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New solutions of the fractional Boussinesq-like equations by means of conformable derivatives

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ABSTRACT

In this paper, the process of the extended direct algebraic method (EDAM) is used to solve two fractional Boussinesq-like equations by means of conformable derivatives. Firstly, these fractional equations are changed into the ordinary differential equations by using the traveling wave transformation. Then new solutions are obtained by using EDAM. This dynamical model plays a key role in engineering and physics. The constructed solitons solution help researchers in understanding the physical phenomenon of this equation. The standard linear stability analysis is utilized and the stability of the model is investigated which substantiate that all results are stable and exact. Graphically, the movements of some solutions are depicted at appropriate values of parameters. The achieved results show simplicity, reliability, and power of the current schemes.

Introduction

There has been considerable interests and significant theoretical developments in fractional calculus used in many fields and in fractional differential equations and its applications [1–8]. There have been much research for nonlinear fractional partial differential equations (FPDEs) which are a specific form of NPDEs. Because FPDEs are important for various analysis due to their recurrent appearing, versatile and potentiality put into operations in nonlinear optics, water wave hypothesis, plasma physics, fluid dynamics, optical fiber, signal processing, quantum mechanics and so on. Numerous authors have efforted to obtain the wave solutions of NPDEs by using a lot of mathematical processes. Ekici et al. [9] used the first integral method by using the fractional derivative of conformable type for getting the soliton solutions, Bhrawy et al. [10] analyzed the Jacobi spectral collocation approximate solution for various Schrödinger equations, Yang et al. [11] found the solutions of the sub-diffusion and wave equations via FVIM, Gao and Yang [12] used the fractional Euler's method with local case to investigate approximate solution of the fractional heat-relaxation equation with local case, Yang et al. [13] obtained the solutions for local fractional KdV equation, Yang et al. [14] found the solutions of two-dimensional fractional Burgers equations and Zhang et al. [15] obtained the solutions of transport equations by using the series expansion method with local fractional derivative, Rezazadeh et al. [16] obtained new exact solutions of nonlinear time-fractional Phi-4 equation with conformable derivative. There are many more researches related to fractional derivatives. Besides, it can be said very easier to work with conformable fractional derivatives [17–20]. Using these methods, the researchers investigate different kinds of traveling wave solutions of different NPDEs and fractional NPDEs, e.g. see [21–27]. Moreover, various recent studies show the richness of another kind of exact solutions called lump solutions and their interactions for PDEs [28–33].

In this work, we analyze two different types of fractional Boussinesq-like equations by means of conformable derivative operators [34–36]. Four different types of the fractional Boussinesq-like equations by means of conformable derivative operators given in the following forms,

$$q_{tt}^{(2\eta)} - q_{xx} - (6q^2q_x + q_{xxx})_x = 0, \tag{1.1}$$

$$q_{tt}^{(2\eta)} - q_{xx} - (6q^2q_x + q_{xtt}^{(2\eta)})_x = 0, \tag{1.2}$$

$$q_{tt}^{(2\eta)} - q_{xt}^{(\eta)} - (6q^2q_x + q_{xxt}^{(\eta)})_x = 0, \tag{1.3}$$

$$q_{tt}^{(2\eta)} - (6q^2q_x + q_{xxx})_x = 0,. \tag{1.4}$$

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where $t>0,\ 0<\eta\leqslant 1,\ q_t^{(\eta)}$ is the conformable derivative operator ($q=q(x,t)$). Eqs. (1.1),(1.2),(1.3),(1.4) are non-integrable equations that are used as model in ocean and coastal sciences when $\eta=1$. Some applications of these equations when they are represented by tsunami wave modeling and mathematical modeling of tidal oscillations. Furthermore, they can be used in studying the dynamics of the thin in viscid layers with free surface, the wave propagation in elastic rods, and in the continuum limit of lattice dynamics or some particular forms of electrical circuits.

Eslami and Mirzazadeh applied the first integral method to look for exact solutions of these equations in [34]. Darvishi et al. investigated soliton solutions for these equations with spatio-temporal dispersion in [35].

The conformable derivative of order $\eta\in(0,1)$ defined as the following expression

$$_tD^{\eta}f(t)=\lim_{\vartheta\to 0}\frac{f(t+\vartheta t^{1-\eta})-f(t)}{\vartheta},\quad f:\left(0,\infty\right)\to\mathbb{R}. \tag{1.5}$$

Some of the features of conformable derivative as follows:

(a) $_tD^{\eta}t^{\alpha}=\alpha t^{\alpha-\eta},\ \forall\,\eta\in\mathbb{R}$,
(b) $_tD^{\eta}(fg)=f_tD^{\eta}g+g_tD^{\eta}f$,
(c) $_tD^{\eta}(fog)=t^{1-\eta}g'(t)f'(g(t))$,
(d) $_tD^{\eta}\left(\frac{f}{g}\right)=\frac{g_tD^{\eta}f-f_tD^{\eta}g}{g^2}$.

