# Modeling the Shear Strength of Reinforced Aerated Concrete Slabs via Support Vector Regression 

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#### Abstract

Autoclaved aerated concrete (AAC) attracts attention as it provides superior material characteristics such as high thermal insulation and environmentally friendly properties. Apart from non-structural applications, AAC is being considered as a structural material thanks to its characteristics such as lighter weight compared to normal concrete, resulting in lower design costs. This study focuses on the feasibility of support vector regression (SVR) in predicting the shear resistance of reinforced AAC slabs. An experimental dataset with 271 data points extracted from eleven sources is used to develop models. Based on random selection, the dataset is divided into two portions, $75 \%$ for model development and $25 \%$ for testing the validity of the model. Two SVR model types (epsilon and $N u$ ) and four kernel functions (linear, polynomial, sigmoid and radial basis) are used for model development and the results of each model and kernel type is presented in terms of correlation coefficient ( $R^{2}$ ) and mean squared error ( $M S E$ ). Results show that epsilon model type with radial basis function yields the best SVR model.


Keywords Autoclaved aerated concrete, reinforced concrete slab, shear strength, support vector regression, modelling.

## 1. Introduction

Autoclaved aerated concrete (AAC) is made of cement or lime mortar which contains air voids entrapped in the matrix by means of an expansion agent. AAC has been used in the construction industry for non-structural and structural applications since mid-1920s. The main property of AAC is high porosity, i.e., up to above $70 \%$ of the volume contains air voids, resulting in lower density which minimizes the design cost [1]. AAC is considered to be environmentally friendly material as it reduces $70 \%$ and $40 \%$ energy per material volume as compared to normal concrete and bricks, respectively. It also provides high thermal insulation [2, 3].

Production of AAC panel elements with reinforcement can offer an alternative for low-rise precast construction. $60 \%$ of new building constructions in Europe are built with different types of AAC elements [4]. In the housing industry
in China, reinforced AAC materials for exterior walls are preferred to other materials [4].

Shear resistance of reinforced normal concrete or AAC slabs without shear reinforcement is a complex phenomenon. It is known that the shear resistance depends not only on the concrete properties but also on the shear-span-to-depth ( $\mathrm{a} / \mathrm{d}$ ) ratio as well as the presence of tensile reinforcement (Fig. 1). Aroni and Cividini (1989) proposed a formulation (Eq. 1a, Eq.1b) for the shear strength of reinforced AAC slabs with a modification to the formulation available for normal concrete slabs [5]. Fig. 2 shows a typical shear resistance test setup of reinforced AAC slab.


Fig. 1. Schematic of typical test setup
$\tau_{u}=0.035 f_{c}+1.163 \rho(d / a)-0.053$ within the normal
range
$\tau_{u}=0.039 f_{c}+0.82 \rho(d / a)-0.075$ outside the normal
(1b)
where $\tau_{u}$ is the ultimate shear stress in $\mathrm{MPa}\left(\tau_{u}=V_{u} / b d\right), f_{c}$ is the compressive strength of AAC in MPa, $\rho$ is reinforcement ratio $\left(100 A_{s} / b d\right), d$ is the effective depth in mm , a is the shear span in mm .


Fig. 2. Test setup
In this study, a novel machine learning based regression method, namely support vector regression, is implemented to produce predictive models for the shear resistance of reinforced AAC slabs.

## 2. Experimental Data

The experimental data consist of 271 data points extracted from previously published papers [6-15]. Table 1 summarizes the origins and product types for the tests. All data points were included in the modeling process. Data inputs are $f c$ (compressive strength), $d / a$ (span-to-depth ratio) and $\rho$ (reinforcement ratio), the output is $\tau$ (ultimate shear stress, $V / b d$ ). Table 2 presents the statistical variations of input and output parameters. Some specimens contained compression reinforcement consisting of two or three bar. Possible contributions of these bars in shear strength have been neglected.

Table 1 References and types of test product

| Series <br> No. | Reference | Product type |
| :---: | :--- | :--- |
| 1 | Bernon [14] (France) | Siporex |
| 2 | Blaschke [13] (Germany) | Ytong |
| 3 | Briesemann [12] (Germany) | Hebel |
| 4 | Cividini [11] (Yugoslavia) | Siporex, Ytong |
| 5 | Dalby [10] (Sweden) | Siporex |
| 6 | Edgren [10] (Sweden) | Siporex |
| 7 | Kanoh '66 [9] (Japan) | Siporex |
| 8 | Kanoh '69 [8] (Japan) | Hebel |
| 9 | Matsamura [7] (Japan) | ALC |
| 10 | Newarthill [6] (UK) | Siporex |
| 11 | Regan [15] (UK) | Durox |

