



Fractal analysis of high speed rail geometry data: A case study of Ankara-Eskişehir high speed rail

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ARTICLE INFO

Article history:

Received 24 January 2020

Received in revised form 27 May 2020

Accepted 13 June 2020

Available online 30 June 2020

Keywords:

Fractal analysis

Railway track geometry irregularities

High speed rail

Track geometry index

ABSTRACT

By considering railway track geometry as a fractal pattern, the irregularity of the track geometry can be expressed numerically with the help of fractal dimensions. In this study, a novel algorithm based on ruler method has been developed and a program was written to determine the fractal dimensions of railway track geometry graphs. Four different fractal dimensions of D_{R1} , D_{R2} , D_{R3} , and D_{R4} were proposed to determine the geometric irregularity of the railway track by using the data of Ankara-Eskişehir High-Speed Railway track. D_{R1} and D_{R2} were used to quantify short wavelength irregularities while D_{R3} and D_{R4} were used to quantify medium wavelength irregularities. At the end of the study, the relationship between fractal dimensions and standard deviation values, which is the quality indicator according to EN 13848-6, was investigated. According to this, there is a strong relationship between the D_{R3} and the standard deviation.

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1. Introduction

Railway tracks deteriorate over time with repeated traffic loads and environmental effects. The degradation is occurred as a structural degradation of track components (rail, sleeper, ballast bed) or a deterioration of railway track geometry. The railway track geometry, which expresses the location of the rails in spatial planes, consists of several parameters. These are the alignment and gauge in the horizontal plane and the longitudinal level, twist and cross level parameters in the vertical plane. These parameters are defined and described in detail in EN13848-1 [1]. Each track geometry parameter has its own design values. Deviations in design values result in a rough track geometry pattern. Rough track geometry leads to degradation of rail vehicles and track components, passenger discomfort, decrease in train operational speed, and derailment in later phases [2]. Due to these negative effects, track geometry should be rehabilitated with maintenance and repair works. Maintenance costs of railway tracks constitute a large part of total track lifecycle costs [3–5]. It is estimated that European countries allocate approximately 15 to 25 billion EUR each year for the

maintenance and renovation of railway systems [6,7]. The size of the allocated budgets has revealed the necessity to carry out maintenance and repair works in a certain program and as much as necessary. In order to determine the type and date of maintenance and repair works, track geometry inspections are made with automatic track recording cars and the condition of the track is tried to be determined. Track condition is usually expressed numerically by a quality index derived from track geometry measurement data. To date, using various statistical or empirical methods, several track quality index (TQI) has developed throughout the world by researchers of universities, technology firms, and railway organizations.

Many quality indexes used by railway organizations have been developed using statistical methods. In the method developed by the European Rail Research Center (ORE) in France the direct standard deviation (SD) approach is recommended [8]. The SD of the geometry parameters (profile, alignment, cross level and gauge) is calculated for a 1000 m section using 18.9 m mid-chord offsets [9]. According to this method SD of geometry parameters are represented the track condition. The J index developed by Polish researchers, the Q index used by Swedish National Railways, the Track Geometry Index (TGI) used by Indian railways, track quality index (TQI) of Canadian National Railway Company (CN), Q index used by ProRail of Netherlands, Chinese track quality index, The K-value used in Sweden and the Running Roughness (R^2) developed by the American railway company Amtrak, are derived from

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the SD of the track geometry parameters [9–20]. Five-parameter track defectiveness (w5), developed by the Austrian Railway, considers the arithmetic mean of vertical and horizontal irregularities and defectiveness of the three other geometry parameters (gauge, twist and cant) [9]. The American Federal Railroad Administration (FRA) also developed a set of objective track quality indexes (TQIs) from track geometry data in an annex to the Federal Track Safety Standards (FTSS). This index uses the space curve length to indicate the track quality. In this method, TQI values are calculated for each profile, gauge, cross level and twist [21]. The indexes mentioned above are summarized in Table 1.

Railway companies in Europe have been considering the EN-13848 standard series in the assessment of track geometry measurements in recent years. Also, at the high-speed railways in Turkey, EN-13848-5 standard is used to determine track quality. In these standard, three indicators which are for assessment of the track geometric quality are described. These are; extreme values of isolated defects, SD of the geometry values for the 200 m long track segments and the mean values. According to this norm three main levels have to be considered [23]:

1- Immediate Action Limit (IAL): Refers to the value which, if exceeded, requires taking measures to reduce the risk of derailment to an acceptable level. This can be done either by closing the line, reducing speed or by correction of track geometry;

2- Intervention Limit (IL): Refers to the value which, if exceeded, requires corrective maintenance in order that the immediate action limit shall not be reached before the next inspection;

3-Alert Limit (AL): Refers to the value which, if exceeded, requires that the track geometry condition is analyzed and considered in the regularly planned maintenance operations [23].

The values of these main levels are given as a function of to the speed. Furthermore, the threshold values of the main levels for the alignment and longitudinal level parameters also vary according to the wavelength (λ) of the defects. According to EN 13848-6, SD is linked to the energy of the signal in a given wavelength range [24]. SD is commonly calculated for the longitudinal level D1 and Alignment D1 parameters. In EN 13848, three wavelengths are defined: D1 ($3\text{ m} < \lambda \leq 25\text{ m}$), D2 ($25\text{ m} < \lambda \leq 70\text{ m}$) and D3 ($70\text{ m} < \lambda \leq 150\text{ m}$ for longitudinal level and $70\text{ m} < \lambda \leq 200\text{ m}$ alignment). Furthermore, in the 2019 version of EN13848-1, in order to short wavelength defect, the D0 ($1\text{ m} < \lambda \leq 5\text{ m}$) wavelength is defined [1]. The wavelengths of the irregularities are important for track-vehicle interaction. Given its nature, medium wavelength track irregularities have a negative impact on the comfort of passengers, while short wavelength irregularities create more vibration on the axles and wheels [25].

There are other indexes in the literature suggested by researchers. In his study, Iranian researcher Sadeghi developed an index by

Table 1
Current track geometry indices [9–20,22].

Index	Developer	Formula
Standard Deviation	ORE	$SD = \sqrt{\frac{\sum_{i=1}^N (X_i - \bar{X})^2}{N-1}}$ - The number of values in the sample X_i - The current value of a signal \bar{X} - The mean value of a signal
J Index	Polish Railway	$J = \frac{S_y + S_w + S_z + 0.55S_z}{3.5} S_z$ - Gauge standard deviation S_y - Horizontal irregularities standard deviation S_w - Twist standard deviation S_z - Vertical irregularities standard deviation
Q Index	Swedish National Railway	$Q = 150 - 100 \left[\frac{\sigma_H}{\sigma_{Hlim}} + 2X \frac{\sigma_s}{\sigma_{slim}} \right] / 3$ - Average standard deviations of right and left level σ_s - Average standard deviations of alignment and cant σ_{Hlim} - Allowable value of σ_H based on track categories σ_{slim} - Allowable value of σ_s as per track categories
TGI	Indian Railways	$TGI = \frac{2UI + TI + GI + 6AI}{10}$ - Measure parameter standard deviation $UI, TI, AI = 100 \frac{SDn - SDu}{SDn}$ - Measure parameter standard deviation SDn - New track standard deviation SDu - Track urgent maintenance standard deviations
Track Quality Indeks (TQI)	Canadian National Railway Company	$TQI = 1000 - C * \sigma_i^2$ - Constant (for the main line tracks 700) σ_i - SD of track geometry parameters
Q Index	ProRail Netherlands	$N = 10 * 0.675^{\sigma_i / \sigma_i^{80}}$ - Q index for a quality parameter over a 200m long track segment σ_i - SD for the quality parameter σ_i^{80} - 80th percentile of SD's for 200 m long segments in a maintenance section ranging in length from 5 to 10 km
Track Quality Indeks (TQI) Running Roughness (R ²)	Chinese National Railroads Amtrak (USA)	$TQI = \sum_{i=1}^7 \sigma_i$ - SD of track geometry parameters $R^2 = \frac{\sum_{i=1}^n d_i^2}{n}$ - Total number of measurements in length d_i - Amount of deviation measured over the 20 m mid-chord offsets (profile, alignment, cross level, and gauge)
Five Parameter Track Defectiveness (W5)	Austrian Railways	$w_5 = 1 - (1 - w_e)(1 - w_g)(1 - w_w)(1 - w_y)(1 - w_z)$ $W = \frac{\sum_{i=1}^L w_e}{n}$ - Track gauge defectiveness w_g - Cant defectiveness w_w - Twist defectiveness w_y - Calculated average horizontal irregularities w_z - Calculated average vertical irregularities
Track Quality Indeks (TQI)	The American Federal Railroad Administration (FRA)	$TQI = \left[\frac{L_s}{L_0} - 1 \right] \times 10^6$ - Space curve traced length L_0 - Track segment (theoretical length)
K-Value	Sweden Railways	$K = \frac{\sum_{i=1}^L l}{L} * 100\%$ - l-the total length of the track with standard deviations below the comfort limits L - Total length of track

making use of the statistical distribution of track geometry data. In this study, Sadeghi calculated individual indexes for each geometry parameter (gauge, profile, alignment and twist). He then determined the significance coefficient of each index and proposed the overall track geometry index (OTGI), which represents the geometric condition of the track [9]. Hamid and Gross proposed five different TQIs, namely Gage-roughness index, Wide-Gage index, Surface index, Line index, and superelevation index, by using the appropriate statistics such as standard deviation and mean of the raw track geometry data [26]. Sadeghi et al. investigated the effect of the rail cant in the general condition of the track geometry. They proposed new indices based on the allowable limits for each of the alignment, profile, twist, gauge, and rail cant parameters. After creating an index for each geometry parameter, the researchers appropriately combined the indexes of all geometry parameters to create a general TGI indicating the total track geometry condition [27]. In another study, Sadeghi et al. proposed a new railway track conditions index which takes into account passenger ride comfort [5]. Faiz and Singh examined the geometry parameters used in the track maintenance process in the UK and identified the relationship between different track geometry measurements using the linear regression technique [28]. El-Sibaie and Zhang examined the relationship between the track quality index of FRA and the different track classes proposed by FTSS and they proposed a new set of TQI. Their results show that new TQIs can quantitatively define the relative condition of track surface geometries. In addition, new TQIs have been shown to correlate well with FTSS for each class and provide a relative quality indicator in each class [29]. Li and Xu proposed an integral maintenance index (IMI) that considers all the single track geometry irregularities (profile, alignment, cross level, and twist), acceleration and history maintenance works. They also developed the application of IMI in making track maintenance plan [30].

