2\left(\mu+d_2\right)k+9\sigma_0^2 c_2\right.\\
&\left.-3\omega\right)\sigma_2+18\left(\kappa_2\kappa_1\left(k^2+\frac{10}{3}\kappa_2\kappa_0+\frac{5}{6}\kappa_1^2\right)b_2+\frac{1}{9}\left(\left(\mu+d_2\right)k+\frac{3}{2}c_2\right)\sigma_0\sigma_1\right)\sigma_1=0,\\
\phi(\xi)^3 &: \frac{1}{3}\left(180\kappa_1\kappa_2\left(k^2+\frac{22}{3}\kappa_2\kappa_0+\frac{13}{6}\kappa_1^2\right)b_2+12\left(\left(\mu+d_2\right)k+\frac{3}{2}c_2\right)\sigma_0\sigma_1\right)\sigma_2\\
&+12\left(\kappa_2^2\left(k^2+\frac{10}{3}\kappa_2\kappa_0+\frac{25}{6}\kappa_1^2\right)b_2+\frac{1}{18}\left(\left(\mu+d_2\right)k+\frac{3}{2}c_2\right)\sigma_1^2\right)\sigma_1=0,\\
\phi(\xi)^4 &: 2\left(\left(\mu+d_2\right)k+\frac{3}{2}c_2\right)\sigma_0\sigma_2^2+\frac{1}{3}\left(\left(108k^2\kappa_2^2+720\kappa_0\kappa_2^3+990\kappa_1^2\kappa_2^2\right)b_2\right.\\
&\left.+6\left(\left(\mu+d_2\right)k+\frac{3}{2}c_2\right)\sigma_1^2\right)\sigma_2+60\kappa_2^3\sigma_1 b_2\kappa_1=0,\\
\phi(\xi)^5 &: 2\left(\left(\mu+d_2\right)k+\frac{3}{2}c_2\right)\sigma_1\sigma_2^2+336\kappa_2^3\sigma_2 b_2\kappa_1+24\kappa_2^4\sigma_1 b_2=0,\\
\phi(\xi)^6 &: \frac{2}{3}\sigma_2\left(\left(\left(\mu+d_2\right)k+\frac{3}{2}c_2\right)\sigma_2^2+180\kappa_2^4 b_2\right)=0.
\end{aligned} \tag{28}$$

By using computer algebra software, solving the system in Eq. (28), we get the following solution set:

$$\mathbf{SET}_3:\left\{\begin{array}{c}
\omega=-\frac{219}{25}b_2k^4,\ \kappa_0=\frac{\left(\left(-2\mu-2d_2\right)k-3c_2\right)\sigma_1^2+432b_2k^2\kappa_2^2}{1440b_2\kappa_2^3},\\
\kappa_1=\frac{\sigma_1\left(2d_2k+2k\mu+3c_2\right)}{6\kappa_2\sqrt{-10b_2\left(2d_2k+2k\mu+3c_2\right)}},\ \kappa_2=\kappa_2,\ \sigma_1=\sigma_1,\\
\sigma_0=-\frac{1}{24}\frac{432b_2k^2\kappa_2^2-2d_2k\sigma_1^2-2k\mu\sigma_1^2-3c_2\sigma_1^2}{\sqrt{-10b_2\left(2d_2k+2k\mu+3c_2\right)}\kappa_2^2},\\
\sigma_2=\frac{6\sqrt{-10b_2\left(2d_2k+2k\mu+3c_2\right)}\kappa_2^2}{2d_2k+2k\mu+3c_2}
\end{array}\right\}. \tag{29}$$

Substituting the obtained set in Eq. (29) into Eq. (26) by considering the solutions in Eq. (17), Eq. (18) and the constraint conditions in Eq. (13), then, applying the wave transformation in Eq. (10), we obtain the following solutions of Eq. (2):

$$u_{2,1}(x,t)=\left(\sigma_0+\sigma_1\left(\phi_1(x,t)\right)+\sigma_2\left(\phi_1(x,t)\right)^2\right)e^{i(-kx+\omega t+\psi_0)}, \tag{30}$$

$$u_{2,2}(x,t)=\left(\sigma_0+\sigma_1\left(\phi_2(x,t)\right)+\sigma_2\left(\phi_2(x,t)\right)^2\right)e^{i(-kx+\omega t+\psi_0)}, \tag{31}$$