Table 2 Statistics of experimental data

|  | $f c$ <br> $(M P a)$ | $d / a$ | $\rho$ | $(M p a)$ |
| :--- | :---: | :---: | :---: | :---: |
| Minimum | 2.3 | 0.08 | 0.12 | 0.107 |
| Maximum | 7.8 | 0.766 | 1.349 | 0.836 |
| Mean | 3.78 | 0.24 | 0.41 | 0.24 |
| Standard deviation | 1.31 | 0.16 | 0.26 | 0.14 |
| Coeff. of variation | 0.35 | 0.66 | 0.62 | 0.56 |

## 3. Support vector machines

Support vector machines (SVMs) were first identified by Boser et al. (1992) is an artificial intelligence learning technique developed to solve the classification problem [16]. However, researchers began using SVM to solve regression problems, and this method was named support vector regression (SVR).

SVM has performed well in many applications such as text analysis, face recognition, image processing and bioinformatics, as well as a strong digital basis in statistical learning theory. This shows that SVM is one of the most modern methods of machine learning and data mining, along with other methods such as neural networks and fuzzy systems [17].

### 2.1. Support vector regression (SVR)

In SVR, the main purpose is to obtain a function whose actual output value is estimated with the maximum deviation of epsilon and to get two parallel planes for this function. The distance between these planes must be minimized. [18].

For the training data set presented in SVR, the main objective is to find a function with the difference from specific target. At the same time, the function should be flattest with errors less than a certain amount without excess deviation [18]. The (linear) $\varepsilon$-insensitive loss function $L(x, y$, $f$ ) is described as
$L^{\varepsilon}(x, y, f)=|y-f(x)|_{\varepsilon}=\left\{\begin{array}{cc}0 & i f|y-f(x)| \leq \varepsilon \\ |y-f(x)|-\varepsilon & \text { otherwise }\end{array}\right.$
where $f$ is a real-valued function on a $x$ and the quadratic $\varepsilon$ insensitive loss is defined by
$L_{2}^{\varepsilon}(x, y, f)=|y-f(x)|_{\varepsilon}^{2}$

Fig. 3 demonstrates the linear and quadratic $\varepsilon$-insensitive loss function for zero and non-zero $\varepsilon$.


Fig. 3 The form of linear and quadratic $\varepsilon$-insensitive loss function for zero and non-zero $\varepsilon$.

The loss function defines the accuracy performance. Performing linear regression in the high-dimension feature space by the use of $\varepsilon$-insensitive loss function, SVM attempts to reduce the model complexity by performing the minimization of $\|\omega\|^{2}$. By introducing slack variables $\xi_{j} \xi_{i}^{*} i=1, . . n$

$$
\begin{gather*}
L(y, f(x, \omega))=|y-f(x)|_{\varepsilon}^{2} \\
L_{2}^{\varepsilon}(x, y, f)=|y-f(x)|_{\varepsilon}^{2} \tag{3c}
\end{gather*}
$$

to determine the deviation of training data outside $\varepsilon$-zone. Following formulation is implemented for the minimization of SVM regression:

$$
\begin{equation*}
\frac{1}{2}\|\omega\|^{2}+c \sum_{i=1}^{n}\left(\xi_{i+} \xi_{i}^{*}\right) \text { subject to } \xi_{j,} \xi_{i}^{*} i=1, . . n \tag{3d}
\end{equation*}
$$

$$
\begin{equation*}
\xi_{j}, \xi_{i}^{*} i=1, . . n \tag{3e}
\end{equation*}
$$

The solution of this optimization problem can be found by transforming it into the dual problem:

$$
\begin{gather*}
f(x)=\sum_{i=1}^{n_{5 v}}\left(\alpha_{j}-\alpha_{i}^{*}\right) K\left(x_{j}, x\right)+b \text { subject to }  \tag{3f}\\
0 \leq \alpha_{i}^{*} \leq C, 0 \leq \alpha_{j} \leq C
\end{gather*}
$$

where $n_{s v}$ is the number of support vectors (SVs), $a_{i}{ }^{*}$ and $a_{j}$ are the Lagrange multipliers and $K\left(x_{j}, x\right)$ is a kernel function and $b$ is the bias term. The generalization of SVM depends on the appropriate settings of meta-C, $\varepsilon$, and kernel parameters. Available software applications generally have the option for manual specification of meta-parameters [19].