The reason for implementing statistical methods is to control larger track geometry deviations that often cause larger vehicle vibration responses due to the balance of the train can be guaranteed when the deviation values are within acceptable limits. However, in all cases statistical approaches are not enough. Some smaller sizes of the deviation may show a high correlation with the car body vertical acceleration, and the track geometry wavelength is the effective factor in this phenomenon [31]. In studies used power spectral density approach, the track geometry wavelength was taken into consideration in evaluating the quality of track. Corbin et al., Iyengar et al. and Chen et al. used this approach to diagnose and classify the quality of track geometry [31–34]. Li and Xiao proposed an integrated index called generalized energy index (GEI) [35]. According to them, since the GEI can consider different track irregularity wavelength and speed, the GEI can capture various effects of different wavelength components of track irregularity to the vehicle dynamic response [25]. The authors also stated that, according to the results, GEI is better than TQI for the assessment of track irregularity [35]. In their study, Li et al. stated that the track geometry wavelength factor should be taken into consideration in order to evaluate the track geometry quality more effectively. They proposed an approach to evaluate track geometry quality based on the intrinsic mode function (IMF) derived from track geometry data measured by characteristic wavelength scales [31]. In 2000, Hyslip used the fractal analysis method to determine the condition of the track geometry. Fractal analysis methodology is based on the fact that vertical track geometry can be derived from a sum of many parts of harmonic irregularities with different wavelengths and their amplitudes [36]. In this study, which is based on the characterization of the rough and wavy appearance of the vertical geometry of the conventional railway track, Hyslip calculated the fractal dimensions of the vertical track geometry and obtained the

indicative values for the track quality. According to Hyslip, fractal analysis provides good indicators to quantify the irregularity of the geometry data and fractal analysis has the potential to evaluate track substructure condition [2]. Afterwards, Landgraf and Hansmann implemented Hyslip's methodology to data from the Austrian Federal Railways [36].

In the current work, we suggest a methodology to evaluate high speed rail track geometry irregularities based on fractal analysis. As stated in previous studies, track geometry irregularity wavelengths can be quantified with the fractal analysis method. However, the proposed approach has the some differences compared to previous studies. First of all, fractal analysis was made on track geometry graphs of 200-meter-long track inspection sections. The main feature of geometry graphics is that they have a scale of 1/5000 in the horizontal plane and 1/1 in the vertical plane. This scaling makes track geometry irregularities more visible on the graph and irregularities in different wavelengths could be recognised. That is, track geometry graphs have appropriate patterns to quantify by fractal analysis. Then, to calculate fractal dimensions a new algorithm was developed and four different fractal dimensions of D_{R1} , D_{R2} , D_{R3} , and D_{R4} were proposed to quantify different wavelength irregularities. The track geometry data were used in this study, obtained from the track inspections conducted by Turkish State Railways (TCDD) High-Speed Rail (HSR) department on Ankara-Eskişehir HSR track. Track geometry graphs were drawn using these data. Fractal analysis was made for alignment and longitudinal level parameters since they represent horizontal and vertical geometry. According to the comparative results, track geometry irregularities in short and medium wavelengths could be quantified with the proposed approach.

2. Material and method

2.1. Fractal analysis

Fractal analysis is a mathematical method used to characterize and quantify irregular, random looking patterns [37]. Fractal dimension is an indication of the complexity or roughness of the structure. The fractal dimension of a pattern varies depending on the degree of irregularity of the pattern and is different for each pattern [38]. Fractal dimension of highly detailed complex patterns has larger values. Fractal analysis method which has important effects on mathematics in the last century has applications in various fields such as basic sciences, various engineering branches, architecture and medicine.

One of the most commonly used fractal dimension calculation methods is the ruler method. Mandelbrot's work to measure the coastline of England is one of the best known examples of this method [37]. In this method, the length of the pattern is measured with rulers of different lengths to calculate the fractal dimension of a pattern. Fig. 1 shows how the ruler method can be applied on a rough pattern. As shown in Fig. 1, the rulers with varied lengths are progressed on the pattern step by step to intersect each point at a time. The total length of the pattern ($L(r)$) is simply calculated by multiplying the length of the ruler (r) by the number of steps (N). As the ruler size used in the measurement decreases, the measured total length of the pattern increases. Since, as the ruler size becomes smaller, the rulers intersect with the rough pattern at more points and the measurement accuracy increases. Then the graph $\log(r)$ and $\log L(r)$ are plotted. The fractal dimension (D_R) value of the pattern is calculated by using the slope (m) of the trend line which shows the relationship between the points in the graph as shown in Fig. 1. [2]

Real fractals are self-similar structures that show the same pattern, no matter how much the scale is enlarged, and have one frac-

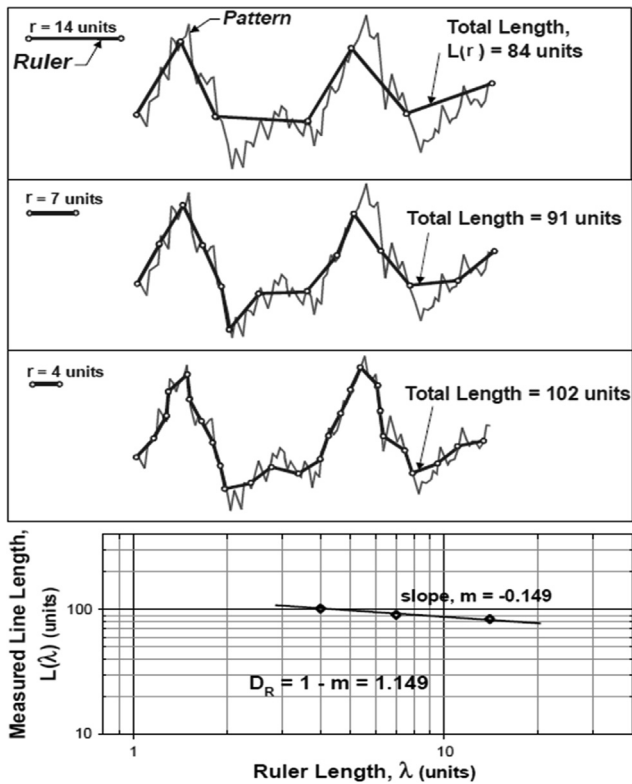


Fig. 1. Calculation of fractal dimension with ruler method [2].

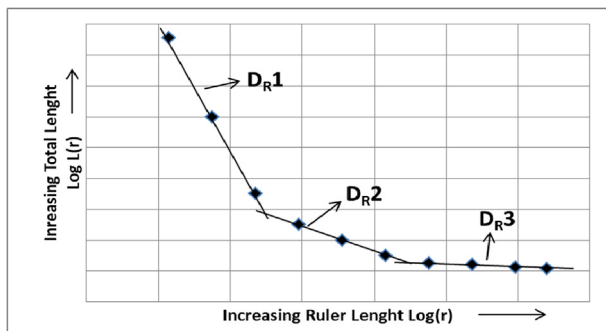


Fig. 2. Multi-fractal structure [24]

tal dimension. Natural fractals, on the other hand, are structures with similar characteristics within a certain limit value. As natural fractal patterns are enlarged, different patterns appear or become noticeable [39]. Because of these properties, they have many fractal dimensions and the fractal dimension has a different value at each examination level (Fig. 2). This is called a multi-fractal feature. Each fractal dimension of a multi-fractal structure is indicative of the irregularity of the pattern at each examination level.

As Hyslip shows in his study, railway track geometry is considered a natural fractal structure because it shows different details at different scales.

2.2. High-Speed rail geometry data

Ankara-Eskişehir high-speed rail was opened in 2009 as a Turkey's first HSR track. The length of the railway that was designed as a double line is 245 km. In the superstructure of this high speed rail, basalt-origin ballast materials, UIC 60 type long welded rails

and B 70 type prestressed concrete sleepers were used. Track gauge is 1435 mm, the axle load is 22.5 tons and the maximum operating speed is 250 km/h. The track recording vehicle used for track inspections on this railway has the capacity to measure track geometry parameters every 0.25 m at a speed of 250 km/h. The measurements are made and evaluated according to EN-13848 standards. Measurements are made separately for two lines. Only passenger trains serve on this high-speed railway. At the date when the data which is used in this study were collected 10 reciprocal travels a day were performing on the HSR. Therefore, the HSR track had low traffic volume.