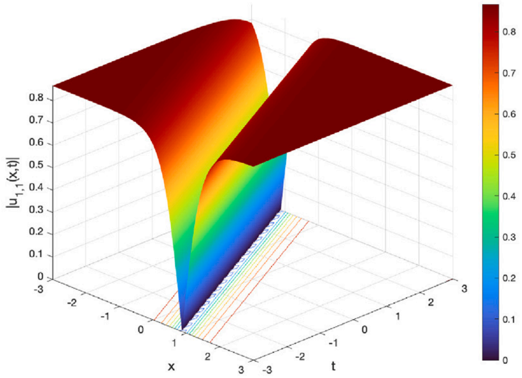
where $\phi_1(x,t)$ and $\phi_2(x,t)$ are defined as in Eqs. (17) and (18). In Eqs. (30) and (31), we preferred to give the general form of the solution functions rather than the very long form. Also $\sigma_0,\sigma_1,\sigma_2,\kappa_0,\kappa_1,\kappa_2$ and ω are obtained from Eq. (29).

4. Results and discussion

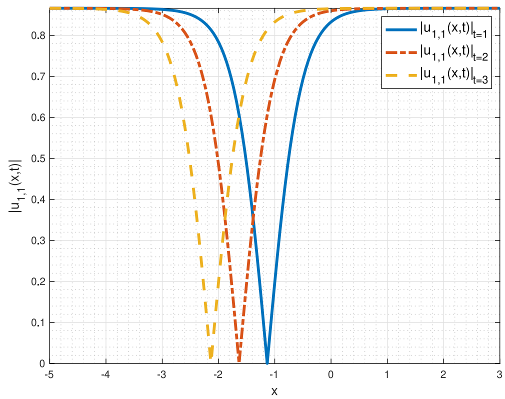
In this section, we depicted some plots of obtained results and investigated the wave behavior depending on the parameter changes.

In Fig. 1, we depicted some plots of $u_{1,1}(x,t)$ in Eq. (22). Fig. 1(a), shows 3D plots of $|u_{1,1}(x,t)|$ with $\sigma_1=2,\kappa_0=-1,\kappa_1=\kappa_2=1,r_1=2,r_2=0.8,\upsilon=-0.5,\alpha=1,c_1=0.8$ and $a_1=0.8$. Fig. 1(b), shows 2D plots of $|u_{1,1}(x,t)|$ at $t=1,2,3$ in Eq. (22) with aforementioned values. According to Figs. 1(a) and 1(b), $|u_{1,1}(x,t)|$ has dark soliton solution and the soliton travels through positive-x direction due to time. On the other hand, Fig. 1(c), represents 3D plot of $|u_{1,1}(x,t)|$ with aforementioned values when $r_2=2.8$. Also, Fig. 1(d) shows the 2D plots of the solution in Fig. 1(c) at $t=1,2,3$. The traveling direction of the singular soliton in Fig. 1(c) is positive-x direction as shown in Fig. 1(d). As a result, Figs. 1(c) and 1(d) show that the change in the parameter r_2, transforms the solution into a singular soliton solution.

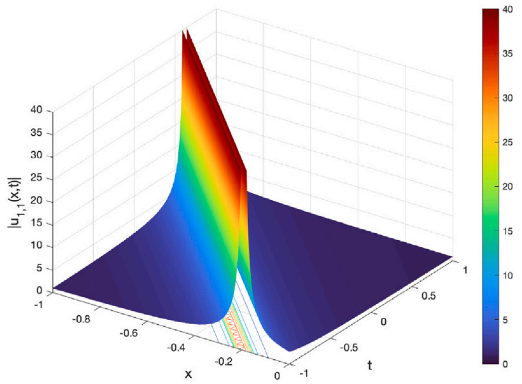
Fig. 2 illustrates some views of $u_{1,2}(x,t)$ in Eq. (23). Fig. 2(a), shows the 3D plot of $|u_{1,2}(x,t)|$ and Fig. 2(b), shows the 2D plots of $|u_{1,2}(x,t)|$ at $t=1,2,3$. Figs. 2(a) and 2(b) are plotted with $\sigma_1=2,\kappa_0=1,\kappa_1=\kappa_2=2,r_3=1,r_4=0.5,\upsilon=-0.5,\alpha=1,c_1=0.8$ and $a_1=0.8$ values. According to Figs. 2(a) and 2(b), $|u_{1,2}(x,t)|$ has a periodic-singular soliton solution and the traveling direction is positive-x direction due to time.



