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New soliton solutions of the fractional Regularized Long Wave Burger equation by means of conformable derivative

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ABSTRACT

In this paper, the practice of the extended direct algebraic method (EDAM) is used to solve fractional Regularized Long Wave Burgers (RLW-Burgers) equation by means of the conformable derivative. Firstly, this fractional equation is changed into the ordinary differential equation by using the traveling wave transformation. Then new soliton solutions are obtained by using EDAM. This presented form is important in physics and engineering. The created soliton solutions play a major task for scientists about an agreement the physical event of this equation. The graphics of some solutions are drawn at fitting values of parameters. The obtained outcomes appear clarity, accuracy, and potentiality of the presented scheme.

1. Introduction

Nonlinear differential equations have been briefly studied in the literature since they act as a bridge between mathematics and physics [1–12]. Recently, there has been considerable interests and significant theoretical developments in fractional calculus used in many fields and in fractional differential equations and its applications [13–36]. In [13]; M. Ekici et al. used the first integral method by using fractional derivative of conformable type for getting the soliton solutions, Yang et al. found the solutions of the sub-diffusion and wave equations via FVIM in [14], Yang et al. obtained the solutions for local fractional KdV equation in [15], in [16]; is found the solutions of two-dimensional fractional Burgers equations and Zhang et al. obtained the solutions of transport equations by using the series expansion method with local fractional derivative in [17] and in [18], H. Rezazadeh et al. obtained some new solutions of nonlinear time fractional Phi-four equation with conformable derivative. There are many more researches related to fractional derivatives.

In this work, we analyze the time fractional RLW-Burger equation by means of conformable derivative operator [19,20] to form solitons using the EDAM. When arguments affect this process are accepted to be specific values we can achieve the solitary wave solutions which are deduced from other methods such as, the (G'/G)–expansion method [21], auxiliary equation method [22], the direct algebraic method [23,24] and so on [25–30]. Difference between the direct algebraic method and extended direct algebraic method are special functions used in the solution. It is clear that this extended direct algebraic method, by using characteristic calculation, contributes a more influential mathematical tool for several other solving fractional differential equations.

The time fractional RLW-Burgers equation with conformable derivative is presented as follow [19,20]:

$$q_t^{(\eta)} + \delta q_x + \varepsilon q q_x + \lambda q_{xx} + \theta q_{xxt} = 0, \quad t > 0, \quad 0 < \eta \leqslant 1. \tag{1.1}$$

where $q_t^{(\eta)}$ is the conformable derivative operator $(q = q(x, t))$; δ, ε, $\lambda$and $\theta$are real valued constants. In [19] Korkmaz A. used the modified Kudryashov method obtain to construct the solution of this equation. Zhao and Xuan [20] analyse the existence and convergence cases of solutions for the RLW-Burgers equation. In [26]; scientists studied integer ordered type of this equation in 1981 to investigate surface water waves propagation in a channel. In [27], are analyzed oscillatory and monotone kink type waves for the presented equation. Some exact solutions as hyperbolic and trigonometric type are obtained via some expansion methods [28]. In [29], are found some complex solutions of this equation by the using direct algebraic method.

The conformable fractional derivative was proposed in [30], which can rectify the shortcomings of the previous definitions. This derivative is the simplest and most natural and efficient definition of the fractional derivative of order $\eta \in (0,1]$. We should remark that the definition can be generalized to include any η. However, the case $\eta \in (0,1]$ is the most

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important one.

The conformable derivative of order $\eta \in (0, 1)$ defined as the following expression [30]

$$ {}_tD^{\eta}f(t) = \lim_{\vartheta \to 0} \frac{f(t + \vartheta t^{1-\eta}) - f(t)}{\vartheta}, f{:}\left(0, \infty\right) \to \mathbb{R}. \tag{1.2}$$

The definition seems to be a natural extension of normal derivatives. But the form of the definition shows that it is the most natural definition, and the most fruitful one. The definition for $0 \leqslant \eta < 1$ coincides with the classical definitions on polynomials (up to a constant).

Some of the features of the conformable derivative as follows [30,31].

a) ${}_tD^{\eta}t^{\alpha} = \alpha t^{\alpha-\eta}, \forall \eta \in \mathbb{R}$,

b) ${}_tD^{\eta}(fg) = f_tD^{\eta}g + g_tD^{\eta}f$,

c) ${}_tD^{\eta}(fog) = t^{1-\eta}g'(t)f'(g(t))$,

d) ${}_tD^{\eta}\left(\frac{f}{g}\right) = \frac{g_tD^{\eta}f - f_tD^{\eta}g}{g^2}$.