In this work, we analyze the first and second fractional Boussinesq-like equations by means of the conformable derivative to obtain solutions using the extended direct algebraic method (EDAM) [16,36]. Rezazadeh studied new solitons solutions of the complex Ginzburg-Landau equation with Kerr law nonlinearity by using EDAM in [36].

This study is organized as follows: Firstly, we give the process of EDAM for Boussinesq-like equations with conformable derivatives. Also, these fractional equations are changed into the ordinary differential equations by using the traveling wave transformation. Finally, we obtain new soliton solutions for the model problem by using EDAM.

Analysis of the extended direct algebraic method

Assume the general nonlinear partial differential equation,

$$A(q,q_t^{(\eta)},q_x,q_{xx},q_{tt}^{(2\eta)},\ldots)=0. \tag{2.1}$$

where q is an unknown function depending on x and t, A is a polynomial in $q=q(x,t)$ and the sub-indices represent the partial fractional derivatives.

- Suppose the traveling wave variable:

$$q\left(x,t\right)=u(\phi),\quad \phi=x-Q\frac{t^{\eta}}{\eta}, \tag{2.2}$$

Then, from Eq. (2.2), Eq. (2.1) is turn to an ordinary differential equation for $u(\phi)$:

$$B(u,u_{\phi},u_{\phi\phi},u_{\phi\phi\phi},\ldots)=0. \tag{2.3}$$

where the sub-indices represent the ordinary derivatives with respect to ϕ.

- Consider the solution of Eq. (2.3),

$$u(\phi)=\sum_{i=0}^{N}\alpha_iG^i(\phi), \tag{2.4}$$

where $a_n\neq 0$ and $G(\phi)$ can be expressed as follows:

$$G'(\phi)=\ln(A)(fG^2(\phi)+gG(\phi)+h),\ A\neq 0,1, \tag{2.5}$$

where h,g,f are arbitrary constants. The general solution to the basis Eq. (2.5) is given by the formulas (40)–(42) in [37].

- N is found by balancing between the nonlinear terms and the highest order derivatives in Eq. (2.3).
- Replacing Eq. (2.4) together with Eq. (2.5) into the Eq. (2.3), then equating each coefficient of the polynomials to zero, give a set of algebraic equations for $\alpha_i(i=1,2,\ldots,N)$, f, g, h and Q.
- Solving the obtained system, we obtain values for $\alpha_i(i=1,2,\ldots,N)$ and Q. Then, solutions of Eq. (2.3) are obtained.

Where some special solutions of Eq. (2.3) as follows;

(1) When $\Psi=g^2-4hf<0$ and $f\neq 0$,

$$G_1(\phi)=-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\tan_A\left(\frac{\sqrt{-\Psi}}{2}\phi\right),$$
$$G_2(\phi)=-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\cot_A\left(\frac{\sqrt{-\Psi}}{2}\phi\right),$$
$$G_3(\phi)=-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}(\tan_A(\sqrt{-\Psi}\phi)\pm\sqrt{\Delta\Omega}\sec_A(\sqrt{-\Psi}\phi)),$$
$$G_4(\phi)=-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}(-\cot_A(\sqrt{-\Psi}\phi)\pm\sqrt{\Delta\Omega}\csc_A(\sqrt{-\Psi}\phi)),$$
$$G_5(\phi)=-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\left(\tan_A\left(\sqrt{\frac{-\Psi}{4}}\phi\right)-\cot_A\left(\sqrt{\frac{-\Psi}{4}}\phi\right)\right).$$

(2) When $\Psi=g^2-4hf>0$ and $f\neq 0$,

$$G_6(\phi)=-\frac{g}{2f}-\frac{\sqrt{\Psi}}{2f}\tanh_A\left(\frac{\sqrt{\Psi}}{2}\phi\right),$$
$$G_7(\phi)=-\frac{g}{2f}-\frac{\sqrt{\Psi}}{2f}\coth_A\left(\frac{\sqrt{\Psi}}{2}\phi\right),$$
$$G_8(\phi)=-\frac{g}{2f}+\frac{\sqrt{\Psi}}{2f}\left(-\tanh_A\left(\sqrt{\Psi}\phi\right)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(\sqrt{\Psi}\phi\right)\right),$$
$$G_9(\phi)=-\frac{g}{2f}+\frac{\sqrt{\Psi}}{2f}(-\coth_A(\sqrt{\Psi}\phi)\pm\sqrt{\Delta\Omega}\,\mathrm{csch}_A(\sqrt{\Psi}\phi)),$$
$$G_{10}(\phi)=-\frac{g}{2f}+\frac{\sqrt{\Psi}}{4f}\left(\tanh_A\left(\frac{\sqrt{\Psi}}{4}\phi\right)+\coth_A\left(\frac{\sqrt{\Psi}}{4}\phi\right)\right).$$