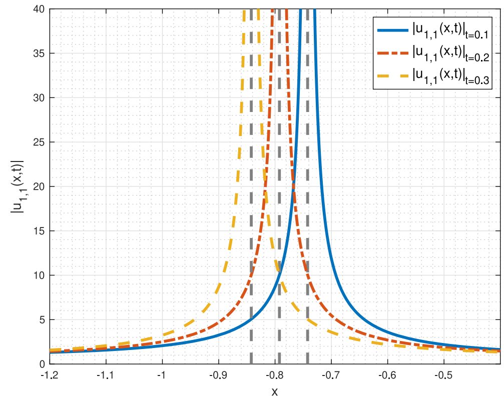
(a) 3D plot of $|u_{1,1}(x,t)|$



(b) 2D plots of $|u_{1,1}(x,t)|$

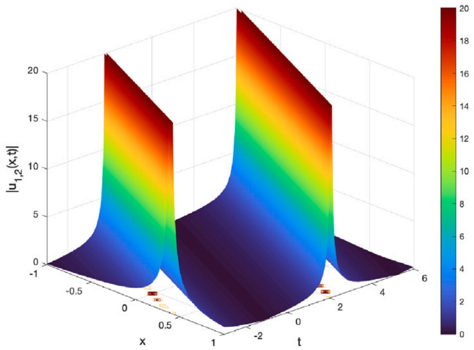


(c) 3D plot of $|u_{1,1}(x,t)|$

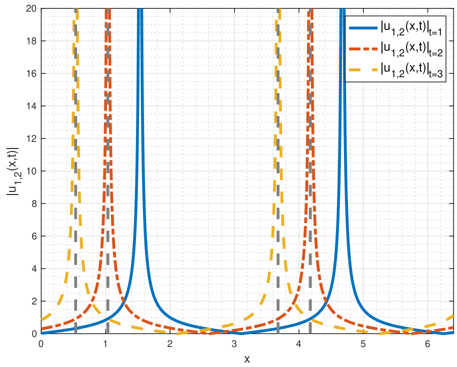


(d) 2D plots of $|u_{1,1}(x,t)|$

Fig. 1. Various plots of $|u_{1,1}(x,t)|$ in Eq. (22).