It's very easier to work with this fractional derivative. Recently, several studies have been done related to the conformable type of fractional calculations [31–35].

2. Analysis of the extended direct algebraic method

Assume the general nonlinear partial differential equation,

$$A(q, q_t^{(\eta)}, q_x, q_{xx}, q_{tt}^{(2\eta)}, ...) = 0. \tag{2.1}$$

where q is an unknown function depending on x and t, A is a polynomial in $q = q(x, t)$ and the sub-indices represent the partial fractional derivatives.

- Suppose the traveling wave variable:

$$q\left(x, t\right) = u(\phi), \quad \phi = x - Q\frac{t^{\eta}}{\eta}, \tag{2.2}$$

Then, from Eq. (2.2), Eq. (2.1) is turn to an ordinary differential equation for $u(\phi)$:

$$B(u, u_{\phi}, u_{\phi\phi}, u_{\phi\phi\phi}, ...) = 0. \tag{2.3}$$

where the sub-indices represent the ordinary derivatives with respect to ϕ.

- Consider the solution of Eq. (2.3),

$$u(\phi) = \sum_{i=0}^{N} \alpha_i G^i(\phi), \tag{2.4}$$

where $a_n \neq 0$ and can be expressed as follows:

$$G'(\phi) = \ln(A)(fG^2(\phi) + gG(\phi) + h), \quad A \neq 0, 1, \tag{2.5}$$

where h, g, f are arbitrary constants.

- N is found by balancing between the nonlinear terms and the highest order derivatives in Eq. (2.3).
- Replacing Eq. (2.4) together with Eq. (2.5) into the Eq. (2.3), then equating each coefficient of the polynomials to zero, give a set of algebraic equations for $\alpha_i(i = 1, 2, ..., N)$, f, g, h and Q.
- Solving the obtained system, we obtain values for $\alpha_i(i = 1, 2, ..., N)$ and Q. Then, solutions of Eq. (2.3) are obtained.

Where some special solutions of Eq. (2.3) as follows;

1)When $g^2 - 4hf < 0$ and $f \neq 0$,

$$G_1(\phi) = -\frac{g}{2f} + \frac{\sqrt{-(g^2 - 4hf)}}{2f}\tan_A\left(\frac{\sqrt{-(g^2 - 4hf)}}{2}\phi\right),$$

$$G_2(\phi) = -\frac{g}{2f} + \frac{\sqrt{-(g^2 - 4hf)}}{2f}\left(-\cot_A\left(\sqrt{-(g^2 - 4hf)}\,\phi\right) \pm \sqrt{\Delta\Omega}\csc_A\left(\sqrt{-(g^2 - 4hf)}\,\phi\right)\right),$$

$$G_3(\phi) = -\frac{g}{2f} + \frac{\sqrt{-(g^2 - 4hf)}}{2f}\left(\tan_A\left(\sqrt{\frac{-(g^2 - 4hf)}{4}}\phi\right) - \cot_A\left(\sqrt{\frac{-(g^2 - 4hf)}{4}}\phi\right)\right).$$

2)When $g^2 - 4hf > 0$ and $f \neq 0$,

$$G_4(\phi) = -\frac{g}{2f} - \frac{\sqrt{g^2 - 4hf}}{2f}\tanh_A\left(\frac{\sqrt{g^2 - 4hf}}{2}\phi\right),$$

$$G_5(\phi) = -\frac{g}{2f} + \frac{\sqrt{g^2 - 4hf}}{2f}(-\tanh_A(\sqrt{g^2 - 4hf}\phi) \pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A(\sqrt{g^2 - 4hf}\phi)),$$

$$G_6(\phi) = -\frac{g}{2f} + \frac{\sqrt{g^2 - 4hf}}{4f}\left(\tanh_A\left(\frac{\sqrt{g^2 - 4hf}}{4}\phi\right) + \coth_A\left(\frac{\sqrt{g^2 - 4hf}}{4}\phi\right)\right).$$