(3) When $hf>0$ and $g=0$,

$$G_{11}(\phi)=\sqrt{\frac{h}{f}}\tan_A\left(\sqrt{hf}\phi\right),$$
$$G_{12}(\phi)=-\sqrt{\frac{h}{f}}\cot_A\left(\sqrt{hf}\phi\right),$$
$$G_{13}(\phi)=\sqrt{\frac{h}{f}}(\tan_A(2\sqrt{hf}\phi)\pm\sqrt{\Delta\Omega}\sec_A(2\sqrt{hf}\phi)),$$
$$G_{14}(\phi)=\sqrt{\frac{h}{f}}(-\cot_A(2\sqrt{hf}\phi)\pm\sqrt{\Delta\Omega}\csc_A(2\sqrt{hf}\phi)),$$
$$G_{15}(\phi)=\frac{1}{2}\sqrt{\frac{h}{f}}\left(\tan_A\left(\frac{\sqrt{hf}}{2}\phi\right)-\cot_A\left(\frac{\sqrt{hf}}{2}\phi\right)\right).$$

(4) When $hf<0$ and $g=0$,

$$G_{16}(\phi)=-\sqrt{-\frac{h}{f}}\tanh_A\left(\sqrt{-hf}\phi\right),$$
$$G_{17}(\phi)=-\sqrt{-\frac{h}{f}}\coth_A\left(\sqrt{-hf}\phi\right),$$
$$G_{18}(\phi)=\sqrt{-\frac{h}{f}}\left(-\tanh_A\left(2\sqrt{-hf}\phi\right)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(2\sqrt{-hf}\phi\right)\right),$$
$$G_{19}(\phi)=\sqrt{-\frac{h}{f}}(-\coth_A(2\sqrt{-hf}\phi)\pm\sqrt{\Delta\Omega}\,\mathrm{csch}_A(2\sqrt{-hf}\phi)),$$
$$G_{20}(\phi)=-\frac{1}{2}\sqrt{-\frac{h}{f}}\left(\tanh_A\left(\frac{\sqrt{-hf}}{2}\phi\right)+\coth_A\left(\frac{\sqrt{-hf}}{2}\phi\right)\right).$$

(5) When $h=f$ and $g=0$,

$G_{21}(\phi)=\tan_A(h\phi)$,
$G_{22}(\phi)=-\cot_A(h\phi)$,
$G_{23}(\phi)=\tan_A(2h\phi)\pm\sqrt{\Delta\Omega}\sec_A(2h\phi)$,
$G_{24}(\phi)=-\cot_A(2h\phi)\pm\sqrt{\Delta\Omega}\csc_A(2h\phi)$,
$G_{25}(\phi)=\frac{1}{2}\left(\tan_A\left(\frac{h}{2}\phi\right)-\cot_A\left(\frac{h}{2}\phi\right)\right)$.

(6) When $h=-f$ and $g=0$,

$G_{26}(\phi)=-\tanh_A(h\phi)$,
$G_{27}(\phi)=-\coth_A(h\phi)$,
$G_{28}(\phi)=-\tanh_A(2h\phi)\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A(2h\phi)$,
$G_{29}(\phi)=-\coth_A(2h\phi)\pm\sqrt{\Delta\Omega}\,\mathrm{csch}_A(2h\phi)$,
$G_{30}(\phi)=-\frac{1}{2}\left(\tanh_A\left(\frac{h}{2}\phi\right)+\coth_A\left(\frac{h}{2}\phi\right)\right)$.

(7) When $g^2=4hf$,

$$G_{31}(\phi)=-2h\frac{g\phi\ln(A)+2}{g^2\phi\ln(A)}.$$

(8) When $g=k$, $h=mk(m\neq 0)$ and $f=0$,

$G_{32}(\phi)=A^{k\phi}-m$.

(9) When $g=f=0$,

$G_{33}(\phi)=h\phi\ln(A)$.

(10) When $g=h=0$,

$$G_{34}(\phi)=-\frac{1}{f\phi\ln(A)}.$$

(11) When $h=0$ and $g\neq 0$,

$G_{35}(\phi)=-\frac{\Delta g}{f(\cosh_A(g\phi)-\sinh_A(g\phi)+\Delta)}$,
$G_{36}(\phi)=-\frac{g(\sinh_A(g\phi)+\cosh_A(g\phi))}{f(\sinh_A(g\phi)+\cosh_A(g\phi)+\Omega)}$.