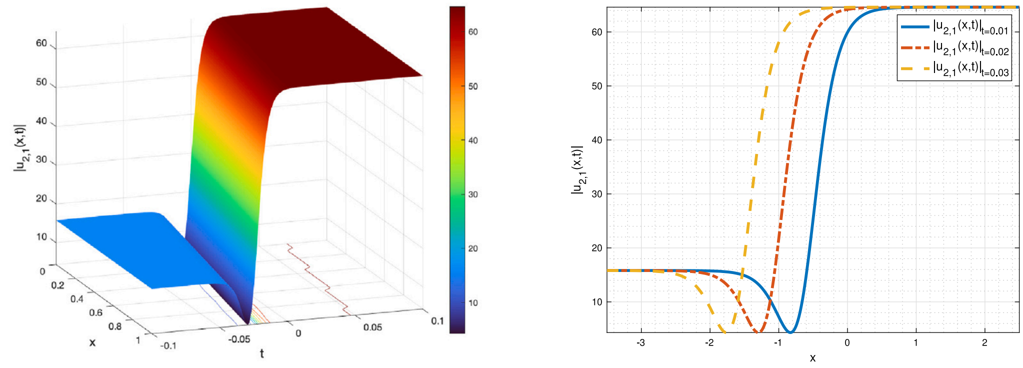


(a) 3D plot of $|u_{1,2}(x,t)|$



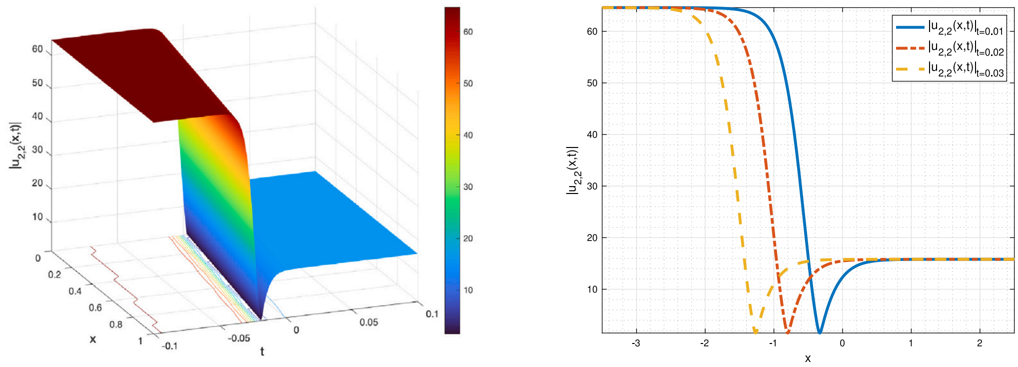
(b) 2D plots of $|u_{1,2}(x,t)|$

Fig. 2. Various plots of $|u_{1,2}(x,t)|$ in Eq. (23).



(a) 3D plot of $|u_{2,1}(x,t)|$ (b) 2D plots of $|u_{2,1}(x,t)|$

Fig. 3. Graphical demonstrations of $|u_{2,1}(x,t)|$ in Eq. (30).



(a) 3D plot of $|u_{2,2}(x,t)|$ (b) 2D plots of $|u_{2,2}(x,t)|$

Fig. 4. Some portraits of $|u_{2,2}(x,t)|$ in Eq. (31).

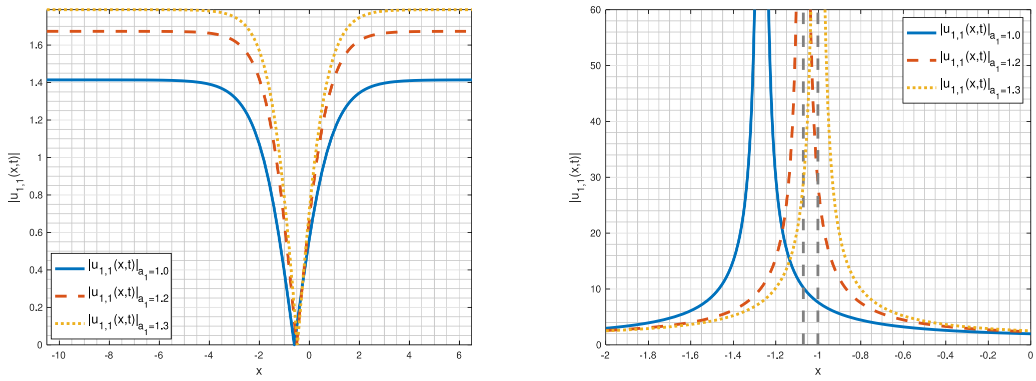
In Fig. 3, we plotted some graphs of $u_{2,1}(x,t)$ in Eq. (30). Fig. 3(a), represents 3D plot of $|u_{2,1}(x,t)|$ with $b_2 = 1, \kappa_2 = 3, k = 1.8, \mu = -2, d_2 = -3.9, c_2 = 1, r_1 = 2, r_2 = 1, \psi = 2$ and $\sigma_1 = 1$ values. On the other hand, Fig. 3(b) shows 2D plots of $|u_{2,1}(x,t)|$ in Eq. (30) with aforementioned parameters at $t = 1, 2, 3$. According to Figs. 3(a) and 3(b), the solution $|u_{2,1}(x,t)|$ is a mixed kink–dark soliton solution and travels through the negative-x direction.

In Fig. 4, we depicted some plots of $|u_{2,2}(x,t)|$ in Eq. (31). Fig. 4(a), shows the 3D plot and Fig. 4(b) shows the 2D plots of $|u_{2,2}(x,t)|$ in Eq. (31) with $b_2 = 1, \kappa_2 = 3, k = 1.8, \mu = -2, d_2 = -3.9, c_2 = 1, r_3 = 2, r_4 = 1, \psi = 2$ and $\sigma_1 = 1$ values at $t = 0.01, 0.02, 0.03$. Also, Figs. 4(a) and 4(b) show that the solution is a mixed kink-dark soliton solution and travels along to the negative-x direction.

