Mathematical Model of the Traveling Salesman Problem with Delivery and Draft Limit

Kadir Mersin*, Cemile Nur**

International Logistics and Transportation, Faculty Of Economics, Administrative And Social Sciences

Istanbul Gelisim University 34315 Avcilar, Istanbul, Turkey

**Computer Engineering, Engineering Faculty, Yalova University,

(kmersin@gelisim.edu.tr, cemile_nur@yahoo.com)

‡ Corresponding Author; Kadir Mersin Istanbul Gelisim University 34315 Avcilar, Istanbul, Turkey

+90 212 422 7000

kmersin@gelisim.edu.tr

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Abstract-In this paper, we determine the order of the ports to be reached with the help of graph theory and establish Hamilton cycles. We obtain sub-solutions and solution sets with the help of these cycles. Moreover, we prove some relations about the draft limit and the demand of the port. Finally we present some examples**.**

Keywords Traveling Salesman Problem, Draft Limit, Maritime Transportation.

1. Introduction

In maritime transportation, to determine the optimal route has a crucial importance. Many constraints affect the optimal route. Therefore, there are many works about determining the most optimal route in maritime transportation. One of the most interesting approaches is to transform the ship routing problem into the traveling salesman problem. Maritime routing and scheduling, draft limits have been considered in a lot of papers. For example Hennig [6], Song and Furman [10], Christiansen et al.(2011) but draft limit was not the main subject. The other area is vehicle routing problem. Several problems was described by combining routing and loading problems. Petersen and Madsen [8], Felipe et al.[5], Erdogan et al.[4] Cordeau et al.[2], Iori and Martello [7], Fagerholt and Christiansen [3] but none of them considered draft limit as a constraint. Finally Rakke J. et al.[9] approached this situation from a different perspective and transform this problem into travelling salesman problem. They built a model by accepting the ports as vertices.

The draft of a vessel is the distance between the water level and the base of the vessel. The draft limit is the height from the seabed to the water level. Each port has a draft limit. This means that vessels with values above a certain draft cannot enter the port. Some ports have low draft limits, causing the arriving vessels to run aground. For example, since the Kota Kinabalu bulk freight port in Malaysia has a draft limit of 9.2 meters, it is impossible for a fully loaded Panamax to reach the port with draft of about 12 meters. In this paper, we further develop a model and obtain sub-solution sets. We find the number of Hamilton cycles to be generated and in this way we obtain solution sets and develop a new approach. Moreover, we prove some relations about the draft limit and the demand of the port. (See Proposition 1, Proposition 2, Proposition 3, Corollary 1 and Corollary 2) Finally we present some examples. We are interested only in the delivery case of the vessels.

2. Graph Theory

A graph G is an ordered pair (V,E), where V is some set and E is a set of 2-point subsets of V. The elements of the set V are called vertices of the graph G and the elements of E edges of G. The number of edges of G containing the vertex v is denoted by the symbol deg(v) or |v|. The number deg(v) is called the degree of v in the graph G. A graph G is connected if there is a path between any two of its vertices. A graph G is said to be complete if every vertex in G is connected to every other vertex in G. Thus a complete graph G must be connected. The complete graph with n vertices is denoted by Kn. G and G' are called subgraph of K_n , If K_n=G∪G' . And G' is called complement of G.

A graph is said to be traversable if it can be drawn without any breaks in the curve and without repeating any edges, that is, if there is a path which includes all vertices and uses each edge exactly once. Such a path must be a trail (since no edge is used twice) and will be called a traversable trail. A Hamiltonian circuit in a graph G is a closed path that visits every vertex in G exactly once. (Such a closed path must be a cycle.) A graph G is called an Eulerian graph if there exists a closed traversable trail, called an Eulerian trail. If G does admit a Hamiltonian circuit, then G is called a Hamiltonian graph. There is no sufficent and necessery condition to have Hamilton circuit for any graph.