The model complexity and the degree, to which deviations larger than $\varepsilon$ are tolerated, are controlled by a parameter $C$ controls in optimization formulation. Parameter $\varepsilon$ describes the width of $\varepsilon$-insensitive zone, which is utilized to fit the training data. Value of $\varepsilon$ can affect the number of support vectors used to form the regression function. On the other hand, greater $\varepsilon$-insensitive values cause more 'flat' predictions. Although in different ways, both $C$ and $\varepsilon$ values affect model complexity (flatness) [19].

Several kernel functions are used in machine learning. Four functions used in this study are:

Linear function:

$$
\begin{equation*}
K\left(x_{i}, x\right)=x_{i} x \tag{4a}
\end{equation*}
$$

Polynomial function:

$$
\begin{equation*}
K\left(x_{i}, x\right)=\left(x_{i}(x+1)\right)^{d} \tag{4b}
\end{equation*}
$$

Radial-based function:

$$
\begin{equation*}
K\left(x_{i}, x\right)=\exp \left[-\frac{\left(\mathrm{x}_{\mathrm{i}}-x\right)\left(\mathrm{x}_{\mathrm{i}}-x\right)}{2 \sigma^{2}}\right] \tag{4c}
\end{equation*}
$$

Sigmoid function:

$$
\begin{equation*}
K\left(x_{i}, x\right)=\tanh \left(x_{i}(x+1)\right) \tag{4d}
\end{equation*}
$$

where $x_{i}$ and $x$, are the training and test inputs, respectively, $\sigma$ is the Gaussian kernel function and $d$ is the polynomial degree of kernel function.

## 4. Model Development

Experimental data (three inputs and one output) is divided into two portions, i.e., $75 \%$ of the data is used as model training set, $25 \%$ is used for testing the validity of the model. SVR models are developed by optimizing the meta parameters $C$ and $\varepsilon$ or $N u$, by performing a grid search along a pre-specified range. The model with best correlation coefficient ( $R^{2}$ ) is selected for each model type and kernel function. Correlation coefficient $\left(R^{2}\right)$ measure the relationship between predicted and experimental data, in
which $R^{2}=1$ means significant correlation and $R^{2}=0$ means no correlation. Eq. 5.1 and Eq. 5.2 are used for calculating correlation coefficient ( $R^{2}$ ) and mean squared error (MSE), respectively. Fig. 4 shows the correlation coefficient $\left(R^{2}\right)$ values for eight SVR models developed using two model types and four kernel functions. SVR models developed with Radial Basis kernel appear to yield better fitting results as compared to other kernel types. Epsilon model type with radial basis kernel gives the best correlation coefficient (total set: 0.936 , training set: 0.945 , testing set: 0.901 ).

$$
\begin{gather*}
R^{2}=1-\left(\frac{\sum_{i=1}^{N}\left(o_{i}-t_{i}\right)^{2}}{\sum_{i=1}^{N}\left(o_{i}-o^{\prime}\right)^{2}}\right)  \tag{5.1}\\
M S E=\frac{\sum_{i=1}^{N}\left(o_{i}-t_{i}\right)^{2}}{N} \tag{5.2}
\end{gather*}
$$

where $o_{i}$ is the experimental value of $i t h$ data, $t_{i}$ is the predicted value of ith data, $N$ is the number of data used for training and testing of SVR models.


Fig. 4 Correlation coefficient of SVR models
Fig. 5 shows mean squared error (MSE) values calculated for each SVR model type, using Eq. 5.2. SVR models produced with sigmoid kernel appear to yield significantly large errors while models with radial basis kernel produces less MSE. Table A.1. lists the support vectors generated by the SVR-Eps-Rad model.


Fig. 5 Mean squared error of SVR models
Fig. 6 compares the experimental and estimated values of SVR-Eps-Rad model both for training and testing datasets.


Fig. 6 Experimental data versus predictions of SVR-Eps-Rad model

According to [20], if the correlation coefficient $R^{2}$ is greater than 0.8 and the error values are at a desirable range, there is a strong correlation between predicted and real values. Regarding Fig. 7, proposed SVR-Eps-Rad model has a $R^{2}$ value of 0.931 for whole set and the error is acceptable, as seen in Fig. 5.