(12) When $g=k$, $h=0$ and $f=mk(m\neq 0)$,

$$G_{37}(\phi)=\frac{\Delta A^{k\phi}}{\Omega-m\Delta A^{k\phi}}.$$

Remark. The generalized triangular and hyperbolic functions are defined as [36];

$\sin_A(\phi)=\frac{\Delta A^{i\phi}-\Omega A^{-i\phi}}{2i}$, $\cos_A(\phi)=\frac{\Delta A^{i\phi}+\Omega A^{-i\phi}}{2}$,
$\tan_A(\phi)=-i\frac{\Delta A^{i\phi}-\Omega A^{-i\phi}}{\Delta A^{i\phi}+\Omega A^{-i\phi}}$, $\cot_A(\phi)=i\frac{\Delta A^{i\phi}+\Omega A^{-i\phi}}{\Delta A^{i\phi}-\Omega A^{-i\phi}}$,
$\sec_A(\phi)=\frac{2}{\Delta A^{i\phi}+\Omega A^{-i\phi}}$, $\csc_A(\phi)=\frac{2i}{\Delta A^{i\phi}-\Omega A^{-i\phi}}$
$\sinh_A(\phi)=\frac{\Delta A^{\phi}-\Omega A^{-\phi}}{2}$, $\cosh_A(\phi)=\frac{\Delta A^{\phi}+\Omega A^{-\phi}}{2}$,
$\tanh_A(\phi)=\frac{\Delta A^{\phi}-\Omega A^{-\phi}}{\Delta A^{\phi}+\Omega A^{-\phi}}$, $\coth_A(\phi)=\frac{\Delta A^{\phi}+\Omega A^{-\phi}}{\Delta A^{\phi}-\Omega A^{-\phi}}$,
$\mathrm{sech}_A(\phi)=\frac{2}{\Delta A^{\phi}+\Omega A^{-\phi}}$, $\mathrm{csch}_A(\phi)=\frac{2}{\Delta A^{\phi}-\Omega A^{-\phi}}$.

where ϕ is an independent variable, $\Delta$and $\Omega$are arbitrary constants greater than zero and called deformation parameters.

Fractional Boussinesq-like equations with conformable derivative

First fractional Boussinesq-like equation

By placing Eq. (2.2) into Eq. (1.1), is obtained nonlinear equation as follows,

$$-u''''(\phi)+(Q^2-1)u''(\phi)-12u(\phi)u'(\phi)^2-6u(\phi)^2u''(\phi)=0, \tag{3.1}$$

By integrating twice according to ϕ Eq. (3.1) and by assuming both of the integration constants zero, is obtained nonlinear equation as follows,

$$-u''(\phi)-2u(\phi)^3+(Q^2-1)u(\phi)=0. \tag{3.2}$$

Assumed the solution of Eq. (3.2) is demonstrable as a finite series as follows:

$$u(\phi)=\sum_{N}^{j=0}\alpha_j G^j(\phi) \tag{3.3}$$

where $G(\phi)$ satisfies Eq. (2.5), $\phi=x-Q\frac{t^\eta}{\eta}$ and α_j for $j=\overline{1,N}$ are values to be definited.

By balancing u'' with u^3 in Eq. (3.2), is obtained $N=1$. We can select the solution of Eq. (3.2) as following shape:

$$u(\phi)=\alpha_0+\alpha_1 G(\phi), \tag{3.4}$$

where $G(\phi)$ satisfied Eq. (2.5).

Substituting (3.4) and (2.5) into (3.2), collecting the coefficients of $G(\phi)$, and solving the obtaining system, the following groups of some solutions are found:

One of the six groups of values as follows

$$\alpha_0=\frac{1}{2}ig\ln(A),\ \alpha_1=if\ln(A),$$
$$Q=\sqrt{1-\frac{1}{2}\left(g^2-4fh\right)\ln(A)^2}. \tag{3.5}$$

The solutions of Eq. (1.1) are obtained as follows;

(1) When $\Psi=g^2-4fh<0$ and $f\neq 0$, then the singular periodic solutions are given by

$$q_1(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\tan_A\left(\frac{\sqrt{-\Psi}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_2(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\cot_A\left(\frac{\sqrt{-\Psi}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

$$q_3(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\left(\tan_A\left(\sqrt{-\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.\right.$$
$$\left.\left.\pm\sqrt{\Delta\Omega}\sec_A\left(\sqrt{-\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)\right),$$
$$q_4(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\left(-\cot_A\left(\sqrt{-\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.\right.$$
$$\left.\left.\pm\sqrt{\Delta\Omega}\csc_A\left(\sqrt{-\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)\right),$$
$$q_5(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{-\Psi}}{2f}\left(\tan_A\left(\sqrt{\frac{-\Psi}{4}}\phi\right)-\cot_A\left(\sqrt{\frac{-\Psi}{4}}\phi\right)\right)\right),$$