3)When $hf > 0$ and $g = 0$,

$$G_7(\phi) = -\sqrt{\frac{h}{f}}\cot_A\left(\sqrt{hf}\phi\right),$$

$$G_8(\phi) = \sqrt{\frac{h}{f}}\left(\tan_A\left(2\sqrt{hf}\phi\right) \pm \sqrt{\Delta\Omega}\sec_A\left(2\sqrt{hf}\phi\right)\right),$$

$$G_9(\phi) = \frac{1}{2}\sqrt{\frac{h}{f}}\left(\tan_A\left(\frac{\sqrt{hf}}{2}\phi\right) - \cot_A\left(\frac{\sqrt{hf}}{2}\phi\right)\right).$$

4)When $hf < 0$ and $g = 0$,

$$G_{10}(\phi) = -\sqrt{-\frac{h}{f}}\coth_A\left(\sqrt{-hf}\phi\right),$$

$$G_{11}(\phi) = \sqrt{-\frac{h}{f}}(-\tanh_A(2\sqrt{-hf}\phi) \pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A(2\sqrt{-hf}\phi)),$$

$$G_{12}(\phi) = -\frac{1}{2}\sqrt{-\frac{h}{f}}\left(\tanh_A\left(\frac{\sqrt{-hf}}{2}\phi\right) + \coth_A\left(\frac{\sqrt{-hf}}{2}\phi\right)\right).$$

5)When $h = f$ and $g = 0$,

$$G_{13}(\phi) = \tan_A(h\phi),$$

$$G_{14}(\phi) = -\cot_A(2h\phi) \pm \sqrt{\Delta\Omega}\csc_A(2h\phi)),$$

$$G_{15}(\phi) = \frac{1}{2}\left(\tan_A\left(\frac{h}{2}\phi\right) - \cot_A\left(\frac{h}{2}\phi\right)\right).$$

6)When $h = -f$ and $g = 0$,

$$G_{16}(\phi) = -\coth_A(h\phi),$$

$$G_{17}(\phi) = -\tanh_A(2h\phi) \pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A(2h\phi),$$

$$G_{18}(\phi) = -\frac{1}{2}\left(\tanh_A\left(\frac{h}{2}\phi\right) + \coth_A\left(\frac{h}{2}\phi\right)\right).$$

7)When $g^2 = 4hf$,

$$G_{19}(\phi) = -2h\frac{g\phi\ln(A) + 2}{g^2\phi\ln(A)}.$$

8)When $g = k$, $h = mk(m \neq 0)$ and $f = 0$,

$$G_{20}(\phi) = A^{k\phi} - m.$$

9)When $g = f = 0$,

$$G_{21}(\phi) = h\phi\ln(A).$$

10)When $g = h = 0$,

$$G_{22}(\phi) = -\frac{1}{f\phi\ln(A)}.$$

11)When $h = 0$ and $g \neq 0$,

$$G_{23}(\phi) = -\frac{\Delta g}{f(\cosh_A(g\phi) - \sinh_A(g\phi) + \Delta)},$$

$$G_{24}(\phi) = -\frac{g(\sinh_A(g\phi) + \cosh_A(g\phi))}{f(\sinh_A(g\phi) + \cosh_A(g\phi) + \Omega)}.$$

12)When $g = k$, $h = 0$ and $f = mk (m \neq 0)$,

$$G_{25}(\phi) = \frac{\Delta A^{k\phi}}{\Omega - m\Delta A^{k\phi}}.$$

Remark 1. The generalized triangular and hyperbolic functions are defined as [36];

$$\sin_A(\phi) = \frac{\Delta A^{i\phi} - \Omega A^{-i\phi}}{2i}, \quad \cos_A(\phi) = \frac{\Delta A^{i\phi} + \Omega A^{-i\phi}}{2},$$
$$\tan_A(\phi) = -i\frac{\Delta A^{i\phi} - \Omega A^{-i\phi}}{\Delta A^{i\phi} + \Omega A^{-i\phi}}, \quad \cot_A(\phi) = i\frac{\Delta A^{i\phi} + \Omega A^{-i\phi}}{\Delta A^{i\phi} - \Omega A^{-i\phi}},$$
$$\sec_A(\phi) = \frac{2}{\Delta A^{i\phi} + \Omega A^{-i\phi}}, \quad \csc_A(\phi) = \frac{2i}{\Delta A^{i\phi} - \Omega A^{-i\phi}}$$
$$\sinh_A(\phi) = \frac{\Delta A^{\phi} - \Omega A^{-\phi}}{2}, \quad \cosh_A(\phi) = \frac{\Delta A^{\phi} + \Omega A^{-\phi}}{2},$$
$$\tanh_A(\phi) = \frac{\Delta A^{\phi} - \Omega A^{-\phi}}{\Delta A^{\phi} + \Omega A^{-\phi}}, \quad \coth_A(\phi) = \frac{\Delta A^{\phi} + \Omega A^{-\phi}}{\Delta A^{\phi} - \Omega A^{-\phi}},$$
$$\mathrm{sech}_A(\phi) = \frac{2}{\Delta A^{\phi} + \Omega A^{-\phi}}, \quad \mathrm{csch}_A(\phi) = \frac{2}{\Delta A^{\phi} - \Omega A^{-\phi}}.$$