Fig. 7 Comparison of predicted values and experimental values of Ultimate Shear Stress (MPa)

## 5. Conclusion

This study analyzes the feasibility to use support vector regression method to propose a predictive model for ultimate shear stress of reinforced aerated concrete. Different model types (epsilon and $N u$ ) and kernel function types (linear, sigmoid, polynomial, radial basis) are used for model development to analyze the feasibility. An experimental dataset with 271 data points is implemented to develop models. Dataset is divided into two portions, $75 \%$ for model development and $25 \%$ is for testing the validity of the model, based on random selection. Each model is analyzed statistically to determine the prediction performance. For this, mean squared error ( $M S E$ ) and correlation coefficient $\left(R^{2}\right)$ are used. For epsilon model type, $R^{2}$ values for total set are $0.865,0.865,0.871$ and 0.936 for linear, sigmoid, polynomial and radial basis kernel types, respectively. On the other hand, for $N u$ model type, $R^{2}$ values for total set are $0.869,0.862,0.871$ and 0.931 for linear, sigmoid, polynomial and radial basis kernel types, respectively. Hence, SVR model based on epsilon model type and radial basis kernel function gives the best correlation coefficient values. Sigmoid kernel based models yield largest MSE values while radial basis kernels produce less MSE. Finally, the results confirm that support vector regression (SVR) method has the advantage to be easily applied and yield reasonably accurate prediction performance.

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## Appendix

Table A.1. Support vectors for SVR-Eps-Rad model

| Index | Coefficient | Support Vector (normalized) |
| :---: | :---: | :---: |
| 1 | 88888.9 | -0.745455, -0.892128, -0.674532 |
| 2 | 88888.9 | -0.62, -0.41691, -0.158666 |
| 3 | 88888.9 | -0.527273, -0.833819, -0.563873 |
| 4 | -83099.1 | -1, -0.177843, -1 |
| 5 | -88888.9 | -0.625455, -0.810496, -0.389748 |
| 6 | -79853.3 | -0.745455, -0.77551, -0.558991 |
| 7 | 88745.5 | -0.62, 0.950437, 1 |
| 8 | 88888.9 | -0.610909, -0.909621, -0.554109 |
| 9 | -88888.9 | -0.659273, -0.944606, -0.485761 |
| 10 | -88888.9 | -0.745455, -0.723032, -0.785191 |
| 11 | 67164.1 | -0.8, -0.6793, -0.536208 |
| 12 | -88888.9 | 1, 0.48105, -0.728234 |
| 13 | -88888.9 | -0.549091, -0.795918, -0.607811 |
| 14 | 88888.9 | -0.445091, -0.609329, -0.103336 |
| 15 | 15268.4 | -0.659273, -0.880466, -0.218877 |
| 16 | -82948.9 | 0.272727, -0.653061, -0.853539 |
| 17 | -88888.9 | -0.527273, -0.201166, -1 |
| 18 | 88888.9 | -0.527273, -0.58309, -0.685924 |
| 19 | -88888.9 | 0.272727, -0.0612245, -1 |
| 20 | 88888.9 | -0.527273, -0.921283, -0.661513 |
| 21 | 88888.9 | -0.527273, -0.994169, 0.0903173 |
| 22 | -20775.4 | -0.62, -0.058309, 1 |
| 23 | -88888.9 | -0.445091, -0.623907, - <br> 0.0707893   |
| 24 | -88888.9 | -0.781818, -0.892128, -0.602929 |
| 25 | 88888.9 | -0.927273, -0.708455, -0.567128 |
| 26 | -88888.9 | -0.781818, -0.825073, -0.793328 |
| 27 | -88888.9 | -1, -0.723032, -0.609439 |
| 28 | -88888.9 | -0.781818, -0.725948, -0.79821 |
| 29 | 64448.1 | -0.563636, -0.102041, -0.18633 |
| 30 | 88888.9 | -0.527273, -0.83965, -0.552482 |
| 31 | -88888.9 | -0.781818, -0.825073, -0.593165 |
| 32 | -88888.9 | -0.625455, -0.609329, -0.389748 |
| 33 | 88888.9 | -0.195273, -0.883382, -0.562246 |
| 34 | -56839.3 | $-0.236364,-0.548105,-0.910496$ |