Figs. 3 and 4 are graphically similar and categorically look like a combination of dark and kink soliton, but they are not physically identical. Because if the graphs of Figs. 3 and 4 are examined carefully, the kink soliton image formed in the graph of Fig. 3 is left-oriented and to the left of the dark soliton kink soliton, and the opposite in the graph of Fig. 4.

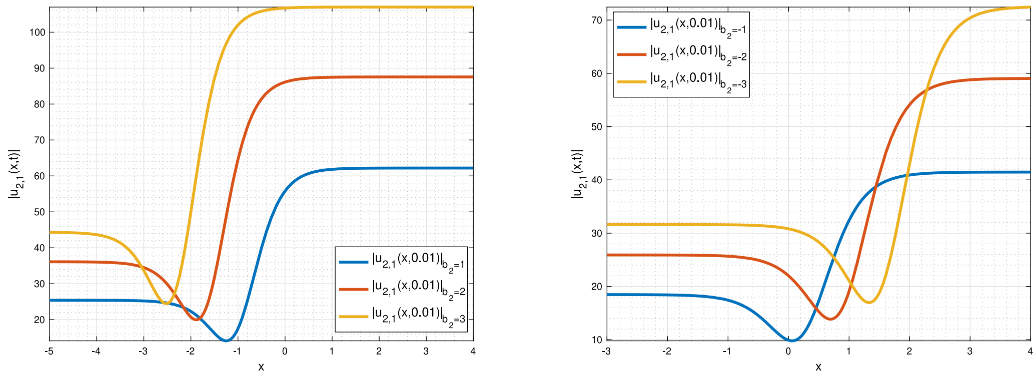
In Fig. 5, we depicted some 2D plots of $u_{1,1}(x,t)$ in Eq. (23). In order to observe the effect of the chromatic dispersion parameter on the wave behavior, we depicted Fig. 5. Fig. 5(a) has three different styled plots. Blue, red and yellow waves are plotted for $a_1 = 1, 1.2$ and $a_1 = 1.3$ values respectively. Depending on the change of a_1 it is seen that the wave amplitude also changes. According to Fig. 5(a), while a_1 takes the positive values and decreasing, wave amplitude increases. On the other hand, in Fig. 5(b). Therefore, it is obvious that the effect originating from the chromatic dispersion term for the Schrödinger–Hirota equation has consequences on the soliton behavior, whether it is dark or singular soliton behavior. The parameter values selected in these graphic presentations are the parameters chosen in line with the requirements of the problem and the applied method. In this context, it is not possible to accept these parameters as an arbitrary real number and give them values. Here, to observe the effect of the chromatic dispersion, one of the most basic elements was to preserve the physical shape of the soliton.

In Fig. 6, we depicted some 2D plots of $|u_{2,1}(x,t)|$ in Eq. (30). The parameter b_2 represents the quartic nonlinearity (FOD) of the model in Eq. (2). Fig. 6(a) shows the different wave behavior due to the parameter b_2 change. Fig. 6(a) has three different styled



(a) 2D plots of $|u_{1,1}(x,t)|$ (b) 2D plots of $|u_{1,1}(x,t)|$

Fig. 5. Various plots of $|u_{1,1}(x,t)|$ in Eq. (24).