where $\Delta > 0$ and $\Omega > 0$ are deformation parameters and $\phi$is an independent variable.

3. The fractional RLW-Burgers equation

By placing Eq. (2.2) into Eq. (1.1), is obtained nonlinear equation as follows,

$$(-Q\theta)u'''(\phi) + \lambda u''(\phi) + \varepsilon u(\phi)u'(\phi) + (\delta - Q)u'(\phi) = 0, \tag{3.1}$$

By integrating once according to ϕEq. (3.1), is obtained nonlinear equation as follows,

$$\left(-Q\theta\right)u''(\phi) + \lambda u'(\phi) + \frac{\varepsilon}{2}u(\phi)^2 + \left(\delta - Q\right)u(\phi) + K = 0, \tag{3.2}$$

where K integration constant.

Assumed the solution of Eq. (3.2) is demonstrable as a finite series as follows:

$$u(\phi) = \sum_{j=0}^{N} \alpha_j G^j(\phi) \tag{3.3}$$

where satisfies Eq. (2.5), $\phi = x - Q\frac{t^\eta}{\eta}$ and α_j for $j = \overline{1, N}$ are values to be definited.

By balancing u'' with u^2 in Eq. (3.2), is obtained $N = 2$.

We can select the solution of Eq. (3.2) as following shape:

$$u(\phi) = \alpha_0 + \alpha_1 G(\phi) + \alpha_2 G(\phi)^2, \tag{3.4}$$

where satisfied Eq. (2.5).

Substituting (3.2) and (2.5) into (3.2), collecting the coefficients of , and solving the obtaining system, the following groups of some solutions are found:

One of the five groups of values as follows

$$\alpha_0 = -\frac{5\delta + 6\lambda g\ln(A) + \frac{\lambda + 12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}, \quad \alpha_1 = \frac{12\lambda f\ln(A)\left(-1 - \frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon},$$
$$\alpha_2 = -\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}, \quad Q = -\frac{\lambda}{5\Psi}, \tag{3.5}$$
$$K = \frac{\lambda^2 - \frac{36\lambda^2\Psi^4}{\theta^2} + 5\delta\Psi\left(2\lambda + 5\delta\Psi\right)}{50\varepsilon\Psi^2},$$

where $\Psi = \sqrt{\theta^2(g^2 - 4fh)\ln(A)^2}$.

The solutions of Eq. (1.1) are obtained as follows;

1)When $g^2 - 4hf < 0$ and $f \neq 0$, the singular periodic solutions are as below

$$q_1(x,t) = -\frac{5\delta + 6\lambda g\ln(A) + \frac{\lambda + 12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon} + \frac{12\lambda f\ln(A)\left(-1 - \frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{g}{2f} + \frac{\sqrt{-(g^2-4hf)}}{2f}\tan_A\left(\frac{\sqrt{-(g^2-4hf)}}{2}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right) - \frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{g}{2f} + \frac{\sqrt{-(g^2-4hf)}}{2f}\tan_A\left(\frac{\sqrt{-(g^2-4hf)}}{2}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)^2$$

,

$$q_2(x,t) = -\frac{5\delta + 6\lambda g\ln(A) + \frac{\lambda + 12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon} + \frac{12\lambda f\ln(A)\left(-1 - \frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{g}{2f} + \frac{\sqrt{-(g^2-4hf)}}{2f}\left(-\cot_A\left(\sqrt{-(g^2-4hf)}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right) \pm \sqrt{\Delta\Omega}\csc_A\left(\sqrt{-(g^2-4hf)}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right) - \frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{g}{2f} + \frac{\sqrt{-(g^2-4hf)}}{2f}\left(-\cot_A\left(\sqrt{-(g^2-4hf)}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right) \pm \sqrt{\Delta\Omega}\csc_A\left(\sqrt{-(g^2-4hf)}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