In this study, data obtained from 5 different track geometry inspections performed August 2011, December 2011, March 2012, July 2012 and November 2012 on HSR were used. The data used in the calculations belonged to the 197 km long section of the railway which is between 518 + 880 and 322 + 211 km. As mentioned before, the high-speed rail platform has double line. In the first superficial examination, it was observed that there was no significant difference between the changes in the geometry of neighbouring lines. For example, it was found that both the value and location of defects in the lines were approximately the same [40]. Therefore, only the measurement data of the so-called "North Track" was used in the study. These data included approximately 800 k point measurement data for each geometry parameter (longitudinal level, alignment, cross level, gauge and twist) in each measurement, taking into account the track length of 197 km. Based on this information, the track was divided into sub-sections with a length of 200 m. In 5 different track inspection studies, it was observed that only 43 of the sub-sections occurred defects according to the limit values specified in EN-13848-5 norm. Therefore, in the research, fractal analysis was performed for the geometry graphs of these segments. The calculations were made for the alignment and longitudinal level graphs as they represent horizontal and vertical geometry.

In order to perform fractal analysis of the track geometry data, first of all it was necessary to draw the geometry graphs of the sub-sections. Although the track geometry graphs were drawn as a result of the inspections carried out by the track recording vehicle, the geometry graphs of the subsections were reconstructed in computer environment using point measurement data for the calculations made in this study. Longitudinal level and alignment values of investigated 43 sub-sections were taken from the collection of point measurement data of each track inspection studies. Using these values, longitudinal level and alignment graphs were drawn separately for the right and left rail in each sub-section. In other words, 4 different graphs were drawn for each sub-section to be 2 longitudinal level and 2 alignment. A total of 860 different graphs were drawn using the data of the inspections performed on 5 different dates. Fig. 3 shows the computer-generated longitudinal level and alignment graph examples. On the top graphs, there is information about sub-section number-measurement time (in year and month) -geometric parameter.

2.3. Development of fractal dimension calculation algorithm

In the continuation of the study, a program was written in MATLAB environment to calculate the fractal dimension of the plotted graphs. In his study, Hyslip calculated the fractal dimension of the geometry parameter by dividing the vertical profile geometry graphs at different step lengths along the x axis. In this study, the ruler method was used again, but a different approach was used to calculate fractal dimensions than Hyslip used. This approach is based on the principle of step by step progression of rulers of different lengths on the graph curve. First, using the raw measurement data, a geometry graph with a scale of 1/5000 in the horizontal plane and 1/1 in the vertical plane was created by the program. The scale values were the same as the scales of the

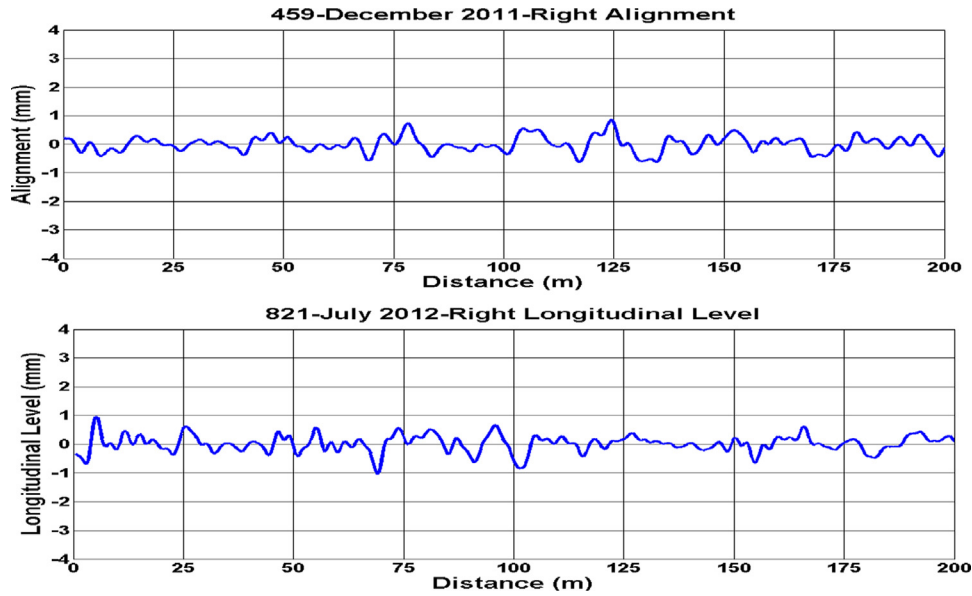


Fig. 3. Examples of track geometry graphs for alignment and longitudinal level parameters.

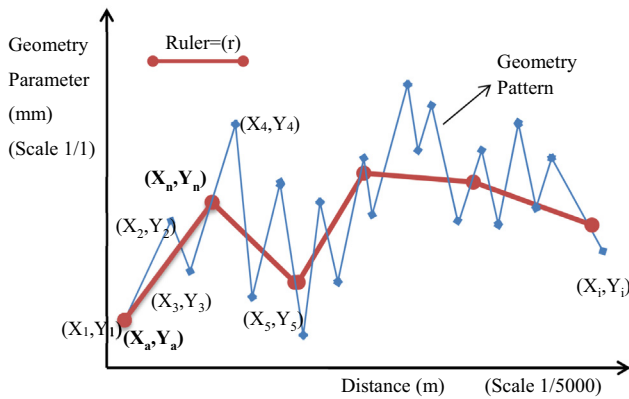


Fig. 4. Step-by-step progression of the ruler in fractal analysis program.

graphs generated by the track recording train. When these graphs were enlarged, it was clearly seen that they consist of line segments attached at the ends. Then a ruler of the specified length was advanced step by step on the geometry graph and the length of the graph curve was measured (Fig. 4). In this process, the starting and end points of the ruler should be placed on the geometry curve at each step. In other words, the coordinates of the points on which the ruler was placed on the graph curve should be determined. A novel algorithm was developed for this [40].

The coordinates of the starting point of the first ruler were named X_a, Y_a , and the coordinates of the end point were named X_n, Y_n (Fig. 4), the following equation was written by using the circle equation for the ruler length (L).

$$L^2 = (X_n - X_a)^2 + (Y_n - Y_a)^2 \quad (1)$$

Since the starting (X_1, Y_1) and end point (X_2, Y_2) of the first line was known, the slope (m) of this line was calculated by the following equation.

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} \quad (2)$$

Since the measurement was started from the first point of the graph, (X_a, Y_a) and (X_1, Y_1) were indicated the same points (Fig. 4). The end point of the ruler was assumed to be on the direc-

tion of the first line segment composing the geometry graph, (X_n, Y_n) was expressed as follows, depending on (X_1, Y_1) and m .

$$Y_n = m * (X_n - X_1) + Y_1 \quad (3)$$

Exponential expression in Eq. (1) was opened and Eq. (4) given below was obtained.

$$X_n^2 - 2X_nX_a + X_a^2 + Y_n^2 - 2Y_nY_a + Y_a^2 - L^2 = 0 \quad (4)$$

The Y_n value was written in Eq. (4) as stated in Eq. (3) and the Eq. (5) given below was obtained.

$$\begin{aligned} X_n^2 - 2X_nX_a + X_a^2 + m^2X_n^2 - 2m(mX_1 - Y_1)X_n + (Y_1 - mX_1)^2 \\ - 2mY_aX_n + 2mX_1Y_a - 2Y_1Y_a + Y_a^2 - L^2 \\ = 0 \end{aligned} \quad (5)$$

Eq. (5) was showed a second-order equation. In Eq. (5), all values except X_n were known values. Considering the unknown X_n expression, the following coefficients were obtained.

$$A = (m^2 + 1)X_n^2 \quad (6)$$

$$B = (-2X_a - 2m(mX_1 - Y_1) - 2mY_a)X_n \quad (7)$$

$$C = X_a^2 + (Y_1 - mX_1)^2 + 2mX_1Y_a - 2Y_1Y_a + Y_a^2 - L^2 = 0 \quad (8)$$

With the help of coefficients A, B and C , the Eq. (5) was became the following equation

$$AX_n^2 + BX_n + C = 0 \quad (9)$$

After this stage the roots of the equation was investigated. In order to find roots, the discriminant (Δ) value of the second order equation was examined [40].

- When $\Delta < 0$, it was found that the value X_n was not on the first line segment. In this case, it was assumed that (X_n, Y_n) was on the next line segment, the same operations were repeated using the coordinates of the next line segment.
- When $\Delta = 0$, it was found that the equation to has two overlapping roots. In this case, it was checked whether the value of X_n was on the first line segment. If $X_2 \geq X_n > X_a$, X_n was on the first line segment. If $X_n > X_2$, X_n was not on the first line segment. In this case (X_n, Y_n) was assumed to be on the next line segment and the same process was repeated.

- When $\Delta > 0$, the equation was found to have two discrete roots. It was checked whether at least one or both of the found root values were on the first line segment of the graph. If $X_2 \geq X_n > X_a$, X_n was on the first line. If the roots were out of this range, X_n and Y_n were assumed to be on the next line segment and the same procedure was repeated [24].

As a result of the processes described above, the end point of the ruler used in the first step was determined. The end point of the ruler determined in the first step was the first point of the

second step. The same approach was applied in the second step and the end point of the second step was determined. Step-by-step progress continued throughout the graph and the number of steps taken from the start point to the end point was determined. The total graph length (L) was calculated by multiplying the number of steps (N) by the length of the ruler (r). The same procedures were repeated with different lengths of rulers and the graphic length was measured with each different ruler. After determining the graphic lengths for different rulers, the fractal dimension was calculated for the geometry graph with the help of Eq. (10) [40–42].