3. Draft Limit

Draft of a ship is the vertical distance between waterline and the bottom of the hull or keel. Every ports have a draft limit. It means, the vessels which has greater draft than the draft of a port, is not able to berth. The draft of a ship is depend on the weight on board so we can assume that draft limit is a function of the weight.

Let l_i, m_{l_i} and n_{l_i} be the port to be visited, the draft of the vessel at l_i port (when entering) and the draft limit of l_i port, respectively, where i=1…k We can write the draft as a function of the load, since the draft of the vessel is related to the load it carries. Based on the assumption that the amount of water sinking is 1 cm per TEU, we define the draft of the vessel (in terms of meters) as follows:

$$m_{l_i} = m_{l_i}(x_{l_i}) = m_0 + \frac{x_{l_i}}{100} \quad \forall i = 1 \dots k$$

where m_0 is the initial draft of the vessel without any load but with full fuel and x_{l_i} is the amount of load (in terms of TEU) on the vessel when entering l_i, port.

It is clear that the draft limit of the port should be greater than the draft of the vessel, i. e.,

$$m_{l_i} \geq n_{l_i} \forall i = 1 \dots k$$

Note that, since the fuel of the vessel decreases as it moves, there will be a small reduction in the initial draft of the vessel m_0. Therefore, the draft of the vessel m_{l_i} at l_i port also decreases in small amounts as it moves. Without loss of generality we assume that

$n_{l_i} \geq n_{l_{i+1}} \forall i = 1 \dots k$ if $\{l_1, l_2, \dots . l_k\}$ is the set of ports. For convenience, we use the pair $< n_{l_i}, d_{l_i} >$ where $d_{l_i}$is the draft effect of the demand of l_i port (in terms of meters). Since there is no delivery at the initial port l_1, we have $d_{l_1} = 0$. It is clear that, in order for the ship to be able to enter the port, the draft limit of the port should be greater than the draft effect of the total load onboard. In other words,

$$\exists n_{i_0} \ni \sum_{i=0}^{k} d_{l_i} \leq n_{l_{i_0}} \quad , i_0 \in \{1,2, \dots, k\}$$

where d_{l_0} is the draft effect of the empty ship with full fuel.

Our aim is to establish an appropriate Hamilton cycle, considering each port as a vertex, according to the draft limits and demands of the ports for the amount of cargo loaded on the vessel. A Hamilton cycle is a path through a graph that starts and ends at the same vertex and visits every other vertex exactly once. If there is a Hamilton cycle containing all the ports $\{l_1, l_2, \dots . l_k\}$, then we say that this set is a solution set. If there is no Hamilton cycle containing all ports, then we break down the graph into the subgraphs containing Hamilton cycles and we say that each subgraph forming a Hamilton cycle is a sub-solution set. For every partition i we denote this sub-solution set by S_i. If there are t partitions then we have

$$S = \bigcup_{i=1}^{t} S_i$$

and the most appropriate partition is the chain $|S_1| \geq |S_2| \geq \cdots \geq |S_t|$ where $|S_i|$ denotes the number of elements of the set S_i. The number of partitions gives us the number of vessels we need for shipment.

Let W_{l_i} be the draft effect of the load on the vessel when entering l_i port. Clearly, if S={l_1,l_2,….l_k } is a solution then

$$W_{l_1} = \sum_{i=1}^{k} d_{l_i}$$

$$W_{l_{i+1}} = W_{l_i} - d_{l_i} \ \forall i = 1 \dots k-1$$

If $\hat{S} = \{l_1, l_2, \dots . l_r\}$ is a subsolution then

$$W_{l_{i+1}} = W_{l_i} - d_{l_i} \ \forall i = 1 \dots r-1$$

where r≤k. If r=k then $\widehat{(\dot{S})} = \dot{S}$

Now we state some of our main results. From now on, we assume that the draft effects of the empty vessel and fuel are included in the draft effect of the total load onboard.