$$q_3(x,t) = -\frac{5\delta + 6\lambda g\ln(A) + \frac{\lambda + 12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon} + \frac{12\lambda f\ln(A)\left(-1 - \frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{g}{2f} + \frac{\sqrt{-(g^2-4hf)}}{2f}\left(\tan_A\left(\sqrt{\frac{-(g^2-4hf)}{4}}\phi\right) - \cot_A\left(\sqrt{\frac{-(g^2-4hf)}{4}}\phi\right)\right)\right) - \frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{g}{2f} + \frac{\sqrt{-(g^2-4hf)}}{2f}\left(\tan_A\left(\sqrt{\frac{-(g^2-4hf)}{4}}\phi\right) - \cot_A\left(\sqrt{\frac{-(g^2-4hf)}{4}}\phi\right)\right)\right)^2.$$

2)When $g^2 - 4hf > 0$ and $f \neq 0$, thus the dark and the singular soliton solutions are as below

$$q_4(x,t) = -\frac{5\delta + 6\lambda g\ln(A) + \frac{\lambda + 12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon} + \frac{12\lambda f\ln(A)\left(-1 - \frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{g}{2f} - \frac{\sqrt{g^2-4hf}}{2f}\tanh_A\left(\frac{\sqrt{g^2-4hf}}{2}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right) - \frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{g}{2f} - \frac{\sqrt{g^2-4hf}}{2f}\tanh_A\left(\frac{\sqrt{g^2-4hf}}{2}\left(x + \frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)^2,$$

$$q_5(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{g}{2f}+\frac{\sqrt{g^2-4hf}}{2f}\left(-\tanh_A\left(\sqrt{g^2-4hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(\sqrt{g^2-4hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{g}{2f}+\frac{\sqrt{g^2-4hf}}{2f}\left(-\tanh_A\left(\sqrt{g^2-4hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm i\sqrt{\delta\Omega}\,\mathrm{sech}_A\left(\sqrt{g^2-4hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

$$q_6(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{g}{2f}+\frac{\sqrt{g^2-4hf}}{4f}\left(\tanh_A\left(\frac{\sqrt{g^2-4hf}}{4}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\coth_A\left(\frac{\sqrt{g^2-4hf}}{4}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{g}{2f}+\frac{\sqrt{g^2-4hf}}{4f}\left(\tanh_A\left(\frac{\sqrt{g^2-4hf}}{4}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\coth_A\left(\frac{\sqrt{g^2-4hf}}{4}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

3)When $hf>0$ and $g=0$, thus the singular periodic solutions are as below

$$q_7(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\sqrt{\frac{h}{f}}\cot_A\left(\sqrt{hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\sqrt{\frac{h}{f}}\cot_A\left(\sqrt{hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)^2$$

$$q_8(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\sqrt{\frac{h}{f}}\left(\tan_A\left(2\sqrt{hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm\sqrt{\Delta\Omega}\sec_A\left(2\sqrt{hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(\sqrt{\frac{h}{f}}\left(\tan_A\left(2\sqrt{hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm\sqrt{\Delta\Omega}\sec_A\left(2\sqrt{hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

$$q_9(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\frac{1}{2}\sqrt{\frac{h}{f}}\left(\tan_A\left(\frac{\sqrt{hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)-\cot_A\left(\frac{\sqrt{hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(\frac{1}{2}\sqrt{\frac{h}{f}}\left(\tan_A\left(\frac{\sqrt{hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)-\cot_A\left(\frac{\sqrt{hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

4)When $hf<0$ and $g=0$, thus the dark, bright and singular soliton solutions are as below

$$q_{10}(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\sqrt{-\frac{h}{f}}\coth_A\left(\sqrt{-hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\sqrt{-\frac{h}{f}}\coth_A\left(\sqrt{-hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)^2,$$

$$q_{11}(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\sqrt{-\frac{h}{f}}\left(-\tanh_A\left(2\sqrt{-hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(2\sqrt{-hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(\sqrt{-\frac{h}{f}}\left(-\tanh_A\left(2\sqrt{-hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(2\sqrt{-hf}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

$$q_{12}(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{1}{2}\sqrt{-\frac{h}{f}}\left(\tanh_A\left(\frac{\sqrt{-hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\coth_A\left(\frac{\sqrt{-hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{1}{2}\sqrt{-\frac{h}{f}}\left(\tanh_A\left(\frac{\sqrt{-hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\coth_A\left(\frac{\sqrt{-hf}}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