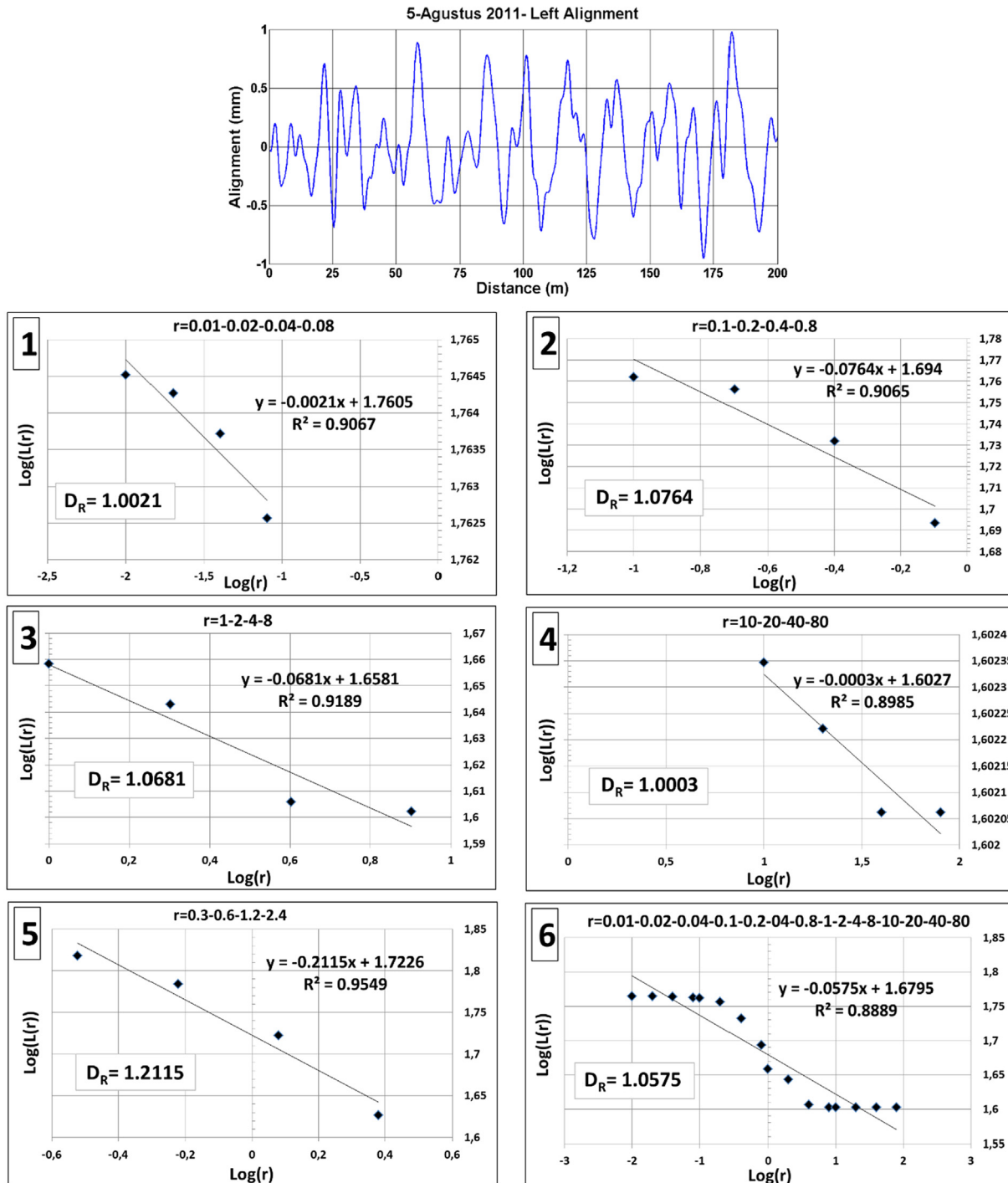


Fig. 5. Calculation of fractal dimensions using rulers of different lengths.

$$D_R = - \frac{\sum (\text{Log}_{10}(N)\text{Log}_{10}(L)) - (\sum \text{Log}_{10}(N) \sum \text{Log}_{10}(L)/J)}{\sum (\text{Log}_{10}(L))^2 - (\sum (\text{Log}_{10}(L))^2/J)} \quad (10)$$

Where D_R is the fractal dimension of the geometry parameter, N is the number of steps of the ruler used in the measurement, r is the length of the ruler used in the measurement, and J is the number of rulers of different lengths.

2.4. Ruler length selection for fractal dimension calculation

Since rail track geometry graphs had multi-fractal dimensions, different fractal dimension values were calculated at each examination level. Therefore, the ruler length had to be selected for the fractal dimension calculation. In this study, the dimensions of geometry graphs were taken into consideration when deciding which length rulers to use. Since the geometry graphs had a scale of 1/5000 on the horizontal and 1/1 on the vertical, the length of the graph was 40 mm on the horizontal axis for 200-meter track sub-sections. The values of the vertical axis indicated the measured values of the geometry parameter in mm and varied with accuracy of 0.01 mm. Therefore, it was decided that the ruler lengths to be used in the fractal dimension calculation should be in varying dimensions in millimeters. In addition the calculation of the fractal dimension, the coefficient of determination (R^2) of regression line indicating the relationship between points in the $\text{Log}(r) - \text{Log}L(r)$ graphs should be close to 1. Therefore, in the selection of the length of ruler R^2 value was taken into account.

Fractal dimension values were calculated using many different ruler lengths. Fig. 5 shows some examples of calculations. On the top of the figure, the left alignment graph of the sub-section-5 is shown. This graph is drawn by using the data of the inspection made in August 2011. Below the graph, $\text{Log}(r) - \text{Log}L(r)$ graphs obtained by using 6 different ruler lengths can be seen. At the

top of each of the $\text{Log}(r) - \text{Log}L(r)$ graphs it is shown which ruler lengths (r) are used in the calculation. Fractal dimension values (D_R) are also indicated on the graphs. In graphs 1-2-3-4 and 5, 4 different ruler lengths were used for fractal dimension calculation and 16 different ruler lengths were used for Graph 6. In the 6 different graphs shown in the figure, R^2 values close to 1 were obtained. As seen in each calculation graph, the fractal dimension value changes as the ruler value range (r) changes.

3. Results

According to the results of the track inspections, the majority of the geometry defects occurring in the Ankara-Eskişehir HSR track were of the AL type. In the alignment parameter, only AL type defects were detected. In the longitudinal level parameter, AL type defects and small amount of IL type defects were detected. IAL type defects were not detected for both geometry parameters (Table 2). This is due to the low traffic volume and the high safety measures taken on the high-speed railway [27]. In addition to the inspections made by the track recording car, the HSR track is continuously controlled by technical personnel. Daily reports of train drivers are also taken into consideration. Therefore, without waiting the results of the track inspection studies, maintenance and repair works are carried out on the track by using tamping machines and stabilizers.

In the continuation of the study, the irregularities of the track geometry graphs were evaluated by using 4 different fractal dimensions named D_{R1} , D_{R2} , D_{R3} and D_{R4} . Four different ruler length ranges were used for the calculation of each fractal dimension. In the calculations, it was determined that the fractal dimension values and the change interval of fractal dimensions changed significantly in each ruler length range. (Table 3)

Table 2 Variation of the amount of deterioration according to the track inspection date [40,43].

Measurement Date	Defect Length in D1 (m)											
	Right Alignment			Left Alignment			Right Long. Level			Left Long. Level		
	AL	IL	IAL	AL	IL	IAL	AL	IL	IAL	AL	IL	IAL
August 2011	8	0	0	3	0	0	4	5	0	11	2	0
December 2011	3	0	0	1	0	0	6	0	0	7	0	0
March 2012	4	0	0	5	0	0	21	9	0	20	6	0
July 2012	1.5	0	0	2	0	0	0	2	0	0	1.5	0
November 2012	3.5	0	0	3	0	0	1.5	0	0	3	0	0
TOTAL	20	0	0	14	0	0	32.5	16	0	51	9.5	0

Table 3 Fractal dimension change interval.

Fraktal Dimension	Ruler Length (mm)	Alignment		Longitudinal Level	
		Min	Max	Min	Max
D_{R1}	0.01	1.00125	1.01317	1.00117	1.01421
	0.02				
	0.04				
	0.08				
D_{R2}	0.1	1.04685	1.17278	1.04638	1.19650
	0.2				
	0.4				
	0.8				
D_{R3}	1	1.02871	1.36324	1.02791	1.59455
	2				
	4				
	8				
D_{R4}	10	1.00001	1.03757	1.00002	1.17813
	20				
	40				
	80				

In order to understand how the fractal dimensions represent irregularity, the alignment and longitudinal level graphs of the 200 m long sub-sections were evaluated. In this study, fractal dimensions were evaluated by considering wavelengths of irregularities in track geometry graphs. As previously mentioned, there is a classification in EN-13848-1 concerning the wavelengths (λ) of irregularities in railway track geometry. These are medium wavelengths D1 ($3\text{ m} < \lambda \leq 25\text{ m}$), long wavelengths D2 ($25\text{ m} < \lambda \leq 70\text{ m}$) and very long wavelengths D3 ($70\text{ m} < \lambda \leq 150\text{ m}$ for longitudinal level and $70\text{ m} < \lambda \leq 200\text{ m}$ alignment). In addition to this, short wavelength (D0) geometry irregularities occurring in 1–5 m band intervals are observed in railway track geometry.