Proposition 3.1. Let Wi be the amount of the cargo on board while berthing the port of i. If $W_1 = \sum_{i=1}^{k} n_i$ and $W_i = W_{i-1} - n_{i-1}$; ∀i∈{2,3,…,r} then

$\tilde{S}$={p_1 , $p_2, p_3, \dots p_r$} is a sub-solution. Moreover, if r=k, then S= $\tilde{S}$

Proposition 3.2. S= $\tilde{S}$ holds if and only if $L_i \leq m_i$

Proof: Straight forward.

Proposition 3.3. Number of port which has same demand and draft limit is at most 1.

Proof: Let ⟨m│m⟩ and ⟨n│n⟩ are two ports in solution set. So, amount of the cargo on board is n+m+k (k>0). In this condition vessel can not berth these two ports. The ship has to transport n+m+k+1 cargo to berth ⟨m│m⟩ and ⟨n│n⟩ but this is contradiction. Because we assume that the amount of the cargo is m+n+k.

Definition 3.5. Let x be a real number then [x] is the maximum integer which is less than x. We will define a function which is called nearest integer function;

f(x):R→Z

f(x)=[x]

Proposition 3.6. Let ⟨m│n⟩ be the draft limit-demand pair. Then the number of the port which has the same draft limit-demand pair is less than [m/n].

Proof: Let number of ⟨m│n⟩ pair is "k" in the solution set. Then, amount of the cargo on board is at least k.n pieces and k.n can not be greater than draft limit because ports belong to solution set. So k.n<m is hold.

Then $k \leq \frac{m}{n}$ and the greatest integer is $\left[\frac{m}{n}\right]$ which holds the inequality.

Proposition 3.7. Let ⟨m│n⟩ be the draft limit-demand pair. Then the maximum number of the port which has the same demand is $\left[\frac{\max_{i\in I}\{m_i\}}{n}\right]$.

Proof: Straightforward.

Corollary 3.4. The graph does not have Hamilton circuit if the number of ports which have same draft limit-demand pair is more than one. Than, the number of sub-solutions are equal the number of ports which have this property.

Corollary 3.5. If the number of ports which have same draft limit-demand pair ⟨m│n⟩ is greater than [m/n], corollary 2.9 is provided.

4. Mathematical Modelling

First we should assign a decision parameter ε_{ij} to form the modelling. This parameter will show that which two elements of the set consisting of the ports as vertices are in the solution set. The decision parameter is defined as follows:

$$\varepsilon_{l_i l_j} = \begin{cases} 1, & if\ there\ is\ a\ path\ connecting\ l_i\ and\ l_j\ ports \\ 0, & otherwise \end{cases}$$

If there are t Hamilton cycles that are formed to find the solution set, then our modelling is as follows:

$$\sum_{l_i \in S_r} \varepsilon_{l_i l_j} = 1\ ;\ l_j \in S_r \tag{1}$$

$$\sum_{l_j \in S_r} \varepsilon_{l_i l_j} = 1\ ;\ l_i \in S_r \tag{2}$$

$$\sum_{l_i \in S_r} W_{l_i l_1} = 0\ ;\ \sum_{l_j \in S_r} W_{l_1 l_j} = \sum_{l_i \in S_r} d_{l_i} \tag{3}$$

$$0 \leq W_{l_i l_j} \leq n_{l_j} . \varepsilon_{l_i l_j} \tag{4}$$

$$\sum_{j=1}^{i-1} \varepsilon_{l_j l_i} = \sum_{j=i+1}^{k} \varepsilon_{l_i l_j}\ ; \tag{5}$$

$$l_i, l_j \in S_r\ ; |S_r| \leq k\ , \varepsilon_{l_i l_j} = \varepsilon_{l_j l_i} = 0\ ;$$