| 35 | 88888.9 | -0.610909, -0.650146, -0.585028 |
| :---: | :---: | :---: |
| 36 | -88888.9 | -0.527273, -0.994169, 0.0903173 |
| 37 | 88888.9 | -0.527273, -0.714286, -0.768918 |
| 38 | 88888.9 | -0.527273, -0.810496, -0.66965 |
| 39 | -88888.9 | -0.549091, -0.376093, -0.607811 |
| 40 | -88888.9 | 1, 0.48105, -0.728234 |
| 41 | 88888.9 | -1, -0.183673, -1 |
| 42 | -88888.9 | -0.563636, -0.568513, -0.121237 |
| 43 | 88888.9 | -0.2, -0.440233, -0.973963 |
| 44 | -88888.9 | -0.236364, -0.696793, -0.495525 |
| 45 | -88888.9 | -0.549091, -0.795918, -0.607811 |
| 46 | -88888.9 | -0.745455, -0.883382, -0.685924 |
| 47 | -88888.9 | -0.625455, -0.915452, -0.389748 |
| 48 | -88888.9 | -0.549091, -0.795918, -0.607811 |
| 49 | 15537.1 | -0.236364, -0.358601, -0.104963 |
| 50 | -88888.9 | -0.781818, -0.810496, -0.710334 |
| 51 | -88888.9 | -0.927273, -0.708455, -0.542718 |
| 52 | -88888.9 | -1, -0.728863, -0.853539 |
| 53 | -88888.9 | -0.549091, -0.376093, -0.607811 |
| 54 | -88888.9 | -0.527273, -0.632653, -0.853539 |
| 55 | -88888.9 | -0.625455, -0.609329, -0.389748 |
| 56 | 88888.9 | 1, 0.300292, -0.728234 |
| 57 | 88888.9 | -0.8, -0.763848, -0.542718 |
| 58 | -88888.9 | -0.527273, -0.623907, -0.853539 |
| 59 | -88888.9 | -0.549091, -0.376093, -0.607811 |
| 60 | 88888.9 | -0.236364, -0.358601, -0.462978 |
| 61 | -88888.9 | -0.236364, -0.381924, -0.332791 |
| 62 | -88888.9 | -0.563636, -0.516035, -0.21725 |
| 63 | -88888.9 | -0.0527273, -0.883382, -0.62083 |
| 64 | -88888.9 | 1, 0.48105, -0.728234 |
| 65 | 88888.9 | 1, -0.0174927, -0.728234 |
| 66 | 88888.9 | -0.236364, -0.381924, -0.576892 |
| 67 | -88888.9 | -0.781818, -0.714286, -0.710334 |
| 68 | -88888.9 | -0.549091, -0.795918, -0.607811 |
| 69 | -88888.9 | -0.527273, -0.705539, -0.775427 |
| 70 | -68863.3 | -0.818182, -0.854227, -0.809601 |
| 71 | 88888.9 | -0.610909, -0.921283, -0.529699 |
| 72 | -88888.9 | -0.781818, -0.825073, -0.793328 |
| 73 | 88888.9 | -0.8, -0.755102, -0.554109 |

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| 74 | -88888.9 | -0.8, -0.460641, -0.570382 |
| :---: | :---: | :---: |
| 75 | 11261.1 | 0.0909091, -0.638484, -0.907242 |
| 76 | 88888.9 | -0.527273, -0.723032, -0.545972 |
| 77 | -88888.9 | -0.808727, -0.629738, -0.601302 |
| 78 | -67882.2 | 1, 0.300292, -0.728234 |
| 79 | 88888.9 | -0.745455, -0.892128, -0.674532 |
| 80 | -45345.3 | -0.62, -0.428571, 1 |
| 81 | -42824 | -0.236364, 0.0408163, -0.495525 |
| 82 | -88888.9 | -0.625455, -0.915452, -0.389748 |
| 83 | 88888.9 | -0.818182, -0.708455, -0.542718 |
| 84 | -88888.9 | -0.345455, -0.793003, -0.915378 |
| 85 | -88888.9 | -0.694909, 0.638484, 0.404394 |
| 86 | 88888.9 | -0.625455, -0.03207, -0.389748 |
| 87 | -88888.9 | -0.745455, -0.801749, -0.668023 |
| 88 | 88888.9 | -0.659273, -0.912536, -0.228641 |
| 89 | -88888.9 | -1, -0.35277, -0.853539 |
| 90 | -88888.9 | -0.195273, -0.947522, -0.663141 |
| 91 | -88888.9 | -0.781818, -0.720117, -0.705452 |
| 92 | 88888.9 | -0.527273, -0.548105, -0.664768 |
| 93 | 88888.9 | -0.818182, -0.690962, -0.809601 |
| 94 | 84913.5 | 1, 0.48105, -0.728234 |
| 95 | -64583.4 | 0.272727, -0.35277, -0.853539 |
| 96 | 88888.9 | -0.302545, -1, 0.977217 |
| 97 | -88888.9 | -0.625455, -0.915452, -0.389748 |
| 98 | 88888.9 | -0.745455, -0.723032, -0.785191 |
| 99 | 26851.2 | -0.62, 0.317784, -0.158666 |
| 100 | 88888.9 | -0.625455, -0.411079, -0.389748 |
| 101 | -45264.7 | -0.527273, -0.373178, -0.853539 |
| 102 | 37881.6 | -0.62, -0.0408163, -0.158666 |
| 103 | -88888.9 | -0.709091, -0.892128, -0.783564 |
| 104 | -88888.9 | -0.781818, -0.822157, -0.697315 |
| 105 | 88888.9 | -1, -0.620991, -0.853539 |
| 106 | 17141.9 | 0.0545455, -0.889213, -0.729862 |
| 107 | -88888.9 | -1, -0.635569, -0.609439 |
| 108 | 88888.9 | -0.563636, -0.580175, -0.103336 |
| 109 | -88888.9 | -0.527273, -0.623907, -0.853539 |
| 110 | 88888.9 | -0.781818, -0.941691, -0.62083 |
| 111 | 70264.4 | -1, -0.35277, -0.853539 |
| 112 | 74879.8 | -0.527273, -0.201166, -1 |