Fig. 6 shows the alignment graphs in which D_{R1} , D_{R2} , D_{R3} and D_{R4} are calculated. On the left side of the figure, the alignment

graphs of the different sub-sections are shown. On the right, the details of a 50-meter section selected from these graphs are shown. The fractal dimension value of each graph is indicated on the graphs on the left. In addition, the detail graphs on the right side contain representations of the wavelengths of irregularities (λ_i). All graphics given in the figures below were drawn in MATLAB and brought together in the Paint 3D

Fig. 6 exhibits the graphs of the sub-sections where the high D_{R1} , D_{R2} , D_{R3} and D_{R4} values are calculated for the alignment parameter. Looking at the figure from top to bottom, the graphs clearly show how the irregularity of the geometry pattern changes. When the graphs in which the high value D_{R1} and D_{R2} are examined, significant geometry changes are not appear in the graphs on the left. However, the changes are clearly seen in the detail graphs

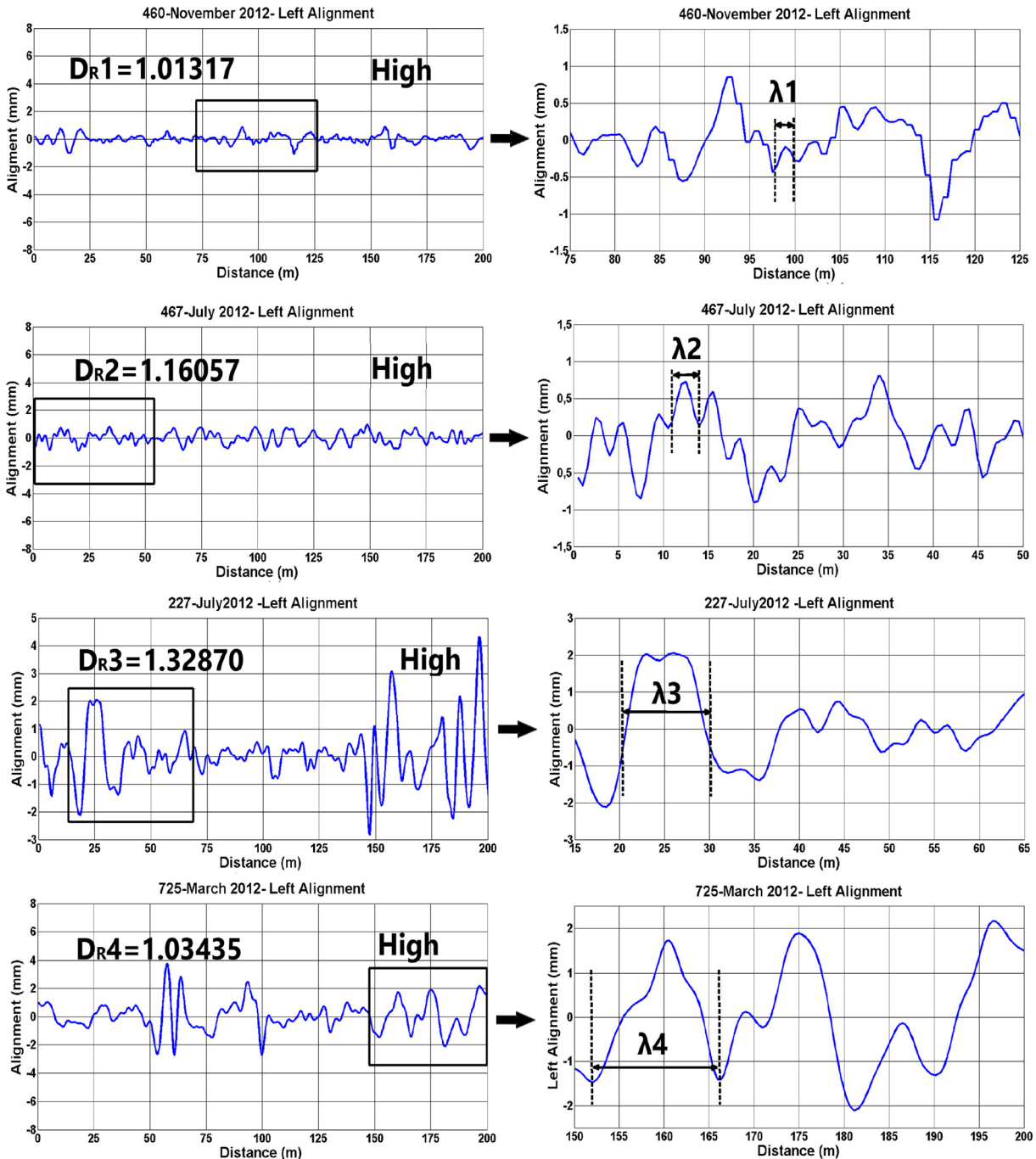


Fig. 6. High fractal dimension graphs for alignment.

on the right. In the sub-sections where high D_{R1} and D_{R2} values are calculated, sudden and small value geometric changes are seen within short distances. These changes are seen both as short (1–5 m length), small amplitude waves (λ_1 and λ_2) and as short, small amplitude waves overlapping longer waves. In other words, short wavelength geometry irregularities are more common in the sub-sections where high D_{R1} and D_{R2} fractal dimensions are calculated. In addition, with respect to these graphs, it can be said that both λ_1 and λ_2 are in the band range of 1–5 m and $\lambda_2 > \lambda_1$.

In the graphs where D_{R3} and D_{R4} values are high, the geometry changes close to the limit values specified in EN 13848–5 are clearly seen on the left (Fig. 6). In these graphs, most of the geometry irregularities are seen as medium length waves (3–25 m long) (λ_3 and λ_4). Nevertheless, these graphs also show short wavelength irregularities or short wavelength irregularities overlapping medium waves. In addition, irregularities in the detail graphs of D_{R3} and D_{R4} appear to form waves with rounded peaks and soft lines. With respect to these graphs, it can be said that $\lambda_4 > \lambda_3$

and both wavelength values are in the band range of 3–25 m. According to Fig. 6, it can be expressed that $\lambda_4 > \lambda_3 > \lambda_2 > \lambda_1$

Fig. 7 demonstrates the graphs of the sub-sections where the high D_{R1} , D_{R2} , D_{R3} and D_{R4} values are calculated for the longitudinal level parameter. In Fig. 7, it is seen that the situation in the alignment graphs (Fig. 6) emerges similarly. In the longitudinal level graphs, irregularities were observed in shorter waves in D_{R1} and D_{R2} values, whereas irregularities were observed in medium-sized waves in D_{R3} and D_{R4} values. Furthermore, according to Fig. 7, it can be stated that $\lambda_d > \lambda_c > \lambda_b > \lambda_a$.

When the fractal dimensions (D_{R1} and D_{R2}) of the alignment and longitudinal level graphs were calculated by using short length rulers, it was calculated that the fractal dimension values were higher in the graphs where the amount of short wavelength geometry irregularities were too much. Because the short rulers intersected more on small details. Smaller details became important as the ruler size became smaller and larger details became important as the ruler length increased. As the number of waves gener-

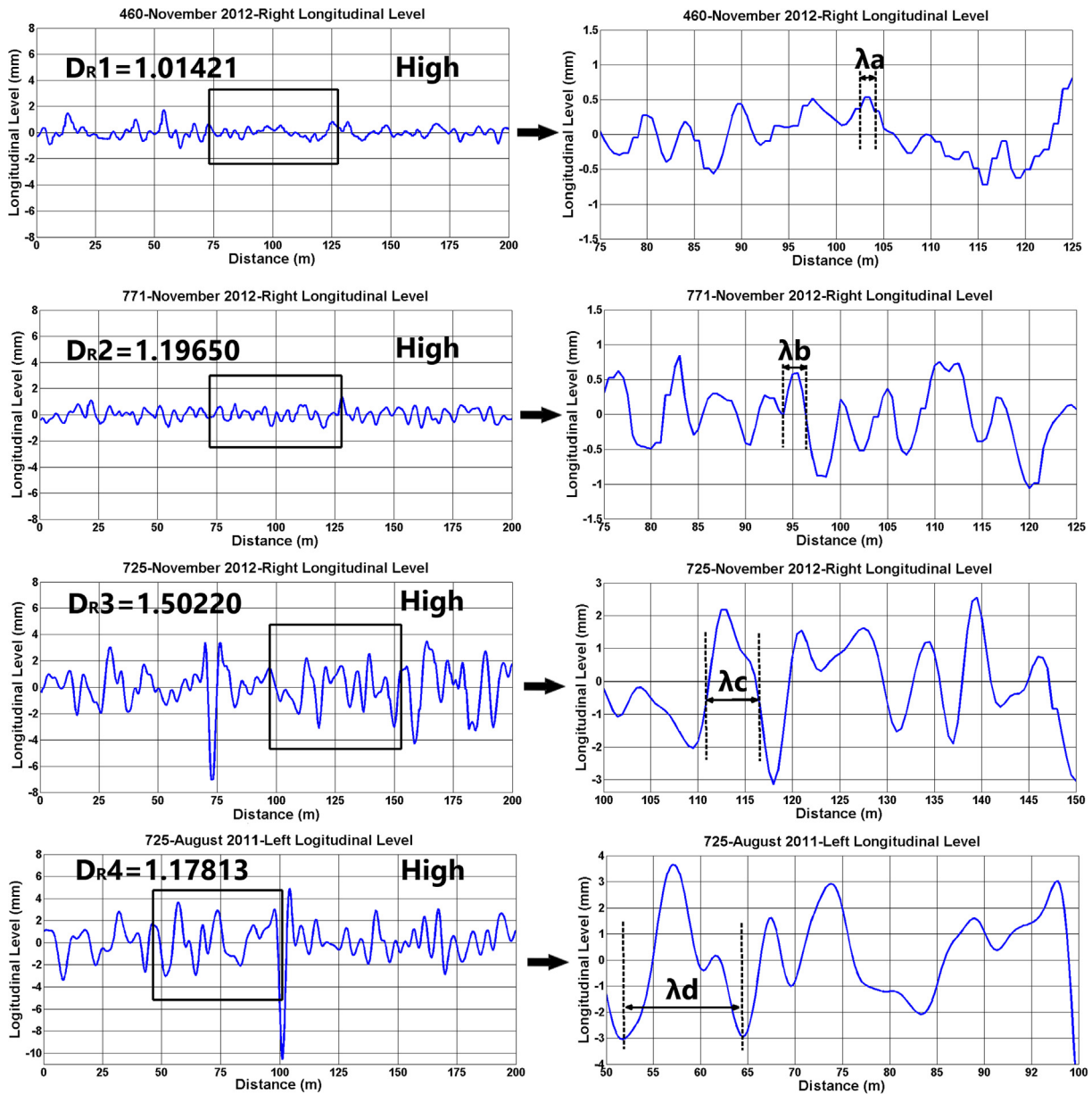


Fig. 7. High fractal dimension graphs for longitudinal level.

ated by the geometry irregularities increased in the graphs, it grew in fractal dimensions.