(2) When $\Psi=g^2-4hf>0$ and $f\neq 0$, then the dark and the singular soliton solutions are given by

$$q_6(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}-\frac{\sqrt{\Psi}}{2f}\tanh_A\left(\frac{\sqrt{\Psi}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_7(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}-\frac{\sqrt{\Psi}}{2f}\coth_A\left(\frac{\sqrt{\Psi}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

$$q_8(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{\Psi}}{2f}\left(-\tanh_A\left(\sqrt{\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.\right.$$
$$\left.\left.\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(\sqrt{\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)\right),$$
$$q_9(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{\Psi}}{2f}\left(-\coth_A\left(\sqrt{\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.\right.$$
$$\left.\left.\pm\sqrt{\Delta\Omega}\,\mathrm{csch}_A\left(\sqrt{\Psi}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)\right),$$
$$q_{10}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g}{2f}+\frac{\sqrt{\Psi}}{4f}\left(\tanh_A\left(\frac{\sqrt{\Psi}}{4}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.\right.$$
$$\left.\left.+\coth_A\left(\frac{\sqrt{\Psi}}{4}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)\right),$$

(3) When $hf>0$ and $g=0$, then the singular periodic solutions are given by

$$q_{11}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\sqrt{\frac{h}{f}}\tan_A\left(\sqrt{hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right),$$
$$q_{12}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\sqrt{\frac{h}{f}}\cot_A\left(\sqrt{hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

$$q_{13}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\sqrt{\frac{h}{f}}\left(\tan_A\left(2\sqrt{hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm\sqrt{\Delta\Omega}\sec_A\left(2\sqrt{hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{14}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\sqrt{\frac{h}{f}}\left(-\cot_A\left(2\sqrt{hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm\sqrt{\Delta\Omega}\csc_A\left(2\sqrt{hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{15}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\frac{1}{2}\sqrt{\frac{h}{f}}\left(\tan_A\left(\frac{\sqrt{hf}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.-\cot_A\left(\frac{\sqrt{hf}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

(4) When $hf<0$ and $g=0$, then the singular, dark and bright soliton solutions are given by

$$q_{16}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\sqrt{-\frac{h}{f}}\tanh_A\left(\sqrt{-hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{17}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\sqrt{-\frac{h}{f}}\coth_A\left(\sqrt{-hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

$$q_{18}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\sqrt{-\frac{h}{f}}\left(-\tanh_A\left(2\sqrt{-hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(2\sqrt{-hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{19}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\sqrt{-\frac{h}{f}}\left(-\coth_A\left(2\sqrt{-hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm\sqrt{\Delta\Omega}\,\mathrm{csch}_A\left(2\sqrt{-hf}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{20}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{1}{2}\sqrt{-\frac{h}{f}}\left(\tanh_A\left(\frac{\sqrt{-hf}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.\right.$$
$$\left.\left.+\coth_A\left(\frac{\sqrt{-hf}}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)\right),$$

(5) When $h=f$ and $g=0$, then the singular periodic solutions are given by

$$q_{21}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\tan_A\left(h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right),$$
$$q_{22}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\cot_A\left(h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

$$q_{23}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(\tan_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm\sqrt{\Delta\Omega}\sec_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{24}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\cot_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm\sqrt{\Delta\Omega}\csc_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{25}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\frac{1}{2}\left(\tan_A\left(\frac{h}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.-\cot_A\left(\frac{h}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

(6) When $h=-f$ and $g=0$, then the singular, dark and bright soliton solutions are given by

$$q_{26}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\tanh_A\left(h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{27}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\coth_A\left(h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$

$$q_{28}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\tanh_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm i\sqrt{\Delta\Omega}\,\mathrm{sech}_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{29}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\coth_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.$$
$$\left.\pm\sqrt{\Delta\Omega}\,\mathrm{csch}_A\left(2h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right),$$
$$q_{30}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{1}{2}\left(\tanh_A\left(\frac{h}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right.\right.$$
$$\left.\left.+\coth_A\left(\frac{h}{2}\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)\right),$$

(7) When $g^2=4hf$, then the rational solution is given by

$$q_{31}\left(x,t\right)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-2h\frac{g\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\ln(A)+2}{g^2\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\ln(A)}\right),$$

(8) When $g=k$, $h=mk(m\neq 0)$ and $f=0$, then the rational solution is given by

$$q_{32}\left(x,t\right)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(A^{k\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)}-m\right),$$

(9) When $g=f=0$, then the rational solution is given by

$$q_{33}\left(x,t\right)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(h\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\ln(A)\right),$$

(10) When $g=h=0$, then the rational solution is given by

$$q_{34}\left(x,t\right)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{1}{f\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\ln(A)}\right),$$

(11) When $h = 0$ and $g \neq 0$, then the bright and dark-like solitons are given by

$$q_{35}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{\Delta g}{f\left(\cosh_A\left(g\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)-\sinh_A\left(g\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)+\Delta\right)}\right),$$

$$q_{36}(x,t)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(-\frac{g\left(\sinh_A\left(g\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)+\cosh_A\left(g\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)\right)}{f\left(\sinh_A\left(g\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)+\cosh_A\left(g\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)\right)+\Omega\right)}\right),$$