5)When $h=f$ and $g=0$, thus the singular periodic solutions are as below

$$q_{13}(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\tan_A\left(h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\tan_A\left(h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)^2,$$

$$q_{14}(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\cot_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm\sqrt{\Delta\Omega}\csc_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right),-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\cot_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm\sqrt{\Delta\Omega}\csc_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)^2$$

$$q_{15}(x,\,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\frac{1}{2}\left(\tan_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)-\cot_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(\frac{1}{2}\left(\tan_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)-\cot_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

6)When $h=-f$ and $g=0$, thus the dark, bright and singular soliton solutions are as below

$$q_{16}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\coth_A\left(h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\coth_A\left(h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)^2,$$

$$q_{17}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\tanh_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\tanh_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(2h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)^2,$$

$$q_{18}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{1}{2}\left(\tanh_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\coth_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{1}{2}\left(\tanh_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\coth_A\left(\frac{h}{2}\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)\right)^2,$$

7)When $g^2=4hf$, thus the rational solution is as below

$$q_{19}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-2h\frac{g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)+2}{g^2\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)}\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-2h\frac{g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)+2}{g^2\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)}\right)^2,$$

8)When $g=k$, $h=mk(m\neq 0)$ and $f=0$, thus the rational solution is as below

$$q_{20}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(A^{k\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)}-m\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(A^{k\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)}-m\right)^2,$$

9)When $g=f=0$, thus the rational solution is as below

$$q_{21}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(h\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)\right)^2,$$

10)When $g=h=0$, thus the rational solution is as below

$$q_{22}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{1}{f\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)}\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{1}{f\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\ln(A)}\right)^2,$$

11)When $h=0$ and $g\neq 0$, thus the dark-like and bright solitons are as below

$$q_{23}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{\Delta g}{f\left(\cosh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)-\sinh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\Delta\right)}\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{\Delta g}{f\left(\cosh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)-\sinh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\Delta\right)}\right)^2,$$

$$q_{24}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(-\frac{g\left(\sinh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\cosh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)}{f\left(\sinh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\cosh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\Omega\right)}\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(-\frac{g\left(\sinh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\cosh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)\right)}{f\left(\sinh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\cosh_A\left(g\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)\right)+\Omega\right)}\right)^2,$$

12)When $g=k$, $h=0$ and $f=mk(m\neq 0)$, thus the rational solution is as below

$$q_{25}(x,t)=-\frac{5\delta+6\lambda g\ln(A)+\frac{\lambda+12\lambda fh\theta\ln(A)^2}{\Psi}}{5\varepsilon}+\frac{12\lambda f\ln(A)\left(-1-\frac{g\theta\ln(A)}{\Psi}\right)}{5\varepsilon}\left(\frac{\Delta A^{k\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)}}{\Omega-m\Delta A^{k\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)}}\right)-\frac{12\lambda f^2\theta\ln(A)^2}{5\varepsilon\Psi}\left(\frac{\Delta A^{k\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)}}{\Omega-m\Delta A^{k\left(x+\frac{\lambda}{5\Psi}\frac{t^\eta}{\eta}\right)}}\right)^2.$$

4. Graphical representation of the solutions

The surface graphics of the obtained solutions are showed below in the figures by using Mathematica. In Figs. 1–3, we drawn some mathematical simulations for $q_1(x,t)$, $q_5(x,t)$, $q_{10}(x,t)$, $q_{15}(x,t)$, $q_{18}(x,t)$ and $q_{23}(x,t)$ in 3D plots when $-5\leqslant x\leqslant 5$ and $-5\leqslant t\leqslant 5$.

We wrote the some of solutions found for the presented fractional RLW-Burgers equation via conformable derivative operator. Besides we showed 3D and 2D graphics for some of solutions in Figs. 1–3. The graphics above were drawn for $A=2.7$, $\delta=1.2$, $\lambda=0.5$, $\theta=1.5$, $\epsilon=0.4$, $\alpha=0.9$, $\Delta=\Omega=1$ and $x=0.5$ (for 2D graphics).