Fig. 8 shows the alignment graphs where small values of fractal dimensions are calculated. When these graphs are examined, it is seen that the geometry distortion is less in the sub-sections where small fractal dimension values are calculated. However, if the graphs are compared, it is noteworthy that the D_{R1} and D_{R2} graphs show irregularities in the medium wavelength, although there are no geometry defects close to the threshold values. In the D_{R3} and D_{R4} graphs, irregularities occur as small waves. Also, the detail graphs are composed of soft lines.

Fig. 9 points out the longitudinal level graphs where low value fractal dimensions are calculated. In this figure, it is seen that there are medium wavelength irregularities in track sub-section graphs where low D_{R1} and D_{R2} values are calculated. In particular, the D_{R2} graph shows a track geometry graph with defects exceeding the threshold values and medium wavelength irregularities although a low fractal dimension value is calculated. Also, as it is clearly seen

from this graph, the D_{R2} dimension is low because of the short wavelength irregularities. D_{R3} and D_{R4} graphs show the track sub-sections where short wavelength irregularities occur as in Fig. 8.

After calculating 4 different fractal dimension values for each of the examination sub-sections, the relationship between these fractal dimensions and the SD of longitudinal level and alignment, which is the quality indicator specified in EN-13848-6, was evaluated (Figs. 10–13).

The most significant relationship was found between D_{R3} dimensions and standard deviation values (Fig. 12). In particular, a very high correlation was obtained between D_{R3} values and standard deviation values for the alignment parameter ($R^2 = 0.91$ and $R^2 = 0.92$).

4. Discussion

The data used in this study was collected every 0.25 m by the track recording car. The frequency of sampling of track geometry

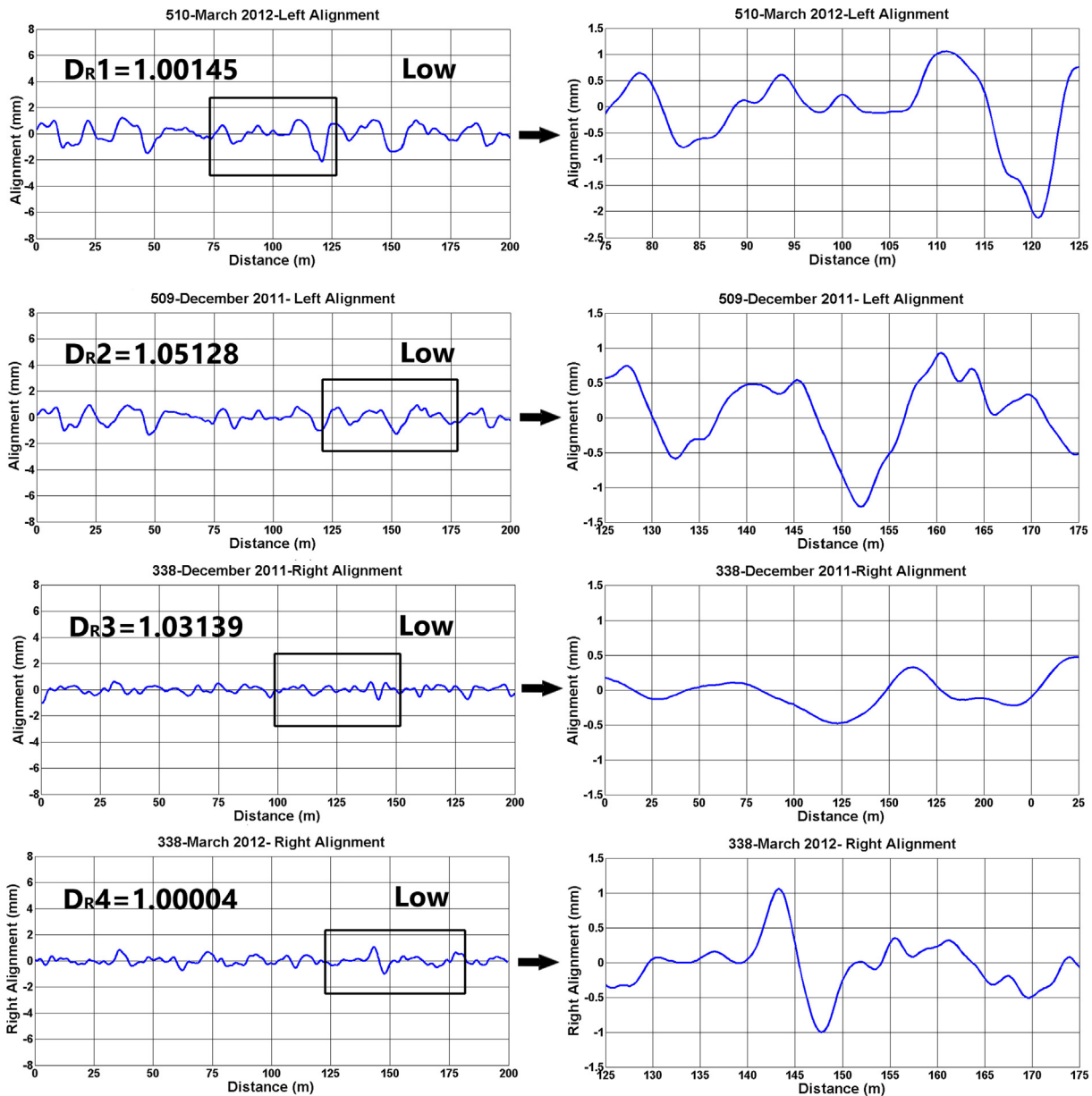


Fig. 8. Low fractal dimension graphs for alignment.

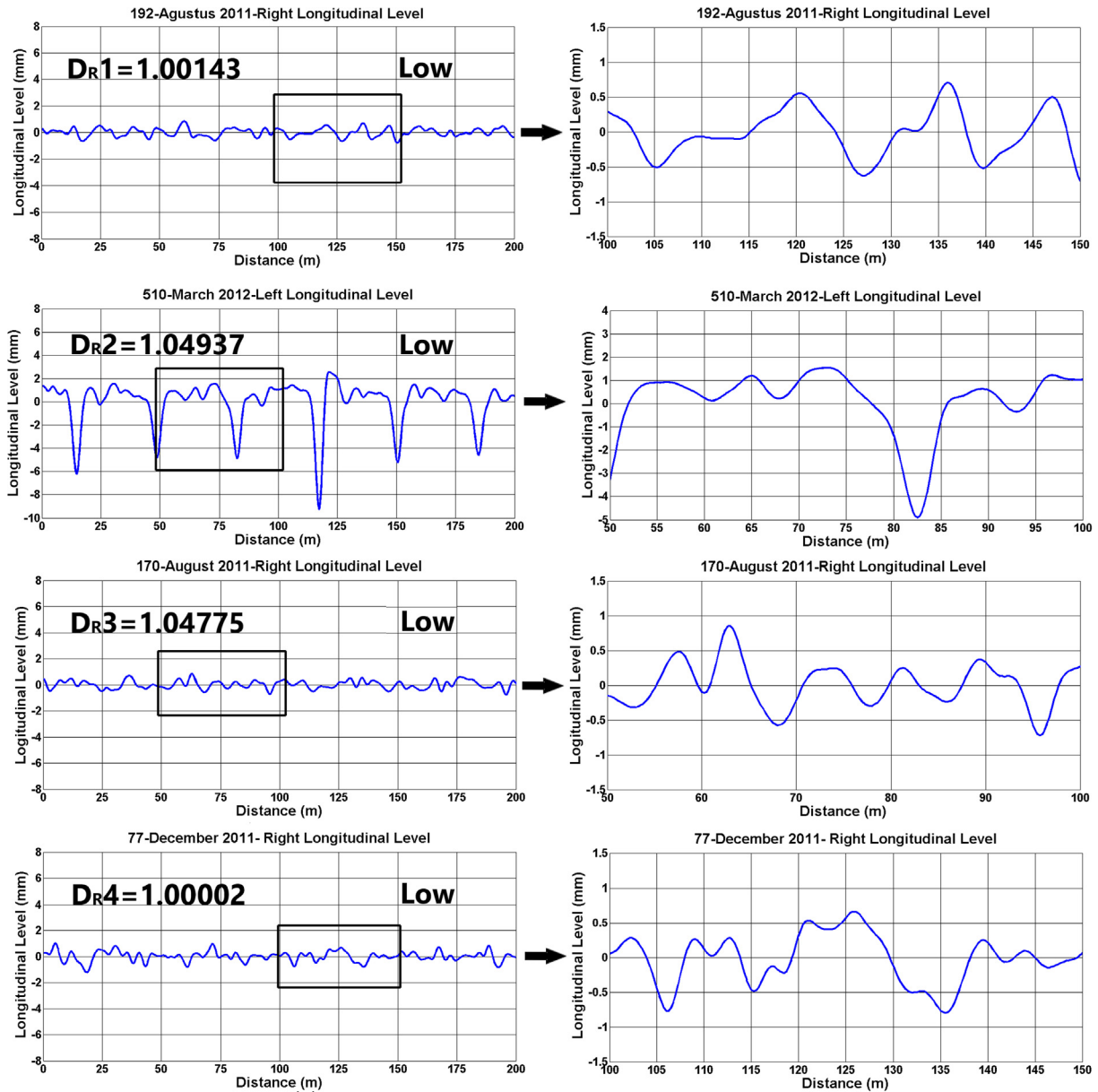


Fig. 9. Low fractal dimension graphs for longitudinal level.

data affects the appearance of the track geometry graph. If the data is not collected at frequent intervals, small details in track geometry may not be visible. Short wavelengths may not be obvious enough. In this case, if fractal dimensions are calculated using short length rulers, there will be no significant differences between the calculated fractal dimensions. Fractal dimension calculation does not make sense with short length rulers. In other words, the frequency of sampling of track geometry data affects the ruler length selection.