(12) When $g = k$, $h = 0$ and $f = mk (m \neq 0)$, then the rational solution is given by

$$q_{37}\left(x,t\right)=\frac{1}{2}ig\ln(A)+if\ln(A)\left(\frac{\Delta A^{k\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)}}{\Omega-m\Delta A^{k\left(x-\sqrt{1-\frac{1}{2}\Psi\ln(A)^2}\frac{t^\eta}{\eta}\right)}}\right).$$

Second fractional Boussinesq-like equation

By placing Eq. (2.2) into Eq. (1.2), is obtained nonlinear equation as follows,

$$-Q^2u''''(\phi)+(Q^2-1)u''(\phi)-12u(\phi)u'(\phi)^2-6u(\phi)^2u''(\phi)=0, \quad (3.6)$$

By integrating twice according to ϕ Eq. (3.6) and by assuming both of the integration constants zero, is obtained nonlinear equation as follows,

$$-Q^2u''(\phi)-2u(\phi)^3+(Q^2-1)u(\phi)=0. \quad (3.7)$$

Assumed the solution of Eq. (3.6) is demonstrable as a finite series as follows:

$$u(\phi)=\sum_{j=0}^{N}\alpha_jG^j(\phi) \quad (3.8)$$

where $G(\phi)$ satisfies Eq. (2.5), $\phi = x - Q\frac{t^\eta}{\eta}$ and α_j for $j=\overline{1,N}$ are values to be definited.

By balancing u'' with u^3 in Eq. (3.7), is obtained $N = 1$. We can select the solution of Eq. (3.7) as following shape:

$$u(\phi)=\alpha_0+\alpha_1G(\phi), \quad (3.9)$$

where $G(\phi)$ satisfied Eq. (2.5).

Substituting (3.9) and (2.5) into (3.7), collecting the coefficients of $G(\phi)$, and solving the obtaining system, the following groups of some solutions are found:

One of the six groups of values as follows

$$\alpha_0=\frac{ig\ln(A)}{\sqrt{4+2(g^2-4fh)\ln(A)^2}},\ \alpha_1=\frac{2if\ln(A)}{\sqrt{4+2(g^2-4fh)\ln(A)^2}}, \quad (3.10)$$

$$Q=\frac{\sqrt{2}}{\sqrt{2+(g^2-4fh)\ln(A)^2}}.$$

The singular, rational, dark and bright soliton solutions of Eq. (1.2) are obtained similar to the first fractional Boussinesq-like equation. Graphs of some of these solutions found were drawn in the next section.

Graphical representation of the solutions

The surface graphics of the obtained solutions are showed below in the figures by using Mathematica. In Figs. 1–6, we present some numerical simulations for $q_1(x,t)$, $q_8(x,t)$, $q_{17}(x,t)$, $q_{25}(x,t)$, $q_{30}(x,t)$ and $q_{35}(x,t)$ in 3D plots when $0 \leqslant x \leqslant 5$ and $0 \leqslant t \leqslant 5$.

We wrote the some of solutions found for the presented first and second fractional Boussinesq-like equation via conformable derivative operator. Besides we showed 3D graphics for some of solutions in Fig. 1–6. The graphics above were drawn for $A = 2.7$, $\eta = 0.9$, $\Delta = \Omega = 1$.

The fig.7 above were drawn for $A = 2.7$, $\Delta = \Omega = 1$, $t = 0.5$.

Conclusion

In this paper, the extended direct algebraic method is used to find new soliton solutions of the first and second fractional Boussinesq-like equation. These solutions consist of twelve different cases. The existences of solutions derived from these functions are all guaranteed through constraint conditions that are also listed beside the solutions. The constructed soliton solutions are helpful to researchers and have important key applications mathematical physics and engineering. By choosing suitable values of parameters, the movements of a few solutions are presented which help the researcher for understanding the

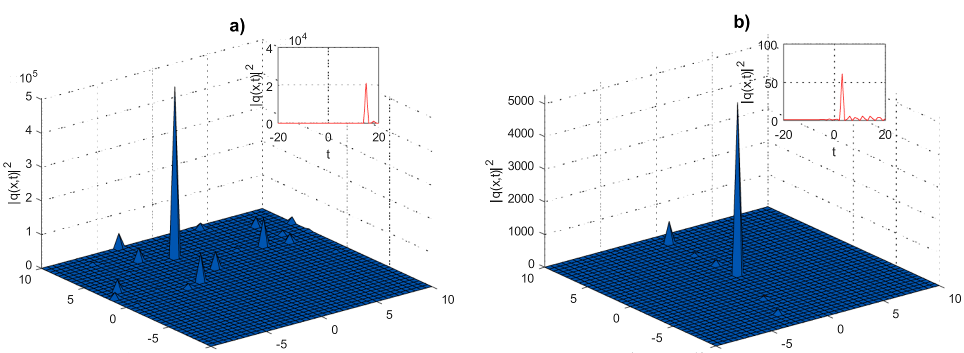

Fig. 1. The 3D and 2D graphics for the $|q(x,t)|^2$ analytical solution of the first fractional Boussinesq-like equation (a) $q_1(x,t) (h = f = 2, g = 1)$, (b) $q_8(x,t)$ $(h = f = 1, g = 3)$.