5. Conclusion

In this paper, the extended direct algebraic method is used to find new soliton solutions of the fractional RLW-Burgers equation. These solutions consist of twelve different cases. The existences of solutions derived from these functions are all guaranteed through constraint conditions that are also listed beside the solutions. The obtained soliton solutions are important for scientists about the agreement the physical event of this equation. By selecting appropriate values of parameters,

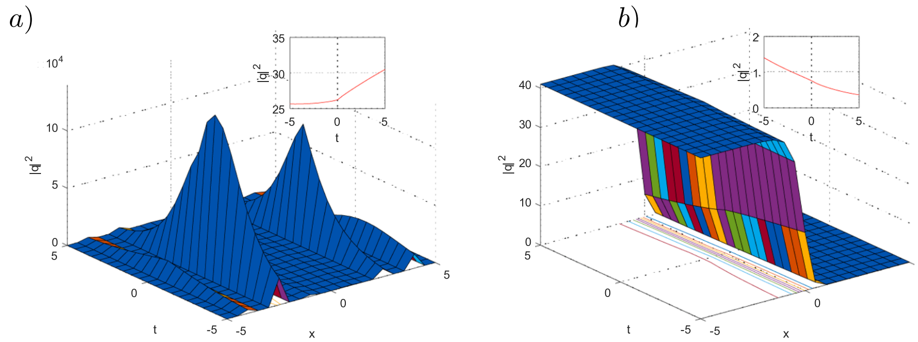

Fig. 1. The surface graphics for the $|q(x, t)|^2$ analytical solution of the fractional RLW-Burgers equation a) $q_1(x, t)(h = f = 2, g = 1)$, b) $q_5(x, t)(h = f = 1, g = 3)$.

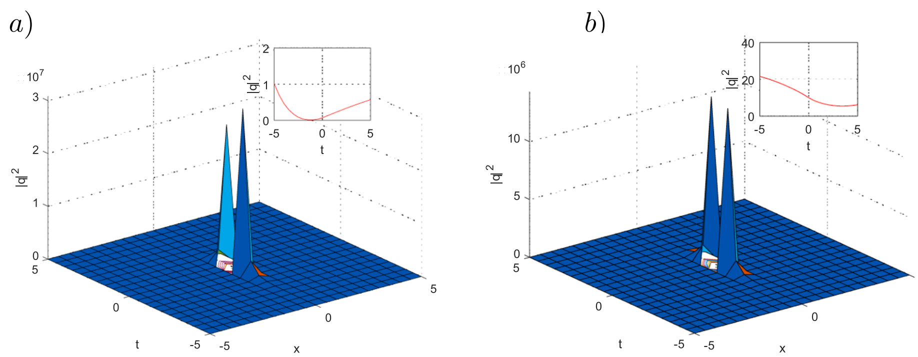

Fig. 2. The surface graphics for the $|q(x, t)|^2$ analytical solution of the fractional RLW-Burgers equation a) $q_{10}(x, t)(h = -2, f = 1, g = 0)$, b) $q_{15}(x, t)(h = f = 1, g = 0)$.

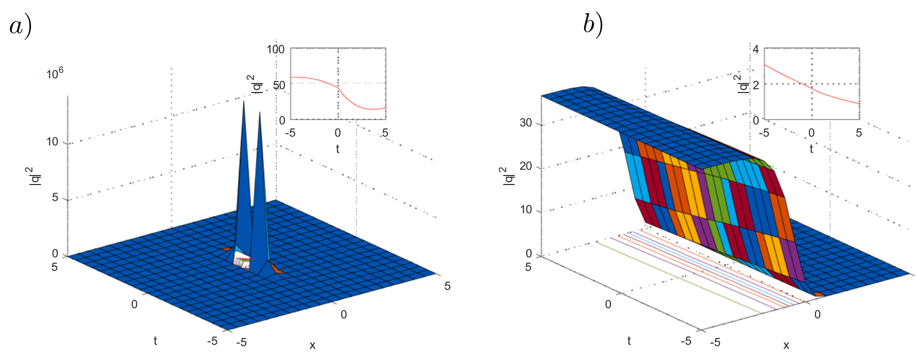

Fig. 3. The surface graphics for the $|q(x, t)|^2$ analytical solution of the fractional RLW-Burgers equation a) $q_{18}(x, t)(h = 1, f = -1, g = 0)$, b) $q_{23}(x, t)$ $(h = 0, f = 1, g = 2)$.

the behaviors of some solutions have been viewed with the help 3D and 2D graphics. We say that the presented method is suitable to examine the many problems located in science and engineering.

Declaration of Competing Interest

There is no conflict of interest in this paper.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, athttps://doi.org/10.1016/j.rinp.2019.102395.

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