$$(l_i, l_j) \notin S_r \times S_r \tag{6}$$

$$\bigcup_{r=1}^{t} S_r = S \tag{7}$$

$$\bigcap_{r=1}^{t} S_r = \{l_1\} \tag{8}$$

where r = 1,2,… t and $W\_(l_i l_j)$ denotes the draft effect of the total load onboard when the vessel goes from $l_i\ to\ l_j$. In this modelling (1) and (2) express that the degree of each port is 1 in the cycle created .The condition (3) states that the ship returns to the initial port l_1 empty and has enough load to reach all the ports in the solution set at the initial port l_1. (4) indicates that the load onboard is suitable for the draft limit of the arrival port. (5) means that the ship visits again none of the visited ports from sub-solutions. Finally, (6), (7), and (8) indicate that the initial port is common to all sub-solutions and that the union of all sub-solutions is equal to the solution set.

Example 1 Let us consider the following table.

Table 1. Ports with draft limits

Port	Demand (TEU)	Draft Limit (meters)
X	0	16
A	300	11
B	300	11
C	400	13
D	200	18
E	300	10

In the table 1 above, for a ship with the initial port X, the demands and the draft limits of the ports to be visited are given. Let us create a Hamilton cycle containing these ports based on the assumption that the amount of sinking water per TEU is 1 cm.

First we need to create the draft limit-the draft effect of the demand pairs, i.e. $\langle l_i \mid d_j \rangle$ pairs.

X: $\langle 16 \mid 0 \rangle$

A: $\langle 11 \mid 3 \rangle$

B: $\langle 11 \mid 3 \rangle$

C: $\langle 13 \mid 4 \rangle$

D: $\langle 18 \mid 2 \rangle$

E: $\langle 10 \mid 3 \rangle$

In accordance with our acceptance, the ship has to return to its starting port empty only by delivering. First of all, if we examine the draft limit-the draft effect of the demand pairs, we see that the ports A and B have the same pair. If there is a solution set, then the number of the ports with the pair $\langle 11 \mid 3 \rangle$ will be at most 3 according to Proposition 2. There is therefore a possibility that there exists a Hamilton cycle. In other words, there is a possibility that our solution set is unique. When we look at our example, the only port is D which the ship can go from the X port. Then the second vertex of the Hamilton cycle will be the port D. When leaving the port D, the total amount of demand on the ship will be 1300 TEU. In this case, the only port to go is the port C. Since the draft limit is equal to the draft effect of the total demand, there is a possibility that the ship may run aground, but the amount of fuel consumed on the road will reduce the draft of the ship so that the ship will be able to enter the port easily. We can take the port C as the third vertex of the Hamilton cycle. The total demand on the ship when leaving the port C is 9000 TEU. This means that the ship will be able to enter every port with a draft limit of more than 9 meters. Since the draft limits of all ports in the table are greater than 9 meters deep, there is no obstacle to establish the Hamilton cycle. In total 3! = 6 solution sets (Hamilton cycles) can be established for the 3 ports left. One of the solution sets is

X →D→ C→ A→ B

Thus, only one ship will be sufficient for delivery. The matrix representation of this graph is given in Figure 1.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Figure1. Matrix representation of example 1

Note that, if $\{l_1, l_2, \dots . l_k\}$ is a solution set, then the matrix representation of the graph $l_1 \to l_2 \to \cdots \to l_k \to l_1$ is given in Figure 2.

$$A_{k\times k} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Figure 2. Matrix representation of solution set.

It is clear from this matrix that $a_{j,j+1} = a_{k,1} = 1$ and the other components of the matrix are zero, for j=1,2,…k+1

Example 2 Let the following table be given in Table 2.