| 113 | 88888.9 | -0.563636, -0.332362, -0.13751 |
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| 114 | 88888.9 | -0.418182, -0.830904, -0.913751 |
| 115 | -88888.9 | -0.418182, -0.833819, -0.910496 |
| 116 | 88888.9 | -0.527273, -0.927114, -0.653377 |
| 117 | 88888.9 | -0.563636, -0.883382, -0.178194 |
| 118 | -88888.9 | -0.62, -0.539359, 1 |
| 119 | 88888.9 | -0.527273, -0.539359, -0.882832 |
| 120 | 88888.9 | -0.527273, -0.74344, -0.889341 |
| 121 | -41855.7 | -0.445091, -0.61516, -0.0919447 |
| 122 | 88888.9 | -0.563636, -0.883382, -0.178194 |
| 123 | 88888.9 | -0.818182, -0.690962, -0.809601 |
| 124 | 81511.7 | 0.272727, -0.0466472, -1 |
| 125 | 88888.9 | -0.527273, -0.620991, -0.762408 |
| 126 | -88888.9 | -0.745455, -0.723032, -0.830757 |
| 127 | -87959.5 | -0.302545, -0.997085, 0.973963 |
| 128 | -63222 | -0.345455, -0.787172, -0.918633 |
| 129 | -88888.9 | -0.659273, -0.906706, -0.493897 |
| 130 | -88888.9 | -0.709091, -0.877551, -0.801465 |
| 131 | 88888.9 | -0.709091, -0.758017, -0.791701 |
| 132 | 88888.9 | 0.272727, -0.626822, -0.853539 |
| 133 | 88888.9 | -0.527273, -0.708455, -0.542718 |
| 134 | 88888.9 | -0.659273, -0.944606, -0.627339 |
| 135 | 88888.9 | -0.8, -0.83965, -0.536208 |
| 136 | -88888.9 | -0.625455, -0.915452, -0.389748 |
| 137 | -88888.9 | -0.527273, -0.816327, -0.664768 |
| 138 | 57766.8 | -0.62, -0.539359, 1 |
| 139 | -88888.9 | -0.625455, -0.810496, -0.389748 |
| 140 | 88888.9 | -0.527273, -0.696793, -0.558991 |
| 141 | 88888.9 | -0.527273, -0.755102, -0.882832 |
| 142 | 88888.9 | -0.8, -0.641399, -0.578519 |
| 143 | 88888.9 | -0.527273, -0.816327, -0.560618 |
| 144 | 88888.9 | -0.563636, -0.819242, -0.13751 |
| 145 | -88888.9 | 0.272727, -0.725948, -0.609439 |
| 146 | -88888.9 | -0.236364, -0.588921, -0.726607 |
| 147 | 88888.9 | -0.527273, -0.819242, -0.555736 |
| 148 | 88888.9 | -0.745455, -0.723032, -0.830757 |
| 149 | 88888.9 | -0.898182, -0.883382, -0.627339 |
| 150 | 88888.9 | -0.527273, -0.54519, -0.66965 |
| 151 | -88888.9 | -0.236364, -0.594752, -0.495525 |