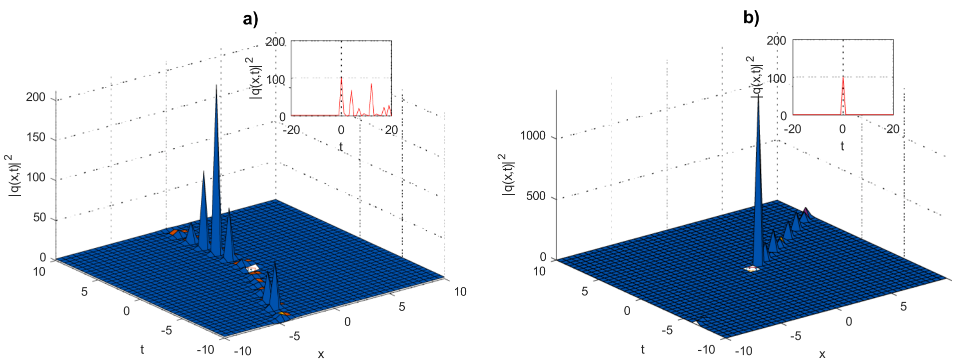

Fig. 2. The 3D and 2D graphics for the $|q(x, t)|^2$ analytical solution of the first fractional Boussinesq-like equation (a) $q_{17}(x, t)(h = -2, f = 1, g = 0)$, (b) $q_{25}(x, t)$ $(h = f = 1, g = 0)$.

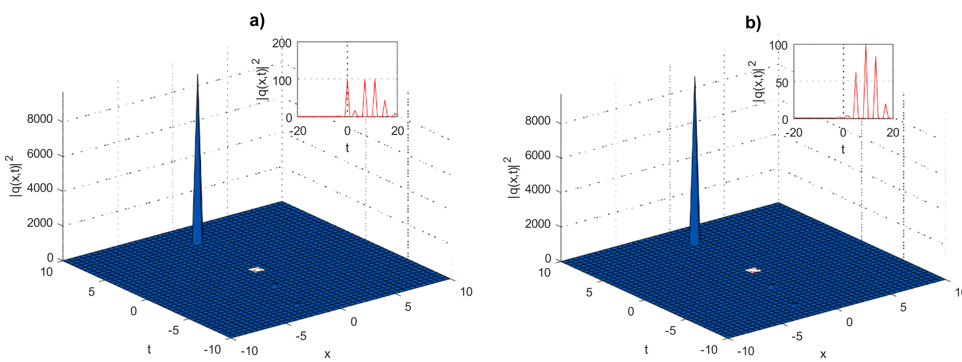

Fig. 3. The 3D and 2D graphics for the $|q(x, t)|^2$ analytical solution of the first fractional Boussinesq-like equation (a) $q_{30}(x, t)(h = 1, f = -1, g = 0)$, (b) $q_{35}(x, t)$ $(h = 0, f = 1, g = 2)$.

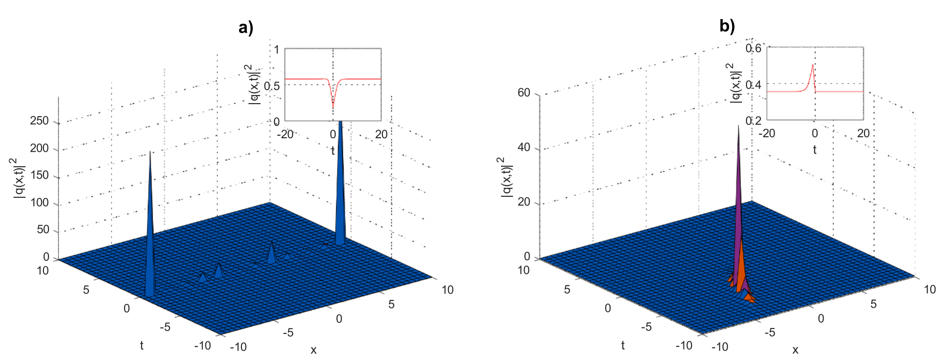

Fig. 4. The 3D and 2D graphics for the $|q(x, t)|^2$ analytical solution of the second fractional Boussinesq-like equation (a) $q_1(x, t)(h = f = 2, g = 1)$, (b) $q_8(x, t)$ $(h = f = 1, g = 3)$.