Table 2. Ports with draft limits-2

Port	Demand(TEU)	Draft Limit (meters)
X	0	25
A	200	9
B	300	14
C	200	9
D	200	9
E	300	17
F	800	8
G	200	11
H	200	9

In the table 2, there are 5 ports with the same demand, 200 TEU, and also there are 4 ports with the same $\langle n_{l_i} \mid d_{l_i} \rangle$ pair, $\langle 9 \mid 2 \rangle$ Moreover, there is a port such that $n_{l_i} = d_{l_i}$. Clearly, there is no Hamilton cycle containing all ports according to the draft limits. For this reason, we need more than one ship. First, we sort the ports according to the draft limits as follows:

$$X : l_1;\ E : l_2;\ B : l_3;\ G : l_4;$$

$$A : l_5;\ C : l_6;\ D : l_7;\ H : l_8;\ F : l_9$$

Let us consider the partitions

$$S_1 = \{l_1, l_2, l_3 l_4 l_9\}$$

$$S_2 = \{l_1, l_5, l_6 l_7 l_8\}$$

It is clear that, there are Hamilton cycles for both S_1 and S_2 and therefore S_1 and S_2 are sub-solution sets. Hence it is necessary two ships for delivery. Another possible partitions are given in Table 3

Table 3. Sub-solutions sets

$S_1 = \{l_1, l_2, l_3 l_4 l_9\}$ $S_2 = \{l_1, l_5, l_6 l_7 l_8\}$ (we built)	$S_1 = \{l_1, l_4, l_5 l_6 l_7 l_8\}$ $S_2 = \{l_1, l_2, l_3 l_9\}$
$S_1 = \{l_1, l_3, l_5, l_6, l_7, l_8\}$ $S_2 = \{l_1, l_2, l_{4,} l_9\}$	$S_1 = \{l_1, l_2, l_4, l_5, l_6, l_7, l_8\}$ $S_2 = \{l_1, l_3, l_9\}$
$S_1 = \{l_1, l_2, l_5, l_6, l_7, l_8\}$ $S_2 = \{l_1, l_3, l_{4,} l_9\}$	$S_1 = \{l_1, l_2, l_3, , l_5, l_6, l_7, l_8\}$ $S_2 = \{l_1, l_4, l_9\}$
$S_1 = \{l_1, l_3, l_4, l_5, l_6, l_7, l_8\}$ $S_2 = \{l_1, l_2, l_9\}$	$S_1 = \{l_1, l_2, l_3, l_4, l_5, l_6, l_7, l_8\}$ $S_2 = \{l_1, l_9\}$

Since there are many route options, the most optimal route can be selected by considering the distance. Note that, for each partition including the ports A, C, D and H, there are 4! = 24 route options since they have the same $\langle 9 \mid 2 \rangle$ pair. In total, there are 24x8 = 192 route options according to the draft limits.

5. An algorithm for the optimal route

It can be seen from Example 2 that there are many route options according to the draft limits. Now we give an algorithm for the optimal route in terms of the cost of the route and draft limit. For this, we use the nearest neighborhood algorithm. (Anderson, 2000)

Step1. Sort the ports according to the draft limits from large to small. The initial port will be l_1 and $n_{l_i} \geq n_{l_{i+1}}$ ∀i=1,2,…k-1, for the set of ports $\{l_1, l_2 \dots l_k\}$

Step2. For t Hamilton cycles, in other words, for t ships for delivery, each ship goes to the nearest convenient (according to the draft limits) port which has the lowest cost from the initial port l_1.

Step3. Each ship goes to another convenient port which has the lowest cost from the second port if any, otherwise the ship returns to the initial port l_1.

Step4. Repeat Step3 until there are no ports left.

In this paper, we have determined the order of the ports to be reached with the help of graph theory and have established Hamilton cycles. We have obtained sub-solutions and solution sets with the help of these cycles and have developed a new approach. Also, we have developed a modelling by the traveling salesman problem with draft limits. We have found the number of Hamilton cycles to be generated and in this way we have determined the number of vessels we need for shipment. Moreover, we have proved some relations about the draft limit and the demand of the port which gives an idea for the optimal route of the ship. Finally we have presented some examples.

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