Fractal dimensions vary according to the length of the ruler used in the fractal dimension calculation. Fractal dimensions calculated using small ruler lengths take lower values, while the ruler length grows, it increases in fractal dimensions. Likewise, the range of fractal dimensions increases or decreases with the ruler length.

According to the results, when fractal dimensions are calculated using rulers suitable for the wavelength of the irregularities in the railway track geometry, these irregularities can be expressed numerically. SD- D_{R3} comparisons supports this view. SD relates to ride quality and passenger comfort and is used to quantify

mid-wavelength irregularities. As seen in Figs. 6 and 7, medium wavelength irregularities are much more in alignment and longitudinal level graphs with a high D_{R3} value. Similarly, medium wavelength irregularities were not observed in track sub-sections with low D_{R3} value (Figs. 8 and 9). Accordingly, the D_{R3} dimension may be an alternative to SD. In addition, the track geometry graphics can be compared by taking D_{R3} dimensions into consideration and maintenance priorities can be planned between the track sections.

No significant results were obtained from the SD- D_{R4} comparisons. According to this result, D_{R4} is not suitable for quantifying medium wavelength irregularities.

Short wavelength irregularities in the railway track are responsible for the dynamic forces applied and thus the track deterioration. According to Figs. 6 and 7, short wavelength irregularities are high in track geometry graphs with high D_{R1} and D_{R2} dimensions. SD- D_{R1} and SD- D_{R2} comparisons support this view. In these comparisons, there was no significant relationship between SD and D_{R1} and D_{R2} dimensions. Since SD is concerned with medium wavelength irregularities.

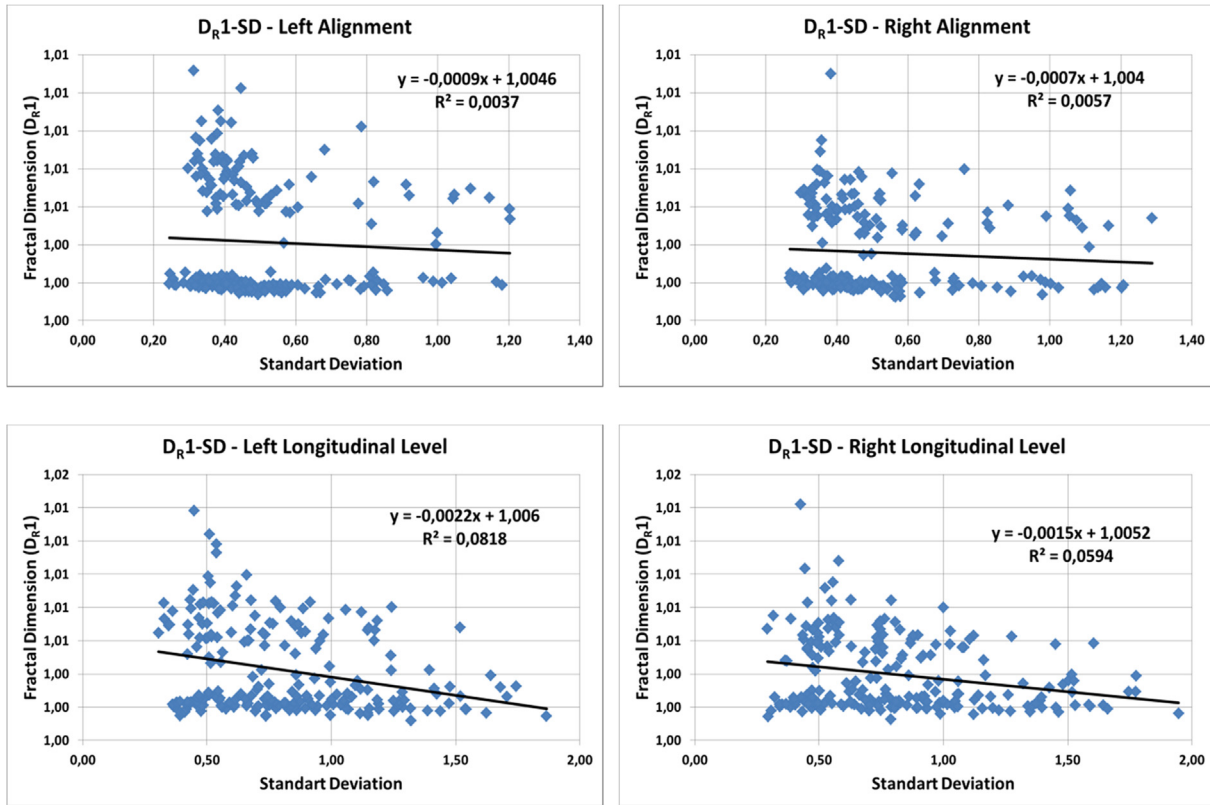


Fig. 10. D_R1-Standard Deviation comparison.

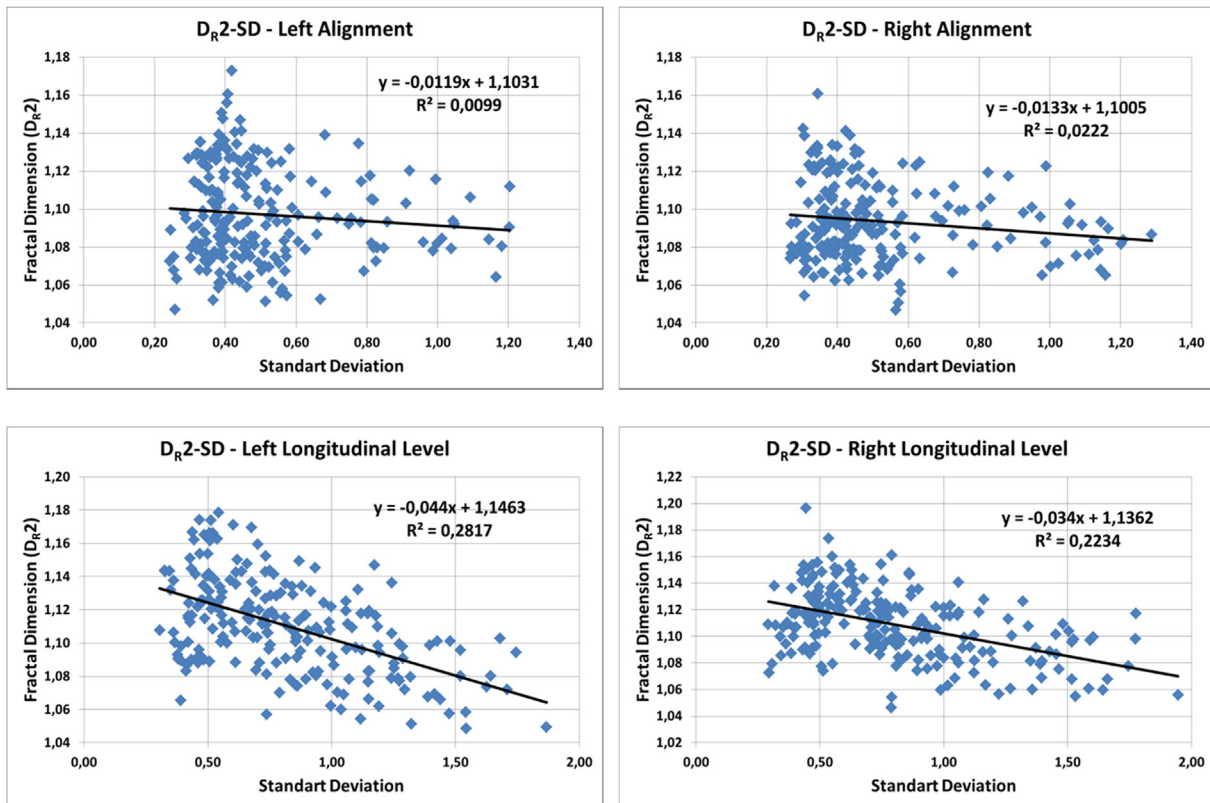


Fig. 11. D_R2-Standard Deviation comparison.