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| 152 | 88888.9 | -0.709091, -0.900875, -0.7738 |
| :---: | :---: | :---: |
| 153 | 88888.9 | -0.818182, -0.854227, -0.809601 |
| 154 | 88888.9 | 0.272727, -0.367347, -0.853539 |
| 155 | -88888.9 | -1, -0.720117, -0.609439 |
| 156 | 88888.9 | -0.8, -0.501458, -0.539463 |
| 157 | -88888.9 | -0.709091, -0.588921, -0.788446 |
| 158 | 88888.9 | -0.610909, -0.705539, -0.521562 |
| 159 | 88888.9 | -1, -0.632653, -0.853539 |
| 160 | 88888.9 | -0.898182, -0.612245, -0.656631 |
| 161 | 88888.9 | $\begin{array}{lll} \hline-0.0527273, & -0.915452, & - \\ 0.557364 \end{array}$ |
| 162 | 88888.9 | -0.745455, -0.77551, -0.558991 |
| 163 | -27416.4 | -1, -1, -1 |
| 164 | -88888.9 | -0.625455, 1, -0.389748 |
| 165 | -5921.62 | -1, -0.0466472, -1 |
| 166 | 88888.9 | -0.62, 0.294461, 1 |
| 167 | -88888.9 | -0.781818, -0.895044, -0.599675 |
| 168 | 88888.9 | -0.236364, -0.597668, -0.889341 |
| 169 | -88888.9 | -0.781818, -0.83965, -0.570382 |
| 170 | 84743.2 | -0.694909, 0.638484, 0.404394 |
| 171 | 88888.9 | -0.709091, -0.758017, -0.791701 |
| 172 | 88888.9 | $\begin{array}{lll} \hline-0.610909, & -0.0932945, & - \\ 0.570382 & \end{array}$ |
| 173 | -88888.9 | -0.625455, -0.609329, -0.389748 |
| 174 | -88888.9 | $\begin{array}{lll} \hline-0.563636, & -0.0670554, & - \\ 0.215622 & & \end{array}$ |
| 175 | -14218 | 0.272727, -0.728863, -0.609439 |
| 176 | -88888.9 | -0.709091, -0.594752, -0.783564 |
| 177 | -80821.4 | -0.62, 0.294461, 1 |
| 178 | -88888.9 | -0.549091, -0.795918, -0.607811 |
| 179 | -88888.9 | -0.527273, -0.591837, -0.677787 |
| 180 | 62825.9 | -0.625455, -0.03207, -0.389748 |
| 181 | -88888.9 | -0.625455, -0.810496, -0.389748 |
| 182 | -88888.9 | -0.781818, -0.825073, -0.593165 |
| 183 | 88888.9 | -0.890909, -0.708455, -0.567128 |
| 184 | -19957.2 | -0.527273, -0.994169, 0.0903173 |
| 185 | -88888.9 | -0.527273, -0.845481, -0.542718 |
| 186 | -51260.4 | -1, -0.626822, -0.609439 |
| 187 | -88888.9 | -0.709091, -0.77551, -0.778682 |
| 188 | -88888.9 | -0.625455, -0.810496, -0.389748 |


| 189 | 88888.9 | -0.236364, -0.358601, -0.283971 |
| :---: | :---: | :---: |
| 190 | -122.445 | 1, 1, 1 |
| 191 | 88888.9 | -0.527273, -0.539359, -0.672905 |
| 192 | 88888.9 | -0.898182, -0.6793, -0.593165 |
| 193 | 88888.9 | -0.527273, -0.553936, -0.874695 |
| 194 | -88888.9 | -0.781818, -0.813411, -0.708706 |
| 195 | -88888.9 | -0.709091, -0.580175, -0.791701 |
| 196 | 88888.9 | 0.272727, -0.731778, -0.609439 |
| 197 | 88888.9 | -0.185455, -0.723032, -0.965826 |
| 198 | 88888.9 | -0.610909, -0.845481, -0.554109 |
| 199 | 88888.9 | 0.0545455, -0.854227, -0.7738 |
| 200 | -88888.9 | -0.563636, -0.311953, -0.163548 |
| 201 | 88888.9 | -0.818182, -0.941691, -0.809601 |
| 202 | 18318.5 | -0.527273, -0.48105, 0.0903173 |
| 203 | -88888.9 | 1, 0.48105, -0.728234 |
| 204 | 88888.9 | 1, 0.48105, -0.728234 |
| 205 | 88888.9 | -0.781818, -0.740525, -0.684296 |
| 206 | 88888.9 | -0.898182, -0.842566, -0.640358 |
| 207 | 58986.2 | -0.236364, -0.594752, -0.495525 |
| 208 | 88888.9 | -0.745455, -0.772595, -0.702197 |
| 209 | 88888.9 | -0.563636, -0.895044, -0.13751 |
| 210 | -88888.9 | -0.62, 0.950437, 1 |
| 211 | 88888.9 | -0.563636, -0.294461, -0.178194 |
| 212 | -88888.9 | -0.62, -0.527697, -0.158666 |
| 213 | -88888.9 | -0.625455, -0.03207, -0.389748 |
| 214 | -68085.4 | 1, -0.0174927, -0.728234 |
| 215 | 88888.9 | -0.236364, -0.212828, -0.495525 |
| 216 | 86645.2 | -0.781818, -0.717201, -0.804719 |
| 217 | 88888.9 | 1, 0.48105, -0.728234 |
| 218 | -88888.9 | -0.745455, -0.778426, -0.697315 |
| 219 | 88888.9 | -0.527273, -0.798834, -0.583401 |
| 220 | -88888.9 | -0.563636, -0.12828, -0.163548 |
| 221 | -5234.68 | -0.709091, -0.769679, -0.783564 |
| 222 | 88888.9 | -0.709091, -0.886297, -0.790073 |
| 223 | -23562.1 | -0.236364, -0.594752, -0.332791 |
| 224 | 88888.9 | -0.527273, -0.921283, -0.661513 |
| 225 | -88888.9 | -0.195273, -0.932945, -0.539463 |
| 226 | -88888.9 | -0.625455, 1, -0.389748 |
| 227 | 88888.9 | -0.781818, -0.938776, -0.624085 |