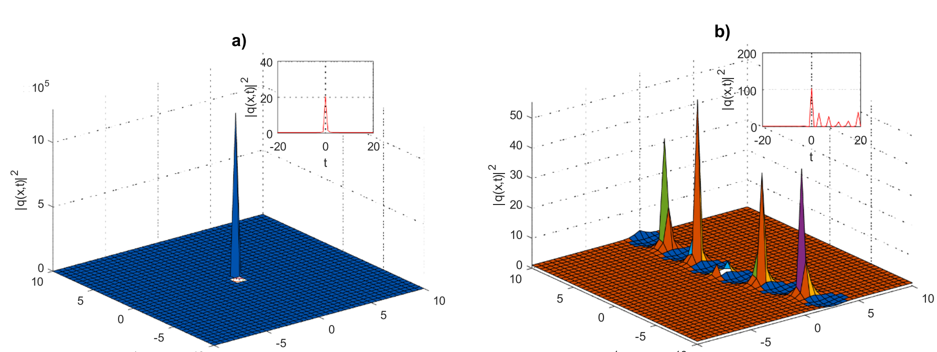

Fig. 5. The 3D and 2D graphics for the $|q(x, t)|^2$ analytical solution of the second fractional Boussinesq-like equation (a) $q_{17}(x, t)(h = -2, f = 1, g = 0)$, (b) $q_{25}(x, t)$ $(h = f = 1, g = 0)$.

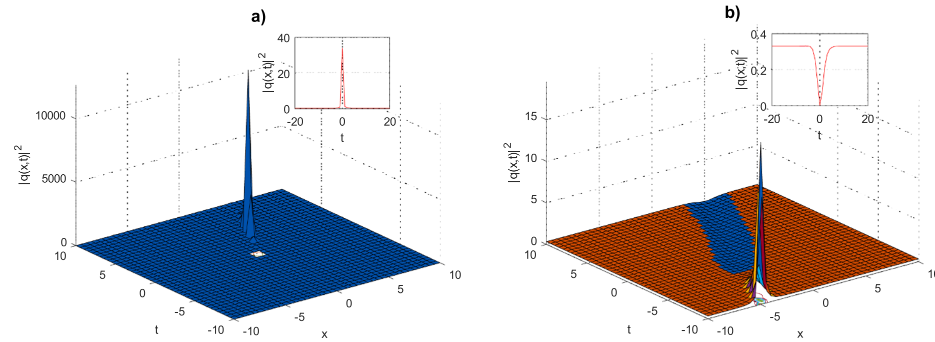

Fig. 6. The 3D and 2D graphics for the $|q(x,\ t)|^2$ analytical solution of the second fractional Boussinesq-like equation (a) $q_{30}(x,\ t)(h = 1, f = -1, g = 0)$, (b) $q_{35}(x,\ t)$ $(h = 0, f = 1, g = 2)$.

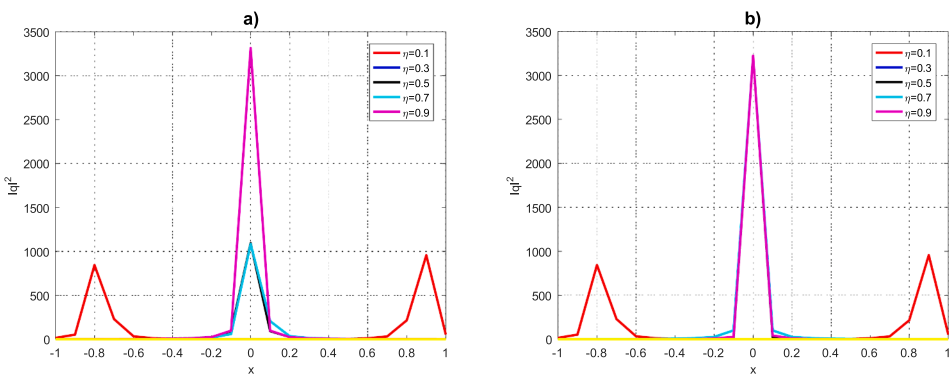

Fig. 7. The 2D graphics for the $|q(x,\ t)|^2$ analytical solution of the fractional Boussinesq-like equation for different value of η. (a) The solution $q_8(x,\ t)$ of first fractional Boussinesq-like equation $(h = f = 1, g = 3)$, (b) The solution $q_{17}(x,\ t)$ of second fractional Boussinesq-like equation $(h = -2, f = 1, g = 0)$.

physical interpretation of this dynamical model. We say that the presented method is suitable to examine the many problems located in science and engineering. We will study the some different applications of this method for different fractional differential operators in future works and we will try to reduce the absolute error.

Declaration of Competing Interest

None.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, athttps://doi.org/10.1016/j.rinp.2019.102339.

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