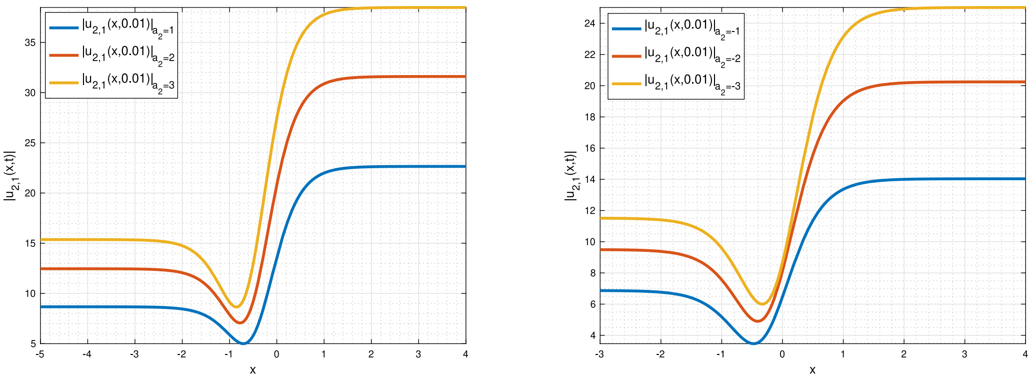


(a) 2D plots of $|u_{2,1}(x,t)|$ (b) 2D plots of $|u_{2,1}(x,t)|$

Fig. 6. Various plots of $|u_{2,1}(x,t)|$ in Eq. (30).

plots (blue, red and yellow demonstrations) for $b_2 = 1, 2, 3$ values respectively. When b_2 is positive and increasing, soliton generally preserves the appearance of dark (left) kink soliton combination, that is, its physical feature. Depending on the increment of b_2, the soliton's position changes to the left and upwards. We can say that this change is directly proportional to the increase of b_2, but we cannot say that it is proportional to the change in amplitude and the change in the position of the signal. The graph Fig. 6(b) represents the graph in which the same examination was made against the negative values of b_2. This graph also shows similar characteristics with Fig. 6(a) in terms of general behavior. In other words, if b_2 takes negative and increasing values, the position of the soliton changes horizontally to the left and vertically upwards. However, the soliton wings in the previous part, which shows the kink soliton behavior gives an image parallel to each other without crossing each other. While b_2 is negative, there is an intersection. This is due to the non-proportional change in the amplitude of the signal.

In Fig. 7, we depicted some 2D plots of $|u_{2,1}(x,t)|$ in Eq. (30). The parameter a_2 represents the cubic nonlinearity (TOD) of the model in Eq. (2). Fig. 7(a) shows the different wave behavior due to the parameter a_2 change. Fig. 7(a) has three different styled plots (blue, red and yellow demonstrations) for $a_2 = 1, 2, 3$ values, respectively. The soliton behavior is generally similar when a_2 takes positive and negative values. In other words, when a_2 is positive and increasing, the soliton preserves its dark–kink combination, and depending on the increase in a_2, the soliton shifts horizontally to the left and vertically upwards. As can be clearly seen, the change in vertical also indicates an increase in distance. However, solitons do not exhibit any intersection. In the case where b_2 is negative and increasing, the same situation is repeated, but the distance between the solitons is closer. Therefore, a model that allows soliton transmission by ignoring the chromatic dispersion term for the CQ-FL equation and adding third and fourth-order dispersion terms instead, has been investigated via Figs. 6 and 7. It has been observed that within the scope of this model, soliton transmission is possible and it is possible to balance the interactions that occur in this transmission with such nonlinear terms. Within the framework of the study conducted for this purpose, the selected terms were also made on the basis of preserving the shape of the soliton obtained by taking into account the restrictions of the investigated models and the method.



(a) 2D plots of $|u_{2,1}(x,t)|$ (b) 2D plots of $|u_{2,1}(x,t)|$

Fig. 7. Various plots of $|u_{2,1}(x,t)|$ in Eq. (30).

5. Conclusion

In this study, we did an investigation on Schrödinger–Hirota equation having chromatic-dispersion and Cubic–Quartic Fokas–Lenells equation by discarding the chromatic dispersion. This study basically serves two main purposes. The first objective, the Schrödinger–Hirota equation with chromatic dispersion term was chosen and the chromatic dispersion effect in soliton transmission was successfully examined on this equation and also graphic presentations were made via 3D and 2D plots. In this sense, the effect of the chromatic dispersion term on soliton transmission is shown. The second aim is to examine the CQ-FL equation, which is one of the new models developed by adding the third and fourth-order nonlinear dispersion term by discarding the chromatic dispersion term, and similarly soliton transmission and its effects on it are examined. Detailed graphic presentations of the examination were made via 2D and 3D plots. Although these studies were conducted on dark, singular and mixed dark–kink soliton types within the scope of the study, it would be possible to expand similar studies for other soliton types. The study will also be beneficial for possible research since the study was investigated in this way, new experimental models, especially quartic soliton solutions, have started to be developed in recent years regarding the chromatic dispersion problem, and the field is still open for extensive research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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