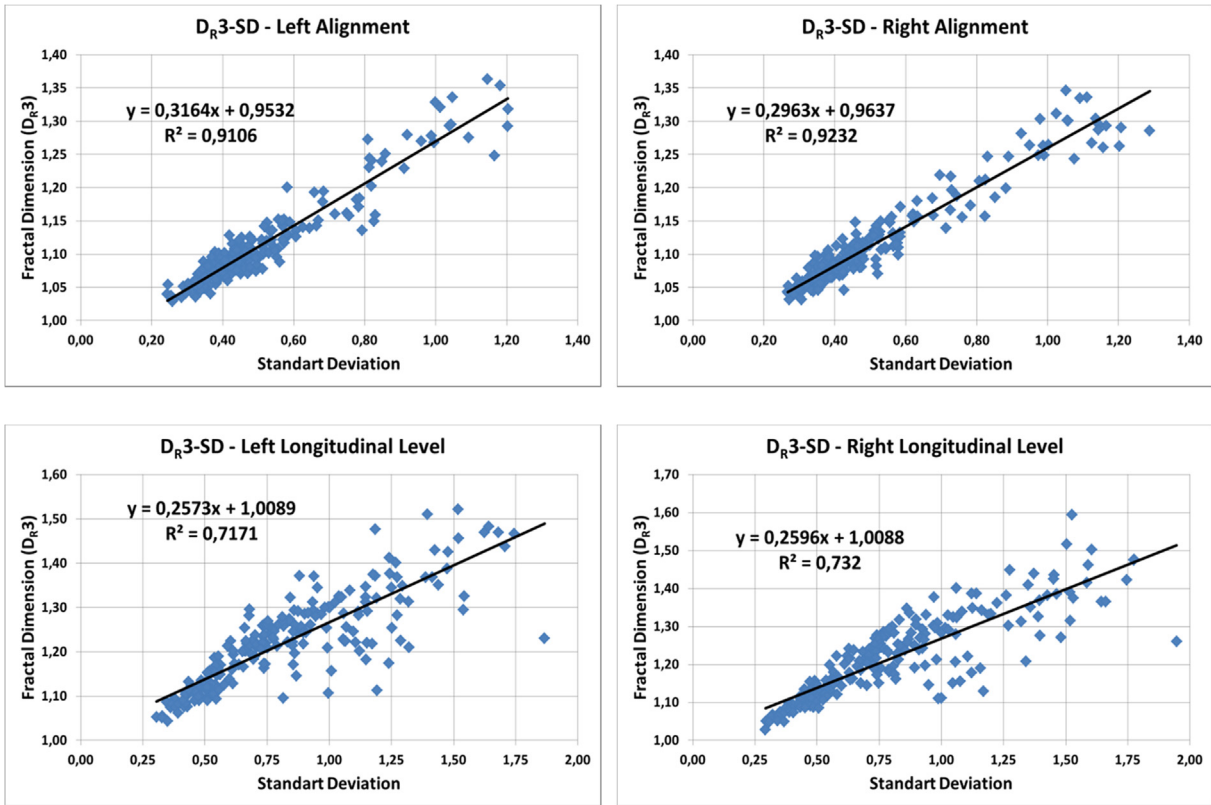


Fig. 12. D_R3-Standard Deviation comparison.

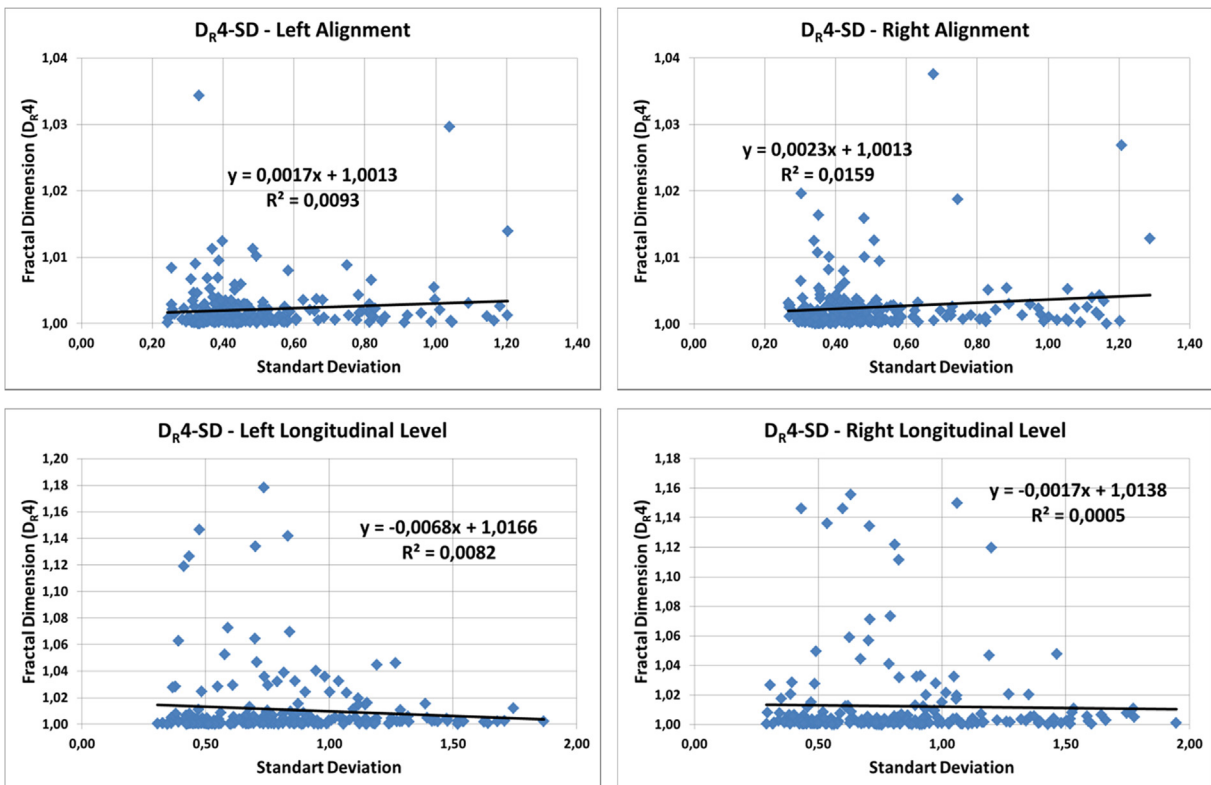


Fig. 13. D_R4-Standard Deviation comparison.

As seen from the graphics (Figs. 6–9), medium and short wavelength irregularities can be seen at the same time in the railway track subsections. In some subsections, medium wavelength irregularities and short wavelength irregularities were seen as combined. With the fractal dimension values obtained for each subsection, numerical data were obtained for both short and medium wavelength irregularities in that subsections. These values can be used in the interpretation of the transition from short wavelength irregularities to medium wavelength irregularities and thus in planning maintenance priorities between track subsections.

5. Conclusions

The indices commonly used to define track condition are usually derived from statistical approaches and focus on the amplitude of the defects, but generally neglect the wavelength of geometrical irregularities. The wavelengths of geometrical irregularities are important for track-vehicle interaction. Therefore, wavelength of geometrical irregularities should be taken into account in determining the track condition. For this purpose, a new approach based on fractal analysis that quantifies the wavelengths of track geometry irregularities is proposed in this study.

Since the standard track geometry graphics have different scales in horizontal and vertical planes, the graphic curve has a wavy pattern. The proposed approach focused on quantify this wavy pattern with fractal dimensions. First of all, HSR track was divided into sub-sections according to the data obtained from the track inspection studies and the geometry defects occurring in these sections were determined. In the continuation of the study, the alignment and longitudinal level graphs of these sub-sections were drawn by using track geometry inspection data. Then a new algorithm was developed for fractal analysis and a program was written. Fractal dimension calculations was made easily and quickly using this program. After determining the appropriate ruler lengths, 4 different fractal dimensions, D_{R1} , D_{R2} , D_{R3} and D_{R4} were proposed.

According to results, it was determined that the general irregularity of the track can be expressed numerically with D_{R3} fractal dimension and especially for the alignment parameter. This means the D_{R3} dimension may be an alternative to SD. Nevertheless, this proposal needs to be supported. As it is seen in the defect analysis, due to the high level of security measures, maintenance and repair works have been carried out continuously on the HSR track by taking the opinions of the train drivers and technical personnel without waiting for the track geometry inspections. This limited the amount of defects that could be observed and the track geometry pattern type. Therefore, it has not been determined how fractal dimensions may change in cases where different types of defects occur.

According to the previous studies, there is a significant relationship between mid-wavelength irregularities and passenger comfort. Therefore, it is expected that the passenger comfort will decrease in track sections where the D_{R3} dimension is high. However, this proposition should support acceleration measurements.

Another important result of this study is that short wavelength geometry changes can be expressed numerically. Short wavelength irregularities produce more vibration on axles and wheels. On the Ankara-Eskişehir HSR track, which is of great importance in terms of safety, detection of this type of vibration by train drivers while driving is often a sufficient reason for even maintenance and repair work. This makes it difficult to plan maintenance and repair work. Therefore, D_{R1} and D_{R2} dimensions can be used in making effective maintenance and repair plans. However, it has not been decided whether D_{R1} or D_{R2} is the appropriate parameter. Making this decision may be the subject of another study in the future.

Based on these results, the proposed fractal dimensions are good indicators for expressing numerically the irregularity of the track geometry. With the proposed fractal dimensions, both medium wavelength irregularities and short wavelength irregularities of any subsection can be quantified numerically. By observing the change of the proposed fractal dimensions in any track section, the tendency of the track geometry deterioration can be evaluated. In addition, by determining the importance coefficients of fractal dimensions, a single track quality index value can be derived from fractal dimensions. Thus, effective maintenance management plans can be made.

CRediT authorship contribution statement

Murat Vergi Tacirođlu: Conceptualization, Methodology, Investigation, Writing - original draft, Writing - review & editing, Visualization, Software. **Mustafa Karasahin:** Investigation, Resources, Supervision, Conceptualization, Writing - original draft. **Mesut Tiđdemir:** Investigation, Writing - original draft, Supervision. **Hakan Iřiker:** Writing - original draft, Software.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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