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| 228 | 88888.9 | -0.527273, -0.723032, -0.545972 |
| :---: | :---: | :---: |
| 229 | -88888.9 | -0.527273, -0.941691, -0.755899 |
| 230 | 88888.9 | 1, 0.48105, -0.728234 |
| 231 | 88888.9 | -0.610909, -0.862974, -0.521562 |
| 232 | -88888.9 | -0.659273, -0.906706, -0.646867 |
| 233 | -88888.9 | -0.625455, -0.03207, -0.389748 |
| 234 | 88888.9 | -0.62, -0.428571, 1 |
| 235 | -88888.9 | -0.8, -0.827988, -0.557364 |
| 236 | 88888.9 | -0.818182, -0.941691, -0.809601 |
| 237 | 72388.5 | -0.927273, -0.708455, -0.542718 |
| 238 | -88888.9 | -0.781818, -0.895044, -0.599675 |
| 239 | -88888.9 | -0.527273, -0.629738, -0.755899 |
| 240 | 88888.9 | -0.781818, -0.708455, -0.567128 |
| 241 | 88888.9 | -0.527273, -0.379009, -0.853539 |
| 242 | -88888.9 | -0.236364, -0.565598, -0.903987 |
| 243 | -88888.9 | -0.625455, -0.03207, -0.389748 |
| 244 | 88888.9 | -0.62, 0.982507, -0.158666 |
| 245 | 88888.9 | 1, -0.227405, -0.728234 |
| 246 | 88888.9 | -0.195273, -0.892128, -0.612693 |
| 247 | -88888.9 | -0.781818, -0.819242, -0.799837 |
| 248 | -88888.9 | -0.527273, -0.734694, -0.752644 |
| 249 | 10221.4 | -1, -0.728863, -0.853539 |
| 250 | -88888.9 | 0.0909091, -0.629738, -0.910496 |
| 251 | 88888.9 | -0.527273, -0.825073, -0.653377 |
| 252 | -88888.9 | -0.527273, -0.93586, -0.76729 |
| 253 | -57612.8 | -0.527273, -0.842566, 0.0903173 |


| 254 | 88888.9 | $-0.625455,1,-0.389748$ |
| :--- | :--- | :--- |
| 255 | -88888.9 | $-0.236364,-0.381924,-0.495525$ |
| 256 | 88888.9 | $-0.610909,-0.137026,-0.545972$ |
| 257 | -80999.7 | $-0.709091,-0.594752,-0.783564$ |
| 258 | -70972.9 | $-0.195273,-0.96793,-0.668023$ |
| 259 | -88888.9 | $-0.781818,-0.71137,-0.809601$ |
| 260 | -88888.9 | $-0.334545,-0.457726,-0.908869$ |
| 261 | -87673.6 | $-0.62,0.982507,-0.158666$ |
| 262 | -1584.33 | $-0.625455,1,-0.389748$ |
| 263 | -66672.8 | $-0.236364,-0.381924,-0.495525$ |
| 264 | 88888.9 | $-0.625455,1,-0.389748$ |
| 265 | -88888.9 | $-0.625455,-0.915452,-0.389748$ |
| 266 | -88888.9 | $1,-0.0174927,-0.728234$ |
| 267 | 88888.9 | $1,0.48105,-0.728234$ |
| 268 | 88888.9 | $-0.527273,-0.749271,-0.887714$ |
| 269 | 88888.9 | $-0.898182,-0.819242,-0.617575$ |
| 270 | -38777 | $1,-0.399417,-0.728234$ |
| 271 | -88888.9 | $1,0.48105,-0.728234$ |
| 272 | 88888.9 | $-0.527273,-0.737609,-0.894223$ |
| 273 | 88888.9 | $-0.527273,-0.717201,-0.532954$ |

